Some words about words

Here is some information about some math words, including a few whose use in mathematics is different from their use in ordinary language. If later in the semester you trip across any examples of misleading or obscure math-language that the book doesn't explain well, and you wish that this document had discussed them, let me know, and I'll include them in future versions of this document.

1. **Zero versus the empty set**
Zero is a *number*; the empty set is a *set* with zero elements. In this class, please write 0 as a circle without a slash, and the empty set as a circle with a slash, so that it's clear which one you mean.

2. **"Positive", "negative"**
A positive number is one that is greater than 0. A negative number is one that is less than 0. 0 is neither positive nor negative.

3. **"Iff"**
"Iff" means if-and-only-if. Example: "You are married if you are a husband" is true, but "You are married iff you are a husband" is false (roughly half of all married people are wives!).

4. **"Divides"**
When a and b are integers with b nonzero, "a divides b" means b/a is an integer. (For instance, 2 and 3 divide 6, but 4 doesn't.) Sometimes people say "a evenly divides b", but don't be misled (by the use of the word "evenly") into thinking that this has something to do with a or b or b/a being even.

Note that "a divides b" says that b divided by a (not a divided by b!) is an integer.

5. **"Let"**
In math, "Let A be a set with $|A| = n$" usually means "Let A be an arbitrary set with $|A| = n$." If the next sentence asks you to prove something about A, you can't just take A to be some particular set with n elements and prove that the claim holds in that one case.

By way of comparison, later on I might ask a question like "Let n be an odd number. Show that $n + 1$ must be even." You would NOT get credit if you wrote "Okay, I'll let $n = 3$, which is odd. Then $n + 1 = 4$, which is even." I would want to see an argument like this: "Suppose n is odd. Then n is of the form $2k + 1$. So then $n + 1$ is of the form $2k + 2$, which can be written as $2(k + 1)$, so it is even." (Here I'm assuming that at that point in the course you would know that a number is even iff it is a multiple of 2 that a number is odd iff it is 1 more than a multiple of 2.)

6. **"Distinct" versus "unique"**

People new to writing math-prose often confuse these two words.

We say a mathematical entity is *unique* when there's nothing else like it. "There exists a unique real number x such that $x^2 = 0$" means there's only one such number.

We say that two mathematical entities are *distinct* when they are unequal to each other. "There exist distinct real numbers x, y such that $x^2 = y^2$" means you can find real numbers $x \neq y$ satisfying $x^2 = y^2$. More generally, we say that n mathematical entities are distinct when no two of them are equal.

Usually we apply the word "unique" to a single thing, and "distinct" to two or more things. But there are exceptions. For instance, we say "There exist unique x, y such that ..." to mean that there's only one ordered pair (x, y) satifying a specified condition.

Here are a few (somewhat artificial) examples to highlight the difference between these two words:

"There exist distinct integers x, y such that $x^2 + y^2 = 1$" is true (just take $x = 0$ and $y = 1$, for instance; there are other solutions but just one will do to prove existence);

"There exist unique integers x, y such that $x^2 + y^2 = 1$" is false (there are four solutions $(x, y) = (0, 1)$, $(0, -1)$, $(1, 0)$, and $(-1, 0)$, though finding just two solutions suffices to prove non-uniqueness);

"There exist distinct integers x, y such that $x^2 + y^2 = 0$" is false; and

"There exist unique integers x, y such that $x^2 + y^2 = 0$" is true.

Moral: "distinct" and "unique" mean very different things.

A symbol we sometimes use to assert "There exists a unique x" is $\exists! x$; e.g., we write $(\exists! x)(\exists! y)\ x^2 + y^2 = 0$.