Math 584, Problem Set #3

(due in class Mon., 11/8/10)

You may use computers for any and all of these problems, but you should include explicit description of the steps you took and the code you entered.

Also: Please write down the number of minutes you spent working on each problem. This will help me keep the work-load at an appropriate level.

A. Grinstead and Snell, problem 11.1.14. You may use experiments to guide your guesses, but you must confirm your guesses rigorously. (That is, you should prove that the nth power of the matrix $\mathbf{P}$ really does converge to the particular matrix $\mathbf{W}$ that it appears to converge to, by appealing to the relevant theorems in section 11.3.) Note that this was formerly problem B on the second assignment.

B. Grinstead and Snell, problem 11.3.5.

C. Grinstead and Snell, problem 11.5.1.

D. Suppose a Markov chain with states $1, 2, ..., n$ is reversible. Must the stationary frequency $w_i p_{ij} p_{jk}$ with which the triple (i, j, k) occurs be equal to the stationary frequency $w_k p_{kj} p_{ji}$ with which the triple (k, j, i) occurs?

E. Grinstead and Snell, problem 11.5.14.

F. Grinstead and Snell, problem 11.5.23.

G. Consider the random walk on $\{1, 2, ..., n\}$ with semi-absorbent barriers at 1 and n: $p_{ij} = p$ if $j = \min(i+1, n)$ and $p_{ij} = 1-p = q$ if $j = \max(i-1, 1)$, with $p_{ij} = 0$ otherwise. Show that the mass distribution that puts mass $(p/q)^k$ at state k is invariant under mass-flow.