Math 584, Problem Set #2
(due in class Mon., 10/4/10)

You may use computers for any and all of these problems, but you should include explicit description of the steps you took and the code you entered.

If you collaborate with anyone else, you must acknowledge who you worked with, and you must do your write-ups separately.

Also: Please write down the number of minutes you spent working on each problem. This will help me keep the work-load at an appopriate level.

A. Find a scheme for using Uniform(0,1) random variables to simulate geometric random variables with parameter p. Apply your scheme to get 10^6 independent pseudorandom samples from the geometric distribution with parameter $p = \pi/10$, and compare your results with the expected number of 1's, 2's, 3's, etc.

B. Grinstead and Snell, problem 11.1.14. You may use experiments to guide your guesses, but you must confirm your guesses rigorously. (That is, you should prove that the nth power of the matrix $\mathbf{P}$ really does converge to the particular matrix $\mathbf{W}$ that it appears to converge to, by appealing to the relevant theorems in section 11.3.)

C. Write a program to pseudorandomly simulate the Markov chain with transition matrix

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ 0 & 1 \end{pmatrix}.$$

Simulate the chain 1000 times to estimate the expected time it takes for the chain to get from state 1 to state 2. Assess your estimate using the standard formulas for the mean and variance of a geometric random variable.

D. Grinstead and Snell, problem 11.2.9.

E. A variant of Grinstead and Snell's problem 11.2.11. Prove the formula given in part (d) of the problem directly, using mathematical induction and the formula for b_k given in part (a).

F. Grinstead and Snell, problem 11.2.13, parts (a), (b), and (c). For parts (a) and (b), express the probability as an exact fraction.

G. Grinstead and Snell, problem 11.2.23.

H. Grinstead and Snell, problem 11.2.24.

I. Grinstead and Snell, problem 11.2.26. Note that we proved in class that the expected time until absorption is finite, so $f(x)$ is finite for all x.

J. (Bollobas) In a game involving 3 gamblers, the ith gambler starts the game with a_i dollars. In each round, two gamblers selected at random make a fair bet, and the winner gets a dollar from the loser. A gambler losing all his money leaves the game. The game continues as long as possible, i.e., until one of the gamblers has all the money. Prove that the expected number of rounds is $\sum_{i<j} a_i a_j$, and that the probability that the ith gambler ends up with all the money is $a_i / \sum_{j=1}^{n} a_j$.