Math 475, Problem Set #7
(due 3/23/06)

A. Let $n$ be a positive integer $\geq 6$. How many different ways are there of rolling $n$ dice so that each of the numbers 1,2,...,6 occurs at least once? (Regard the dice as being distinguishable from one another.)

B. Four married couples are seated around a circular table. How many arrangements are there if no wife sits next to her own husband? (Arrangements that differ only by rotation are to be regarded as identical.) We do *not* require men and women to alternate.

C. Given finite sets $A_1, A_2, ..., A_n$, let $B$ be the set of all $x$ that belong to at least **two** of the $A_i$'s. (For instance, if $n = 3$ with $A_1 = \{T, H, I, S\}$, $A_2 = \{I, S\}$, and $A_3 = \{I, T\}$, then $B = \{S, I, T\}$.) By experimenting with small values of $n$ (or by peeking ahead to part D), find a plausible general formula for the size of $B$ in terms of the sizes $|A_i|$, $|A_i \cap A_j|$, $|A_i \cap A_j \cap A_k|$, etc. To get you started: when $n = 3$,

$$|B| = |A_1 \cap A_2| + |A_1 \cap A_3| + |A_2 \cap A_3| - 2|A_1 \cap A_2 \cap A_3|.$$

D. Prove that for all integers $k \geq 2$,

$$\binom{k}{2} - 2\binom{k}{3} + 3\binom{k}{4} - ... + (-1)^k (k-1)\binom{k}{k} = 1.$$

E. Prove your formula from part C. (Hint: You will probably find the result of part D useful.)

F. Use the formula from part C to obtain a solution to problem 15c in Chapter 6 of Brualdi.