Math 322: Discrete Structures II

Final Exam

75 minutes

Name: ______________________________________

The problems are NOT arranged in order of increasing difficulty, but they're all worth the same amount, so it pays for you to look them all over for a minute or two (instead of just diving in and tackling the problems in the order in which they appear on the exam).

This is a closed book, closed notebook, no-calculator exam. In your solutions you may appeal to any facts that are stated in the text or were discussed in class, unless otherwise instructed. You may use a five-page double-sided cheat-sheet.

Read all questions carefully. If any questions are unclear, request clarification! You will not be given partial credit on the basis of having misunderstood a question.

You must show your work to get full credit.

If you get an answer that doesn't make sense but don't have time to trace down your error, don't just cross out your answer; explain why you think the answer you got looks wrong, and you may get some extra points for showing insight.

If you write on the back of a page, please write "continued on other side" at the bottom of the front of the page.

If you use additional sheets, be sure to write your name on each of them.

Problem 1:

Diagonalize the matrix $\begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix}$.

Problem 2:

Let G be a graph with the adjacency matrix

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}.$$

(a) Draw a sketch of G.
(b) Find the total number of walks of length 1 in G.
(c) Find the total number of walks of length 2 in G.
(d) Find the total number of walks of length 4 in G.

Problem 3:

(a) Is $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ a lattice under the relation "is less than or equal to"? Explain.

(b) Is the set A defined in (a) a lattice under the relation "divides"? Explain.

Problem 4:

Using the rules of Boolean algebra (as discussed in section 13.3), reduce the expression $(x \vee y) \wedge (x \vee \overline{y})$ (or, equivalently, the expression $(x + y) \cdot (x + \overline{y})$) to as simple an expression as possible. (Note: Here $\wedge$ and $\vee$ mean "meet" and "join", not "and" and "or".)

Problem 5:

Let G be the group $\mathbb{Z}_6$.
(a) Do the elements 1 and 4 form a subgroup of G? Explain.
(b) Do the elements 1 and 4 form a coset of G? Explain.