

## “Distinct” versus “unique”

We say a mathematical entity is *unique* when there's nothing else like it. “There exists a unique real number  $x$  such that  $x^2 = 0$ ” means there's only one such number.

We say that two mathematical entities are *distinct* when they are unequal to each other. “There exist distinct real numbers  $x, y$  such that  $x^2 = y^2$ ” means you can find real numbers  $x \neq y$  satisfying  $x^2 = y^2$ . More generally, we say that  $n$  mathematical entities are distinct when no two of them are equal.

Usually we apply the word “unique” to a single thing, and “distinct” to two or more thing. But there are exceptions. For instance, we say “There exist unique  $x, y$  such that ...” to mean that there's only one ordered pair  $(x, y)$  satisfying a specified condition.

Here are a few (somewhat artificial) examples to highlight the difference between these two words:

“There exist distinct integers  $x, y$  such that  $x^2 + y^2 = 1$ ” is true (just take  $x = 0$  and  $y = 1$ , for instance; there are other solutions but just one will do to prove existence);

“There exist unique integers  $x, y$  such that  $x^2 + y^2 = 1$ ” is false (there are four solutions  $(x, y) = (0, 1), (0, -1), (1, 0),$  and  $(-1, 0)$ , though finding just two solutions suffices to prove non-uniqueness);

“There exist distinct integers  $x, y$  such that  $x^2 + y^2 = 0$ ” is false; and

“There exist unique integers  $x, y$  such that  $x^2 + y^2 = 0$ ” is true.

Moral: “distinct” and “unique” mean very different things.

A symbol we sometimes use to assert “There exists a unique  $x$ ” is  $\exists!x$ ; e.g., we write  $(\exists!x)(\exists!y) x^2 + y^2 = 0$ .