

Math 475, Problem Set #3  
(due 2/9/06)

- A. Section 3.6, problem 1. Also: How many of the four-digit numbers being considered satisfy (a) but not (b)? How many satisfy (b) but not (a)? How many satisfy neither (a) nor (b)?
- B. Section 3.6, problem 8. Note that two circular arrangements that differ by a rotation are to be regarded as the same, for purposes of counting.
- C. How many genuinely different necklaces can be made by stringing together a red bead, an orange bead, a yellow bead, a green bead, a blue bead, and a violet bead? These beads are featureless, so the necklace has no discernible “front” or “back”.
- D. A woman invites a nonempty subset of twelve friends to a party.
- (a) In how many ways can she do it, if two of the friends are married to each other and must be invited together or not at all?
  - (b) Repeat part (a) if instead the two friends are recently divorced and cannot both be invited at the same time.
- E. You are dealt a hand of poker, that is, a set of 5 cards from the standard deck of 52 (the order of the cards within a hand does not matter). Assume that all hands are equally likely. Which is larger, the probability of getting a full house or the probability of getting four-of-a-kind? What is the ratio of the probabilities? To answer this, you must compute the number of different hands that contain a full house and the number of different hands that contain four-of-a-kind, see which is larger, and compute the ratio of these numbers. (Reminder: The deck contains 4 cards in each of 13 ranks. A full house is a hand that has three cards of one rank and two cards of a different rank. A four-of-a-kind is a hand that has four cards of one rank and one card of a different rank.)
- F. Twelve new students arrive at a certain wizarding academy, and must be sorted into four different houses, with three going to each house. In how many ways can this be done? Express your answer in terms of

factorials, and cancel all factorials that occur in both the numerator and the denominator. (E.g., if you got the answer  $4!3!/3!2!$ , I would want you to write it as  $4!/2!$ .)