1. (a) Does there exist a polynomial \( p(t) \) of degree 3 such that the linear operator \( p(T) \) annihilates the sequence whose \( n \)th term (for \( n \geq 0 \)) is \( 3^n + 2^n + 1^n \)? Exhibit such a polynomial or explain why none exists.

(b) Same as (a), but with “degree 3” replaced by “degree 4”.

(c) Same as (a), but with “degree 3” replaced by “degree 2”.

2. Let \( F_n \) be the \( n \)th Fibonacci number, as Wilf indexes them (with \( F_0 = F_1 = 1, \ F_2 = 2, \) etc.). Give a simple homogeneous linear recurrence relation satisfied by the sequence whose \( n \)th term is

(a) \( nF_n \);

(b) \( 1F_1 + 2F_2 + ... + nF_n \);

(c) \( nF_1 + (n - 1)F_2 + ... + 2F_{n-1} + F_n \);

(d) \( F_n \) when \( n \) is odd, and \( 2^n \) when \( n \) is even.

In each case, an explanation should be included.

Please be sure to write down how many hours you spent working on the problems, and whom you worked with.