Math 321: Discrete Structures I

Final Exam

75 minutes

Name: ____

The problems are NOT arranged in order of increasing difficulty, but they're all worth the same amount, so it pays for you to look them all over for a minute or two (instead of just diving in and tackling the problems in the order in which they appear on the exam).

This is a closed book, closed notebook, no-calculator exam. In your solutions you may appeal to any facts that are stated in the text or were discussed in class, unless otherwise instructed. You may use a five-page double-sided cheat-sheet.

Read all questions carefully. If any questions are unclear, request clarification! You will not be given partial credit on the basis of having misunderstood a question.

You must show your work to get full credit.

If you get an answer that doesn't make sense but don't have time to trace down your error, don't just cross out your answer; explain why you think the answer you got looks wrong, and you may get some extra points for showing insight.

If you write on the back of a page, please write "continued on other side" at the bottom of the front of the page.

If you use additional sheets, be sure to write your name on each of them.

Problem 1:

Confirm or refute: If A, B, C are subsets of a universe U and $A \cap B \neq \emptyset$ and $B \subseteq C$, then $A \cap C \neq \emptyset$. (To confirm the claim, give a rigorous paragraph-style proof, not just an example or a Venn diagram; to refute the claim, a counterexample will suffice.)

Problem 2:

(a) For what values of c does the matrix $\begin{pmatrix} 1 & 2 \\ 3 & c \end{pmatrix}$ have an inverse?

(b) Give an explicit formula for the inverse of this matrix.

Problem 3:

(a) Define $a \approx b$ to mean $|a - b| \leq 1$. Is $\approx$ an equivalence relation on $\mathbb{Z}$? Which of the defining properties of an equivalence relation (if any) does it satisfy, and which properties (if any) does it not? For each property, explain your reasoning.

(b) Define $a \underline{\ll} b$ to mean $b - a$ is a nonnegative even integer. Is $\underline{\ll}$ a partial ordering on $\mathbb{Z}$? Which of the defining properties of a partial ordering (if any) does it satisfy, and which properties (if any) does it not? For each property, explain your reasoning.

Problem 4:

Define the following functions from the set of nonnegative integers to itself: $f(n) = 2|n-10|$, $g(n) = n+1$, and

$$h(n) = \begin{cases} n-1 & \text{if } n \text{ is odd,} \\ n+1 & \text{if } n \text{ is even.} \end{cases}$$

(a) Which of these three functions are surjections (onto)? Explain each of your yes/no answers.
(b) Which of these three functions are injections (one-to-one)? Explain each of your yes/no answers.

Problem 5:

Find an explicit formula for a_n if $a_0 = 2$, $a_1 = 1$, and $a_{n+2} = a_{n+1} + 6a_n$ for all $n \geq 0$.