02 INFORMATION ABOUT PRINCIPAL INVESTIGATORS/PROJECT DIRECTORS(PI/PD) and co-PRINCIPAL INVESTIGATORS/co-PROJECT DIRECTORS

Submit only ONE copy of this form **for each PI/PD and co-PI/PD** identified on the proposal. The form(s) should be attached to the original proposal as specified in GPG Section II.C.a. Submission of this information is voluntary and is not a precondition of award. This information will not be disclosed to external peer reviewers. ***DO NOT INCLUDE THIS FORM WITH ANY OF THE OTHER COPIES OF YOUR PROPOSAL AS THIS MAY COMPROMISE THE CONFIDENTIALITY OF THE INFORMATION.***

PI/PD Name: James G Propp

Gender: ☒ Male ☐ Female

Ethnicity: (Choose one response) ☐ Hispanic or Latino ☒ Not Hispanic or Latino

Race:
(Select one or more)
☐ American Indian or Alaska Native
☐ Asian
☐ Black or African American
☐ Native Hawaiian or Other Pacific Islander
☒ White

Disability Status:
(Select one or more)
☐ Hearing Impairment
☐ Visual Impairment
☐ Mobility/Orthopedic Impairment
☐ Other
☒ None

Citizenship: (Choose one) ☒ U.S. Citizen ☐ Permanent Resident ☐ Other non-U.S. Citizen

Check here if you do not wish to provide any or all of the above information (excluding PI/PD name): ☐

REQUIRED: Check here if you are currently serving (or have previously served) as a PI, co-PI or PD on any federally funded project ☒

Ethnicity Definition:
Hispanic or Latino. A person of Mexican, Puerto Rican, Cuban, South or Central American, or other Spanish culture or origin, regardless of race.
Race Definitions:
American Indian or Alaska Native. A person having origins in any of the original peoples of North and South America (including Central America), and who maintains tribal affiliation or community attachment.
Asian. A person having origins in any of the original peoples of the Far East, Southeast Asia, or the Indian subcontinent including, for example, Cambodia, China, India, Japan, Korea, Malaysia, Pakistan, the Philippine Islands, Thailand, and Vietnam.
Black or African American. A person having origins in any of the black racial groups of Africa.
Native Hawaiian or Other Pacific Islander. A person having origins in any of the original peoples of Hawaii, Guam, Samoa, or other Pacific Islands.
White. A person having origins in any of the original peoples of Europe, the Middle East, or North Africa.

WHY THIS INFORMATION IS BEING REQUESTED:

The Federal Government has a continuing commitment to monitor the operation of its review and award processes to identify and address any inequities based on gender, race, ethnicity, or disability of its proposed PIs/PDs. To gather information needed for this important task, the proposer should submit a single copy of this form for each identified PI/PD with each proposal. Submission of the requested information is voluntary and will not affect the organization's eligibility for an award. However, information not submitted will seriously undermine the statistical validity, and therefore the usefulness, of information recieved from others. Any individual not wishing to submit some or all the information should check the box provided for this purpose. (The exceptions are the PI/PD name and the information about prior Federal support, the last question above.)

Collection of this information is authorized by the NSF Act of 1950, as amended, 42 U.S.C. 1861, et seq. Demographic data allows NSF to gauge whether our programs and other opportunities in science and technology are fairly reaching and benefiting everyone regardless of demographic category; to ensure that those in under-represented groups have the same knowledge of and access to programs and other research and educational oppurtunities; and to assess involvement of international investigators in work supported by NSF. The information may be disclosed to government contractors, experts, volunteers and researchers to complete assigned work; and to other government agencies in order to coordinate and assess programs. The information may be added to the Reviewer file and used to select potential candidates to serve as peer reviewers or advisory committee members. See Systems of Records, NSF-50, "Principal Investigator/Proposal File and Associated Records", 63 Federal Register 267 (January 5, 1998), and NSF-51, "Reviewer/Proposal File and Associated Records", 63 Federal Register 268 (January 5, 1998).

02 INFORMATION ABOUT PRINCIPAL INVESTIGATORS/PROJECT DIRECTORS(PI/PD) and co-PRINCIPAL INVESTIGATORS/co-PROJECT DIRECTORS

Submit only ONE copy of this form **for each PI/PD and co-PI/PD** identified on the proposal. The form(s) should be attached to the original proposal as specified in GPG Section II.C.a. Submission of this information is voluntary and is not a precondition of award. This information will not be disclosed to external peer reviewers. ***DO NOT INCLUDE THIS FORM WITH ANY OF THE OTHER COPIES OF YOUR PROPOSAL AS THIS MAY COMPROMISE THE CONFIDENTIALITY OF THE INFORMATION.***

PI/PD Name: Thomas Roby

Gender: ☒ Male ☐ Female

Ethnicity: (Choose one response) ☐ Hispanic or Latino ☒ Not Hispanic or Latino

Race: (Select one or more)
- ☐ American Indian or Alaska Native
- ☐ Asian
- ☐ Black or African American
- ☐ Native Hawaiian or Other Pacific Islander
- ☒ White

Disability Status: (Select one or more)
- ☐ Hearing Impairment
- ☐ Visual Impairment
- ☐ Mobility/Orthopedic Impairment
- ☐ Other
- ☒ None

Citizenship: (Choose one) ☒ U.S. Citizen ☐ Permanent Resident ☐ Other non-U.S. Citizen

Check here if you do not wish to provide any or all of the above information (excluding PI/PD name): ☐

REQUIRED: Check here if you are currently serving (or have previously served) as a PI, co-PI or PD on any federally funded project ☒

Ethnicity Definition:
Hispanic or Latino. A person of Mexican, Puerto Rican, Cuban, South or Central American, or other Spanish culture or origin, regardless of race.
Race Definitions:
American Indian or Alaska Native. A person having origins in any of the original peoples of North and South America (including Central America), and who maintains tribal affiliation or community attachment.
Asian. A person having origins in any of the original peoples of the Far East, Southeast Asia, or the Indian subcontinent including, for example, Cambodia, China, India, Japan, Korea, Malaysia, Pakistan, the Philippine Islands, Thailand, and Vietnam.
Black or African American. A person having origins in any of the black racial groups of Africa.
Native Hawaiian or Other Pacific Islander. A person having origins in any of the original peoples of Hawaii, Guam, Samoa, or other Pacific Islands.
White. A person having origins in any of the original peoples of Europe, the Middle East, or North Africa.

WHY THIS INFORMATION IS BEING REQUESTED:

The Federal Government has a continuing commitment to monitor the operation of its review and award processes to identify and address any inequities based on gender, race, ethnicity, or disability of its proposed PIs/PDs. To gather information needed for this important task, the proposer should submit a single copy of this form for each identified PI/PD with each proposal. Submission of the requested information is voluntary and will not affect the organization's eligibility for an award. However, information not submitted will seriously undermine the statistical validity, and therefore the usefulness, of information recieved from others. Any individual not wishing to submit some or all the information should check the box provided for this purpose. (The exceptions are the PI/PD name and the information about prior Federal support, the last question above.)

Collection of this information is authorized by the NSF Act of 1950, as amended, 42 U.S.C. 1861, et seq. Demographic data allows NSF to gauge whether our programs and other opportunities in science and technology are fairly reaching and benefiting everyone regardless of demographic category; to ensure that those in under-represented groups have the same knowledge of and access to programs and other research and educational oppurtunities; and to assess involvement of international investigators in work supported by NSF. The information may be disclosed to government contractors, experts, volunteers and researchers to complete assigned work; and to other government agencies in order to coordinate and assess programs. The information may be added to the Reviewer file and used to select potential candidates to serve as peer reviewers or advisory committee members. See Systems of Records, NSF-50, "Principal Investigator/Proposal File and Associated Records", 63 Federal Register 267 (January 5, 1998), and NSF-51, "Reviewer/Proposal File and Associated Records", 63 Federal Register 268 (January 5, 1998).

List of Suggested Reviewers or Reviewers Not To Include (optional)

SUGGESTED REVIEWERS:
Peter Mann Winkler, Richard Peter Stanley

REVIEWERS NOT TO INCLUDE:
Not Listed

List of Suggested Reviewers or Reviewers Not To Include (optional)

SUGGESTED REVIEWERS:
Not Listed

REVIEWERS NOT TO INCLUDE:
Not Listed

Not for distribution

Corrected : 10/16/2015

COVER SHEET FOR PROPOSAL TO THE NATIONAL SCIENCE FOUNDATION

PROGRAM ANNOUNCEMENT/SOLICITATION NO./CLOSING DATE/if not in response to a program announcement/solicitation enter NSF 15-1 PD 10-7970 10/06/15	FOR NSF USE ONLY NSF PROPOSAL NUMBER
FOR CONSIDERATION BY NSF ORGANIZATION UNIT(S) (Indicate the most specific unit known, i.e. program, division, etc.) DMS - COMBINATORICS	1600651

DATE RECEIVED	NUMBER OF COPIES	DIVISION ASSIGNED	FUND CODE	DUNS# (Data Universal Numbering System)	FILE LOCATION
10/06/2015	2	03040000 DMS	7970	956072490	11/01/2015 7:35am S

EMPLOYER IDENTIFICATION NUMBER (EIN) OR TAXPAYER IDENTIFICATION NUMBER (TIN)	SHOW PREVIOUS AWARD NO. IF THIS IS	IS THIS PROPOSAL BEING SUBMITTED TO ANOTHER FEDERAL AGENCY?
043167352	☐ A RENEWAL ☐ AN ACCOMPLISHMENT-BASED RENEWAL	YES ☐ NO ☒ IF YES, LIST ACRONYM(S)

NAME OF ORGANIZATION TO WHICH AWARD SHOULD BE MADE University of Massachusetts Lowell	ADDRESS OF AWARDEE ORGANIZATION, INCLUDING 9 DIGIT ZIP CODE University of Massachusetts Lowell 600 Suffolk Street Lowell, MA. 018543643
AWARDEE ORGANIZATION CODE (IF KNOWN) 0001966000	
NAME OF PRIMARY PLACE OF PERF University of Massachusetts Lowell	ADDRESS OF PRIMARY PLACE OF PERF, INCLUDING 9 DIGIT ZIP CODE University of Massachusetts Lowell One University Ave Lowell ,MA ,018542827 ,US.

IS AWARDEE ORGANIZATION (Check All That Apply) (See GPG II.C For Definitions)
☐ SMALL BUSINESS ☐ MINORITY BUSINESS
☐ FOR-PROFIT ORGANIZATION ☐ WOMAN-OWNED BUSINESS
☐ IF THIS IS A PRELIMINARY PROPOSAL THEN CHECK HERE

TITLE OF PROPOSED PROJECT **Collaborative Research: Dynamical Algebraic Combinatorics**

REQUESTED AMOUNT	PROPOSED DURATION (1-60 MONTHS)	REQUESTED STARTING DATE	SHOW RELATED PRELIMINARY PROPOSAL NO. IF APPLICABLE
$ 300,884	36 months	07/01/16	

THIS PROPOSAL INCLUDES ANY OF THE ITEMS LISTED BELOW

☐ BEGINNING INVESTIGATOR (GPG I.G.2)
☐ DISCLOSURE OF LOBBYING ACTIVITIES (GPG II.C.1.e)
☐ PROPRIETARY & PRIVILEGED INFORMATION (GPG I.D, II.C.1.d)
☐ HISTORIC PLACES (GPG II.C.2.j)
☐ VERTEBRATE ANIMALS (GPG II.D.6) IACUC App. Date ____
PHS Animal Welfare Assurance Number ____
☒ FUNDING MECHANISM **Research - other than RAPID or EAGER**

☐ HUMAN SUBJECTS (GPG II.D.7) Human Subjects Assurance Number ____
Exemption Subsection ____ or IRB App. Date ____
☒ INTERNATIONAL ACTIVITIES: COUNTRY/COUNTRIES INVOLVED (GPG II.C.2.j)
CA EI FR
☒ COLLABORATIVE STATUS
A collaborative proposal from multiple organizations (GPG II.D.4.b)

PI/PD DEPARTMENT Mathematics	PI/PD POSTAL ADDRESS 1 University Avenue Lowell, MA 01854 United States
PI/PD FAX NUMBER 978-934-3053	

NAMES (TYPED)	High Degree	Yr of Degree	Telephone Number	Email Address
PI/PD NAME James G Propp	PhD	1987	617-999-6790	JamesPropp@gmail.com
CO-PI/PD				
CO-PI/PD				
CO-PI/PD				
CO-PI/PD				

CERTIFICATION PAGE

Certification for Authorized Organizational Representative (or Equivalent) or Individual Applicant

By electronically signing and submitting this proposal, the Authorized Organizational Representative (AOR) or Individual Applicant is: (1) certifying that statements made herein are true and complete to the best of his/her knowledge; and (2) agreeing to accept the obligation to comply with NSF award terms and conditions if an award is made as a result of this application. Further, the applicant is hereby providing certifications regarding conflict of interest (when applicable), drug-free workplace, debarment and suspension, lobbying activities (see below), nondiscrimination, flood hazard insurance (when applicable), responsible conduct of research, organizational support, Federal tax obligations, unpaid Federal tax liability, and criminal convictions as set forth in the NSF Proposal & Award Policies & Procedures Guide,Part I: the Grant Proposal Guide (GPG). Willful provision of false information in this application and its supporting documents or in reports required under an ensuing award is a criminal offense (U.S. Code, Title 18, Section 1001).

Certification Regarding Conflict of Interest

The AOR is required to complete certifications stating that the organization has implemented and is enforcing a written policy on conflicts of interest (COI), consistent with the provisions of AAG Chapter IV.A.; that, to the best of his/her knowledge, all financial disclosures required by the conflict of interest policy were made; and that conflicts of interest, if any, were, or prior to the organization's expenditure of any funds under the award, will be, satisfactorily managed, reduced or eliminated in accordance with the organization's conflict of interest policy. Conflicts that cannot be satisfactorily managed, reduced or eliminated and research that proceeds without the imposition of conditions or restrictions when a conflict of interest exists, must be disclosed to NSF via use of the Notifications and Requests Module in FastLane.

Drug Free Work Place Certification

By electronically signing the Certification Pages, the Authorized Organizational Representative (or equivalent), is providing the Drug Free Work Place Certification contained in Exhibit II-3 of the Grant Proposal Guide.

Debarment and Suspension Certification

(If answer "yes", please provide explanation.)

Is the organization or its principals presently debarred, suspended, proposed for debarment, declared ineligible, or voluntarily excluded from covered transactions by any Federal department or agency? Yes ☐ No ☒

By electronically signing the Certification Pages, the Authorized Organizational Representative (or equivalent) or Individual Applicant is providing the Debarment and Suspension Certification contained in Exhibit II-4 of the Grant Proposal Guide.

Certification Regarding Lobbying

This certification is required for an award of a Federal contract, grant, or cooperative agreement exceeding $100,000 and for an award of a Federal loan or a commitment providing for the United States to insure or guarantee a loan exceeding $150,000.

Certification for Contracts, Grants, Loans and Cooperative Agreements

The undersigned certifies, to the best of his or her knowledge and belief, that:

(1) No Federal appropriated funds have been paid or will be paid, by or on behalf of the undersigned, to any person for influencing or attempting to influence an officer or employee of any agency, a Member of Congress, an officer or employee of Congress, or an employee of a Member of Congress in connection with the awarding of any Federal contract, the making of any Federal grant, the making of any Federal loan, the entering into of any cooperative agreement, and the extension, continuation, renewal, amendment, or modification of any Federal contract, grant, loan, or cooperative agreement.

(2) If any funds other than Federal appropriated funds have been paid or will be paid to any person for influencing or attempting to influence an officer or employee of any agency, a Member of Congress, an officer or employee of Congress, or an employee of a Member of Congress in connection with this Federal contract, grant, loan, or cooperative agreement, the undersigned shall complete and submit Standard Form-LLL, ''Disclosure of Lobbying Activities,'' in accordance with its instructions.

(3) The undersigned shall require that the language of this certification be included in the award documents for all subawards at all tiers including subcontracts, subgrants, and contracts under grants, loans, and cooperative agreements and that all subrecipients shall certify and disclose accordingly.

This certification is a material representation of fact upon which reliance was placed when this transaction was made or entered into. Submission of this certification is a prerequisite for making or entering into this transaction imposed by section 1352, Title 31, U.S. Code. Any person who fails to file the required certification shall be subject to a civil penalty of not less than $10,000 and not more than $100,000 for each such failure.

Certification Regarding Nondiscrimination

By electronically signing the Certification Pages, the Authorized Organizational Representative (or equivalent) is providing the Certification Regarding Nondiscrimination contained in Exhibit II-6 of the Grant Proposal Guide.

Certification Regarding Flood Hazard Insurance

Two sections of the National Flood Insurance Act of 1968 (42 USC §4012a and §4106) bar Federal agencies from giving financial assistance for acquisition or construction purposes in any area identified by the Federal Emergency Management Agency (FEMA) as having special flood hazards unless the:

(1) community in which that area is located participates in the national flood insurance program; and
(2) building (and any related equipment) is covered by adequate flood insurance.

By electronically signing the Certification Pages, the Authorized Organizational Representative (or equivalent) or Individual Applicant located in FEMA-designated special flood hazard areas is certifying that adequate flood insurance has been or will be obtained in the following situations:

(1) for NSF grants for the construction of a building or facility, regardless of the dollar amount of the grant; and
(2) for other NSF grants when more than $25,000 has been budgeted in the proposal for repair, alteration or improvement (construction) of a building or facility.

Certification Regarding Responsible Conduct of Research (RCR)
(This certification is not applicable to proposals for conferences, symposia, and workshops.)

By electronically signing the Certification Pages, the Authorized Organizational Representative is certifying that, in accordance with the NSF Proposal & Award Policies & Procedures Guide, Part II, Award & Administration Guide (AAG) Chapter IV.B., the institution has a plan in place to provide appropriate training and oversight in the responsible and ethical conduct of research to undergraduates, graduate students and postdoctoral researchers who will be supported by NSF to conduct research.
The AOR shall require that the language of this certification be included in any award documents for all subawards at all tiers.

CERTIFICATION PAGE - CONTINUED

Certification Regarding Organizational Support

By electronically signing the Certification Pages, the Authorized Organizational Representative (or equivalent) is certifying that there is organizational support for the proposal as required by Section 526 of the America COMPETES Reauthorization Act of 2010. This support extends to the portion of the proposal developed to satisfy the Broader Impacts Review Criterion as well as the Intellectual Merit Review Criterion, and any additional review criteria specified in the solicitation. Organizational support will be made available, as described in the proposal, in order to address the broader impacts and intellectual merit activities to be undertaken.

Certification Regarding Federal Tax Obligations

When the proposal exceeds $5,000,000, the Authorized Organizational Representative (or equivalent) is required to complete the following certification regarding Federal tax obligations.
By electronically signing the Certification pages, the Authorized Organizational Representative is certifying that, to the best of their knowledge and belief, the proposing organization:
(1) has filed all Federal tax returns required during the three years preceding this certification;
(2) has not been convicted of a criminal offense under the Internal Revenue Code of 1986; and
(3) has not, more than 90 days prior to this certification, been notified of any unpaid Federal tax assessment for which the liability remains unsatisfied, unless the assessment is the subject of an installment agreement or offer in compromise that has been approved by the Internal Revenue Service and is not in default, or the assessment is the subject of a non-frivolous administrative or judicial proceeding.

Certification Regarding Unpaid Federal Tax Liability

When the proposing organization is a corporation, the Authorized Organizational Representative (or equivalent) is required to complete the following certification regarding Federal Tax Liability:

By electronically signing the Certification Pages, the Authorized Organizational Representative (or equivalent) is certifying that the corporation has no unpaid Federal tax liability that has been assessed, for which all judicial and administrative remedies have been exhausted or lapsed, and that is not being paid in a timely manner pursuant to an agreement with the authority responsible for collecting the tax liability.

Certification Regarding Criminal Convictions

When the proposing organization is a corporation, the Authorized Organizational Representative (or equivalent) is required to complete the following certification regarding Criminal Convictions:

By electronically signing the Certification Pages, the Authorized Organizational Representative (or equivalent) is certifying that the corporation has not been convicted of a felony criminal violation under any Federal law within the 24 months preceding the date on which the certification is signed.

AUTHORIZED ORGANIZATIONAL REPRESENTATIVE NAME	SIGNATURE	DATE
Linda Concino	Electronic Signature	Oct 6 2015 10:10AM

TELEPHONE NUMBER	EMAIL ADDRESS	FAX NUMBER
978-934-4723	Linda_Concino@uml.edu	978-934-2027

Not for distribution

COVER SHEET FOR PROPOSAL TO THE NATIONAL SCIENCE FOUNDATION

PROGRAM ANNOUNCEMENT/SOLICITATION NO./CLOSING DATE/if not in response to a program announcement/solicitation enter NSF 15-1 **PD 10-7970** **10/06/15**	FOR NSF USE ONLY NSF PROPOSAL NUMBER **1600661**
FOR CONSIDERATION BY NSF ORGANIZATION UNIT(S) (Indicate the most specific unit known, i.e. program, division, etc.) **DMS - COMBINATORICS**	

DATE RECEIVED	NUMBER OF COPIES	DIVISION ASSIGNED	FUND CODE	DUNS# (Data Universal Numbering System)	FILE LOCATION
10/06/2015	2	03040000 DMS	7970	614209054	11/01/2015 7:35am S

EMPLOYER IDENTIFICATION NUMBER (EIN) OR TAXPAYER IDENTIFICATION NUMBER (TIN)	SHOW PREVIOUS AWARD NO. IF THIS IS	IS THIS PROPOSAL BEING SUBMITTED TO ANOTHER FEDERAL AGENCY?
060772160	☐ A RENEWAL ☐ AN ACCOMPLISHMENT-BASED RENEWAL	YES ☐ NO ☒ IF YES, LIST ACRONYM(S)

NAME OF ORGANIZATION TO WHICH AWARD SHOULD BE MADE **University of Connecticut**	ADDRESS OF AWARDEE ORGANIZATION, INCLUDING 9 DIGIT ZIP CODE **University of Connecticut** **438 Whitney Road Ext.** **Storrs, CT. 062691133**
AWARDEE ORGANIZATION CODE (IF KNOWN) **0014175000**	
NAME OF PRIMARY PLACE OF PERF **University of Connecticut**	ADDRESS OF PRIMARY PLACE OF PERF, INCLUDING 9 DIGIT ZIP CODE **University of Connecticut** **CT ,062691133 ,US.**

IS AWARDEE ORGANIZATION (Check All That Apply) (See GPG II.C For Definitions): ☐ SMALL BUSINESS ☐ FOR-PROFIT ORGANIZATION ☐ MINORITY BUSINESS ☐ WOMAN-OWNED BUSINESS ☐ IF THIS IS A PRELIMINARY PROPOSAL THEN CHECK HERE

TITLE OF PROPOSED PROJECT **Collaborative Research: Dynamical Algebraic Combinatorics**

REQUESTED AMOUNT	PROPOSED DURATION (1-60 MONTHS)	REQUESTED STARTING DATE	SHOW RELATED PRELIMINARY PROPOSAL NO. IF APPLICABLE
$ 410,904	36 months	07/01/16	

THIS PROPOSAL INCLUDES ANY OF THE ITEMS LISTED BELOW

- ☐ BEGINNING INVESTIGATOR (GPG I.G.2)
- ☐ DISCLOSURE OF LOBBYING ACTIVITIES (GPG II.C.1.e)
- ☐ PROPRIETARY & PRIVILEGED INFORMATION (GPG I.D, II.C.1.d)
- ☐ HISTORIC PLACES (GPG II.C.2.j)
- ☐ VERTEBRATE ANIMALS (GPG II.D.6) IACUC App. Date ______ PHS Animal Welfare Assurance Number ______
- ☒ FUNDING MECHANISM **Research - other than RAPID or EAGER**
- ☐ HUMAN SUBJECTS (GPG II.D.7) Human Subjects Assurance Number ______ Exemption Subsection ______ or IRB App. Date ______
- ☒ INTERNATIONAL ACTIVITIES: COUNTRY/COUNTRIES INVOLVED (GPG II.C.2.j) **AU JA**
- ☒ COLLABORATIVE STATUS **A collaborative proposal from multiple organizations (GPG II.D.4.b)**

PI/PD DEPARTMENT **Mathematics**	PI/PD POSTAL ADDRESS **438 Whitney Road Ext.** **Storrs, CT 062691133** **United States**
PI/PD FAX NUMBER **860-486-6504**	

NAMES (TYPED)	High Degree	Yr of Degree	Telephone Number	Email Address
PI/PD NAME **Thomas Roby**	DPhil	1991	860-486-3622	tom.roby@uconn.edu
CO-PI/PD				
CO-PI/PD				
CO-PI/PD				
CO-PI/PD				

CERTIFICATION PAGE

Certification for Authorized Organizational Representative (or Equivalent) or Individual Applicant

By electronically signing and submitting this proposal, the Authorized Organizational Representative (AOR) or Individual Applicant is: (1) certifying that statements made herein are true and complete to the best of his/her knowledge; and (2) agreeing to accept the obligation to comply with NSF award terms and conditions if an award is made as a result of this application. Further, the applicant is hereby providing certifications regarding conflict of interest (when applicable), drug-free workplace, debarment and suspension, lobbying activities (see below), nondiscrimination, flood hazard insurance (when applicable), responsible conduct of research, organizational support, Federal tax obligations, unpaid Federal tax liability, and criminal convictions as set forth in the NSF Proposal & Award Policies & Procedures Guide,Part I: the Grant Proposal Guide (GPG). Willful provision of false information in this application and its supporting documents or in reports required under an ensuing award is a criminal offense (U.S. Code, Title 18, Section 1001).

Certification Regarding Conflict of Interest

The AOR is required to complete certifications stating that the organization has implemented and is enforcing a written policy on conflicts of interest (COI), consistent with the provisions of AAG Chapter IV.A.; that, to the best of his/her knowledge, all financial disclosures required by the conflict of interest policy were made; and that conflicts of interest, if any, were, or prior to the organization's expenditure of any funds under the award, will be, satisfactorily managed, reduced or eliminated in accordance with the organization's conflict of interest policy. Conflicts that cannot be satisfactorily managed, reduced or eliminated and research that proceeds without the imposition of conditions or restrictions when a conflict of interest exists, must be disclosed to NSF via use of the Notifications and Requests Module in FastLane.

Drug Free Work Place Certification

By electronically signing the Certification Pages, the Authorized Organizational Representative (or equivalent), is providing the Drug Free Work Place Certification contained in Exhibit II-3 of the Grant Proposal Guide.

Debarment and Suspension Certification

(If answer "yes", please provide explanation.)

Is the organization or its principals presently debarred, suspended, proposed for debarment, declared ineligible, or voluntarily excluded from covered transactions by any Federal department or agency? Yes ☐ No ☒

By electronically signing the Certification Pages, the Authorized Organizational Representative (or equivalent) or Individual Applicant is providing the Debarment and Suspension Certification contained in Exhibit II-4 of the Grant Proposal Guide.

Certification Regarding Lobbying

This certification is required for an award of a Federal contract, grant, or cooperative agreement exceeding $100,000 and for an award of a Federal loan or a commitment providing for the United States to insure or guarantee a loan exceeding $150,000.

Certification for Contracts, Grants, Loans and Cooperative Agreements

The undersigned certifies, to the best of his or her knowledge and belief, that:
(1) No Federal appropriated funds have been paid or will be paid, by or on behalf of the undersigned, to any person for influencing or attempting to influence an officer or employee of any agency, a Member of Congress, an officer or employee of Congress, or an employee of a Member of Congress in connection with the awarding of any Federal contract, the making of any Federal grant, the making of any Federal loan, the entering into of any cooperative agreement, and the extension, continuation, renewal, amendment, or modification of any Federal contract, grant, loan, or cooperative agreement.
(2) If any funds other than Federal appropriated funds have been paid or will be paid to any person for influencing or attempting to influence an officer or employee of any agency, a Member of Congress, an officer or employee of Congress, or an employee of a Member of Congress in connection with this Federal contract, grant, loan, or cooperative agreement, the undersigned shall complete and submit Standard Form-LLL, ''Disclosure of Lobbying Activities,'' in accordance with its instructions.
(3) The undersigned shall require that the language of this certification be included in the award documents for all subawards at all tiers including subcontracts, subgrants, and contracts under grants, loans, and cooperative agreements and that all subrecipients shall certify and disclose accordingly.

This certification is a material representation of fact upon which reliance was placed when this transaction was made or entered into. Submission of this certification is a prerequisite for making or entering into this transaction imposed by section 1352, Title 31, U.S. Code. Any person who fails to file the required certification shall be subject to a civil penalty of not less than $10,000 and not more than $100,000 for each such failure.

Certification Regarding Nondiscrimination

By electronically signing the Certification Pages, the Authorized Organizational Representative (or equivalent) is providing the Certification Regarding Nondiscrimination contained in Exhibit II-6 of the Grant Proposal Guide.

Certification Regarding Flood Hazard Insurance

Two sections of the National Flood Insurance Act of 1968 (42 USC §4012a and §4106) bar Federal agencies from giving financial assistance for acquisition or construction purposes in any area identified by the Federal Emergency Management Agency (FEMA) as having special flood hazards unless the:
(1) community in which that area is located participates in the national flood insurance program; and
(2) building (and any related equipment) is covered by adequate flood insurance.

By electronically signing the Certification Pages, the Authorized Organizational Representative (or equivalent) or Individual Applicant located in FEMA-designated special flood hazard areas is certifying that adequate flood insurance has been or will be obtained in the following situations:
(1) for NSF grants for the construction of a building or facility, regardless of the dollar amount of the grant; and
(2) for other NSF grants when more than $25,000 has been budgeted in the proposal for repair, alteration or improvement (construction) of a building or facility.

Certification Regarding Responsible Conduct of Research (RCR)
(This certification is not applicable to proposals for conferences, symposia, and workshops.)

By electronically signing the Certification Pages, the Authorized Organizational Representative is certifying that, in accordance with the NSF Proposal & Award Policies & Procedures Guide, Part II, Award & Administration Guide (AAG) Chapter IV.B., the institution has a plan in place to provide appropriate training and oversight in the responsible and ethical conduct of research to undergraduates, graduate students and postdoctoral researchers who will be supported by NSF to conduct research.
The AOR shall require that the language of this certification be included in any award documents for all subawards at all tiers.

CERTIFICATION PAGE - CONTINUED

Certification Regarding Organizational Support

By electronically signing the Certification Pages, the Authorized Organizational Representative (or equivalent) is certifying that there is organizational support for the proposal as required by Section 526 of the America COMPETES Reauthorization Act of 2010. This support extends to the portion of the proposal developed to satisfy the Broader Impacts Review Criterion as well as the Intellectual Merit Review Criterion, and any additional review criteria specified in the solicitation. Organizational support will be made available, as described in the proposal, in order to address the broader impacts and intellectual merit activities to be undertaken.

Certification Regarding Federal Tax Obligations

When the proposal exceeds $5,000,000, the Authorized Organizational Representative (or equivalent) is required to complete the following certification regarding Federal tax obligations. By electronically signing the Certification pages, the Authorized Organizational Representative is certifying that, to the best of their knowledge and belief, the proposing organization:
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(2) has not been convicted of a criminal offense under the Internal Revenue Code of 1986; and
(3) has not, more than 90 days prior to this certification, been notified of any unpaid Federal tax assessment for which the liability remains unsatisfied, unless the assessment is the subject of an installment agreement or offer in compromise that has been approved by the Internal Revenue Service and is not in default, or the assessment is the subject of a non-frivolous administrative or judicial proceeding.

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AUTHORIZED ORGANIZATIONAL REPRESENTATIVE	SIGNATURE	DATE
NAME Joni L Gould	Electronic Signature	Oct 6 2015 10:46AM

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Project Summary

1 Overview

The PIs propose to conduct research on reversible dynamical systems of algebraic-combinatorial origin, with special emphasis on *periodicity* and *homomesy*. The former phenomenon occurs when repeating an operation a relatively small number of times brings the system back to its original state; that is, orbits in the state-space are small. The latter occurs when some quantity describing the states of the system has the same average value over each orbit.

Homomesies of dynamical systems will be explored in the context of algebraic combinatorics. Discrete probability theory can serve as a guide: when X is a set of combinatorial objects and f is some integer-valued function on the set X whose average over X has small denominator d, the smallness of d may be a hint of the existence of some action of a cyclic group of order d on the set X, such that the function f has the same average on each orbit. That is, in MIT graduate student Sam Hopkins' succinct phrase, "Small denominator in averages are often explained by group actions."

An additional theme the PIs plan to explore is *resonance*. This loosely-described phenomenon is related to periodicity; a combinatorial dynamical system resonates with n when, for nearly every orbit, the ratio of the size of the orbit to the number n is either a whole number or a fraction with small denominator. We suspect that when a system resonates with n, there is a related dynamical system of period n.

2 Intellectual merit

Part of the **intellectual merit** of the proposal lies in its foundational nature: the PIs will be using the ideas of periodicity, homomesy, and resonance to shed new light on many sorts of combinatorial objects, ranging from classical (such as permutations and combinations) to extremely modern (such as alternating-sign matrices). A further part of the intellectual merit of the proposal lies in its interdisciplinary character, with possibilities for transfer of ideas between probability, combinatorics, and algebra.

3 Broader impact

A **broader impact** of the proposed research is the inclusion of undergraduates and graduates as research assistants. The PIs will tightly integrate education with research by training students in fundamental tools pertaining to combinatorics, algebra, and discrete dynamical systems, and then setting the trainees loose on unsolved problems. In so doing, the PIs will develop the students' general skills in mathematical research, with the hope of encouraging many of them to become mathematicians or scientists or ordinary citizens with an appreciation of the nature of the scientific enterprise.

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Appendix Items:

*Proposers may select any numbering mechanism for the proposal. The entire proposal however, must be paginated. Complete both columns only if the proposal is numbered consecutively.

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1 Proposed Research

Dynamical systems theory arose from the study of simple systems like a swinging pendulum, and from the insight that, despite the changing appearance of the system, certain properties of the system are constant. In the case of the pendulum, kinetic energy and potential energy are both changing, but their sum, the total energy, is invariant. Identifying such invariants is a key part of the program of classical mechanics and of dynamics more broadly. A variation on this theme is that certain quantities vary with time but in such a way that their *average* over time equals zero or some other constant independent of the initial state of the system. In the case of the pendulum, regardless of the energy of the pendulum's motion, the average angular displacement of the bob from its rest position is zero. Quantities whose average behavior over time is the same regardless of the initial state of the system, which we dub *homomesies* (Greek for "same middle"), are complementary to the invariants of the system; though they may not be as important as the invariants, they have their own story to tell and deserve attention.

The modern study of dynamical systems is diverse, ramifying into topological dynamics, measure-theoretic dynamics, complex analytic dynamics, and so on, according to the nature of the systems that are studied. In this project we propose to study the concepts of invariance and homomesy as they play off against one another in three (interlinked) types of discrete-time dynamical system: combinatorial, piecewise-linear, and birational. In all three realms we have a common setup: a set X of objects we call the states of the system; a map T from X to itself that evolves the system forward in discrete time (replacing one element of X by another); and a collection of functions $F_1, \ldots, F_k$ from X to $\mathbb{R}$ (or some other field) that correspond to making some measurements of the system. Just as by adding potential to kinetic energy we obtain a quantity that is invariant, we can often find linear combinations of the building blocks $F_1, \ldots, F_k$ that exhibit invariance or homomesy; the invariants (resp. "homomesies") of the dynamical system form a subspace V_{inv} (resp. V_{hom}) of the space V spanned by $F_1, \ldots, F_k$. Since in most cases of interest V is much lower-dimensional than the vector space of all functions on X, it is not *a priori* guaranteed by a dimension-count that any non-constant invariants or homomesies will exist in V, and indeed non-trivial invariants are often hard to find. Yet we have consistently found that homomesies exist in abundance for the sort of combinatorial dynamical systems that occur "in nature".

Combinatorial dynamics is concerned with the iteration of operations on the standard sorts of objects studied in algebraic combinatorics and graph theory: permutations, combinations, partitions, compositions, Young tableaux, independent sets, colorings, spanning trees, parking functions, etc. Birational dynamics is the study of invertible operations on a collection of variables $x_1, x_2, \ldots$ that are expressible by rational multivariate functions in the forward and reverse directions. The gap between these two seemingly unrelated styles of dynamics is bridged by piecewise-linear dynamics, which studies invertible multivariate operations that are continuous and piecewise-linear in the forward and reverse directions. Piecewise-linear dynamics is inherently geometric, as the set X on which the dynamics occurs can often be viewed as a polytope [EP13] [EP14] [St86]. Sometimes a piecewise-linear action can be viewed as a description of the limiting behavior of a birational transformation "at infinity", and facts about piecewise-linear dynamics can be inferred from facts about associated dynamical systems in the birational realm via the process of tropicalization. Furthermore, by restricting a piecewise-linear action to the vertices of (or lattice points in) the polytope, one can often derive consequences about a combinatorial dynamical system.

Our main focus in this project will be combinatorial dynamics. In part this reflects our areas of greatest competence and interest, and in part it reflects our sense of the natural path along which the subject will evolve (with combinatorial systems serving as the scaffolding on which piecewise-linear and birational systems are built). When X is a set of combinatorial objects, the "measurements" we make on X are integer-valued functions that are often called "statistics" in the combinatorial literature. Given a finite set of combinatorial objects, an action on the objects, and a set of numerical statistics (i.e., measurements) of the objects that span a vector space V of statistics (a "feature space"), the subspace V_{hom} consisting of those composite statistics that exhibit homomesy is often nontrivial and sometimes surprisingly large; one natural goal is to characterize V_{hom} by determining its dimension and finding a good basis.

In preparation for the project, we have looked at numerous examples of combinatorial dynamical systems (see [PR]). We have identified five sorts of phenomena that arise in various contexts, and the questions they give rise to:

(1) *Periodicity*: Why does a particular invertible map T from some set X to itself have small order $n \ll |X|$?
(2) *Isomorphism/conjugacy*: Why do two particular maps $T : X \to X$ and $T' : X' \to X'$, viewed as abstract permutations, have the same cycle-structure?
(3) *Cyclic sieving* (see [RSW]): For naturally occurring maps $T : X \to X$ with X carrying some combinatorial structure, why is there so often a simple function $r : X \to \mathbb{Z}$ such that the number of fixed points of T^k is equal to $|\sum_{x\in X} \zeta^{k\,r(x)}|$ (where ζ is a primitive nth root of unity) for all k?
(4) *Invariance*: What are the invariants of $T : X \to X$ in the space V consisting of all linear combinations of a specific family of statistics F_i?
(5) *Homomesy*: What are the homomesies of $T : X \to X$ in the space V consisting of all linear combinations of a specific family of statistics F_i?

Our focus will be on the fifth sort of question, since it is simultaneously the least-explored of the five and the one with the broadest applicability. Prior to work done by us and our collaborators, the only researcher (to our knowledge) who studied homomesies of combinatorial dynamical systems was the algebraic combinatorialist Dmitriy Panyushev, and he did so in a narrowly circumscribed domain (antichains in root-posets of finite-dimensional semi-simple Lie algebras under the action of row-motion). In preliminary work, we have looked at a wide variety of combinatorial objects, actions on those objects, and collections of statistics on those objects, and have found that in an amazing number of cases, nontrivial homomesies exist. For instance, interesting homomesy phenomena are associated with ten of the twelve cells in the Twelvefold Way (a categorization of basic combinatorial constructions popularized by Stanley in [St12]), and also in association with more specialized combinatorial objects like Young diagrams that are of interest to algebraists as well as combinatorialists. Indeed, some of the data from which we have inferred nontrivial homomesies can be found in earlier literature; the researchers who generated the data (e.g., Suter [Su02] and Striker and Williams [SW]) did not notice the patterns that we did. This suggests that it will be very easy for the study of homomesy to spread rapidly through the combinatorial community, inasmuch as many researchers have already been studying systems that (empirically) exhibit homomesy, and have been developing tools that can almost certainly be applied in going from empirical observation to rigorous proof. The recent work of Bloom, Pechenik, and Saracino [BPS], who proved one of the PIs' homomesy conjectures for Young tableaux, is a good example of the sort of work the PIs hope to inspire. Another is the more recent article of Schilling, Thiéry, White and Williams [STWW] (an outgrowth of the 2015 AIM meeting on Dynamical Algebraic Combinatorics, co-organized by the PIs) which gives the first known example "in the wild" of an inherently dihedral homomesy, i.e., a homomesy for a naturally-arising dihedral group action that does not arise from homomesy of a cyclic group subaction. It also validates the idea that homomesy can serve as a tool in showing why certain quantities have unexpectedly simple average values (i.e., fractions with small denominators); in this case, as in many others, the global average coincides with the average over each and every orbit, and since the orbit sizes are small, this sets a bound on how big the denominator can be.

We have done extensive work bringing graduate students into the project and providing mentoring. Mike Joseph, a doctoral student in the UConn mathematics department, has been working with Roby for two years, making crucial contributions to a collaboration incubated at the aforementioned 2015 AIM workshop [EFGJMPR]. David Einstein, a doctoral student in the UMass Lowell computer science department, has worked with Propp over the past seven years, and has made many significant contributions to the work that has already been done [EP13, EP14]. MIT graduate student Darij Grinberg has made important contributions to the subject, by proving a challenging periodicity result in collaboration with Roby [GR1, GR2]. Dartmouth graduate student Shahrzad Haddadan has met with Propp and Roby on several occasions, and has done original work of publishable quality related to the project [Ha, CHHM]. Two more graduate students, Jonathan Bloom at Dartmouth and Oliver Pechenik at the University of Illinois (the former in collaboration with Dan Saracino of Colgate), independently found two different proofs of a conjecture presented by Roby in his FPSAC presentation in Paris in July 2013 (regarding a family of homomesies for semistandard Young tableaux of rectangular shape under the action of promotion) [BPS].

Since the above description is fairly general and abstract, we switch now to a simple (though not intrinsically important) illustration, and a general geometric context into which it and many other examples of homomesy fit.

Homomesy can be defined as the "constant-averages-over-orbits" property. For example, consider the set X of k-element subsets of the set of integers between 1 and n, under the action of the operation T that increases each element by 1 mod n. For $1 \leq i \leq k$, and $x \in X$, let $F_i(x)$ be the ith smallest element of the set x. Haddadan has shown (with a beautiful pictorial proof) that for all $1 \leq i, j \leq k$ with $i+j = k+1$, the function $F_i + F_j$ is homomesic with average $n+1$. E.g., take $n = 5$ and $k = 3$. There are two orbits, each of size 5: one consists of $\{1,2,3\}$, $\{2,3,4\}$, $\{3,4,5\}$, $\{1,4,5\}$, and $\{1,2,5\}$, and the other consists of $\{1,2,4\}$, $\{2,3,5\}$, $\{1,3,4\}$, $\{2,4,5\}$, and $\{1,3,5\}$. The average of F_1 over the first of the two orbits is $(1+2+3+1+1)/5 = 8/5$, while the average of F_1 over the other orbit is $(1+2+1+2+1)/5 = 7/5 \neq 8/5$, so F_1 is not homomesic for this action. Neither is F_3. On the other hand, $F_1 + F_3$ is homomesic, with average 6 over both orbits.

When combinatorial objects have a natural representation as lattice points in a convex polytope K in $\mathbb{R}^n$, a very natural class of actions can arise from *flipping* actions on the polytope. Flipping is an operation whose effect on a point $(x_1, \ldots, x_n)$ alters only the ith coordinate (for some i) and leaves the other $n-1$ alone. If we take L, R (depending on $x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n$) so that $[L, R]$ is the set of values t for which $P_t := (x_1, \ldots, x_{i-1}, t, x_{i+1}, \ldots, x_n)$ belongs to K, then flipping sends P_t to $P_{t'}$ for t' satisfying $t + t' = L + R$; for suitable K, this sends lattice points to lattice points. If we perform flipping in each of the n coordinate directions in succession, we obtain a composite operation T that in general affects all the coordinates of a point in K, and in many cases the resulting composite action exhibits homomesy with respect to various statistics in the span of the coordinate functions. Some of these composite actions are operations (such as Schützenberger promotion of semistandard Young tableaux) that have already been considered in the combinatorial literature (though not described in these geometric terms explicitly); in other cases, one obtains operations T that appear to be new, and that have nice properties (e.g., T has small order) that require explanation.

As an aside, we mention that flipping is closely related to the "toggling" operations studied in [SW, Str]. The (continuous) piecewise-linear form of flipping described here was anticipated by work of Kirillov and Berenstein [KB]. The idea of building up maps as compositions of involutions is highly reminiscent of much recent work in the theory of discrete-time integrable systems.

Here is a short summary of one of the main results obtained to date. We will phrase it in terms of the order polytope $\mathcal{O}(P)$, a convex compact polytope in $\mathbb{R}^{|P|}$ (see [St86] for more details), where P is the poset $[a] \times [b]$ (the product of a chain of length a and a chain of length b). Let x denote a point in $\mathcal{O}(P)$ whose entries we will index as $x_{i,j}$ with $1 \leq i \leq a$ and $1 \leq j \leq b$. For each (i, j) in $[a] \times [b]$ we get a coordinate function $f_{i,j}$ on $\mathcal{O}(P)$; let $V \cong \mathbb{R}^{|P|}$ be the span of these functions. Let π denote the promotion operation on $\mathcal{O}(P)$ (a generalization of Striker and Williams' notion [SW] of promotion of order ideals in RC-embedded posets; as we point out in [EP14], π has connections to Schützenberger promotion on Young tableaux). We have $\pi^n(x) = x$ with $n = a + b$. Let $\overline{x}$ denote the average of the points $\pi^k(x)$ $(0 \leq k \leq n-1)$. Then irrespective of one's choice of x, $\overline{x}$ lies in a particular affine subspace of $\mathbb{R}^{|P|}$ whose codimension is $\lceil (a-1)(b-1)/2 \rceil$. Indeed, for each k between $1-b$ and $a-1$, the sum of the functions $f_{i,j}$ with $i - j = k$ is homomesic, and for each (i,j) in $[a] \times [b]$, the sum $f_{i,j} + f_{a+1-i,b+1-j}$ is homomesic, and these functions span the homomesic subspace V_{hom} of V.

The promotion map π can be written as a composition of flip-maps of the order polytope $\mathcal{O}([a] \times [b])$, where flip-maps of a polytope K are defined as above. Striker and Williams, working at the poset level, proved that composing these flip-maps in a different order yields a map they call rowmotion, whose inverse map has been independently invented numerous times over the past forty years. Rowmotion in $[a] \times [b]$ manifests the periodicity phenomenon (1) discussed above; its nth power (with $n = a + b$) is the identity map. Striker and Williams show more: rowmotion and promotion are actually conjugate to one another (a nice example of phenomenon (2)). Following up on their work, Einstein and Propp lifted the definition of rowmotion and promotion from the poset of order ideals of $[a] \times [b]$ (which is in bijection with the vertex set of $\mathcal{O}([a] \times [b])$) to the order polytope itself, and showed that the homomesies of rowmotion coincide with the homomesies of promotion.

We, along with many collaborators, have begun to study what happens when P is replaced by other posets, or when the order polytope is replaced by the chain polytope (see [St86]). Indeed, Panyushev [Pa] was already taking a step in this direction when he looked at antichains in the root poset of type A_n, since antichains in a poset are in bijection with order ideals. Panyushev's operation on antichains of a general ranked poset P is isomorphic to the inverse of Striker-Williams rowmotion on order ideals of P. Lifted up to the realm of polytopes and piecewise-linear maps, the isomorphism is accomplished by Stanley's transfer map between the order polytope and the chain polytope. However, Stanley's piecewise-linear map is not linear, so it warps the homomesy picture considerably: for the action of rowmotion on antichains, the subspace of homomesies within the ab-dimensional feature space has codimension different from $\lceil (a-1)(b-1)/2 \rceil$. Moreover, in the setting of antichains, rowmotion and promotion do not in general have the same homomesies as one another. For one example of fine work by a graduate student and an undergraduate based on [PR], see [RW].

Our work broke through to a new level of significance when, with assistance from Arkady Berenstein, we "lifted" the action from the piecewise-linear realm to the birational realm. The periodicity and homomesy phenomena seen for the piecewise-linear maps also apply to their birational lifts. In this realm, an additional phenomenon appears that is present but obscured in the combinatorial and piecewise-linear realms: extra symmetries such as the one called reciprocity in [GR2].

We digress to mention that the proposal may also lead to results and conjectures of an analytic nature that will interest ergodic theorists. Some variants of the piecewise-linear and birational maps that arise in our work turn out to have infinite order, yet they exhibit asymptotic versions of homomesy. Here are two conjectures along these lines, one from the birational realm and one from the piecewise-linear realm.

(I) Consider the birational map T from $(\mathbb{R}^+)^4$ to itself sending (w,x,y,z) to (w',x',y',z'), where w',x',y',z' are given by the formulas $ww' = x+z$, $xx' = w'y$, $yy' = x'z/(x'+z)$, and $zz' = w'y'$. Define $F(w,x,y,z) = \log wxyz$. Then we conjecture that for a subset of $(\mathbb{R}^+)^4$ whose complement has measure zero, the average $\frac{1}{n}\sum_{k=0}^{n-1} F(T^k(w,x,y,z))$ converges to 0.

(II) Consider the volume-preserving piecewise-linear map T from $\mathbb{R}^4$ to itself sending (w,x,y,z) to (w',x',y',z'), where $w+w' = \min(x,z)$, $x+x' = w'+y$, $y+y' = \max(x',z)$, and $z+z' = w'+y'$. Define $F(w,x,y,z) = w+x+y+z$. Then we conjecture that for all (w,x,y,z) in $\mathbb{R}^4$, the average $\frac{1}{n}\sum_{k=0}^{n-1} F(T^k(w,x,y,z))$ converges to 0.

Although the combinatorial origin of these maps is not obvious, they arise in a simple way from toggle-operations on the order ideals of the four-element poset $[2]\times[2]$. If conjectures (I) and (II) are true, they are likely to be the very simplest cases of higher-dimensional phenomena.

An intriguing connection has recently been uncovered with the theory of integrable discrete dynamical systems (and cluster algebras in particular). Grinberg and Roby's proof of the periodicity of birational rowmotion on a product of two chains was inspired by Volkov's proof of the Zamolodchikov periodicity conjecture for type AA (after Pylyavskyy and Musiker independently pointed out the similarities between the two dynamical systems). But it was only at the 2015 AIM workshop that a working group (consisting of A. Berenstein, M. Glick, D. Grinberg, G. Musiker, and G. Thomas) discovered a more direct connection between these dynamical systems [Ro, Prop. 6]. Although it takes a certain amount of work, this pathway eventually allows one to obtain periodicity of birational rowmotion on a product of two chains from periodicity of the corresponding Y-system, and vice versa. More importantly, it shows a very strong connection between these two phenomena which is yet to be fully explored. In particular, perhaps homomesy results for birational rowmotion might translate into an interesting property of Y-systems.

Also, the Lyness 5-cycle (the smallest non-trivial cluster algebra) exhibits a non-obvious homomesy: if U is the set of all (x,y) in $\mathbb{R}^2$ with x, y, $x+1$, $y+1$, and $x+y+1$ all nonzero, and $\tau : U \to U$ is the period-5 map sending (x,y) to $(y,(y+1)/x)$, and $f : U \to \mathbb{R}$ is the map sending (x,y) to $\log|x^{-1}+x^{-2}|$, then it can be checked that f is 0-mesic (see section 2.6 of [PR]).

In addition to the aforementioned collaborators, and others whom we hope to recruit in New England during the term of the grant, we should mention that Arkady Berenstein, Jessica Striker, Nathan Williams, and Ben Young have played important roles in this work in the past, and we are hoping to engage their efforts in the future. Indeed, Striker, Williams, Propp, and Roby jointly organized an American Institute of Mathematics workshop, held in March 2015, on the theme of Dynamical Algebraic Combinatorics; the overlap between the goals of that workshop and the topics discussed in this research proposal is substantial. Additionally, the four of us have applied to the Banff International Research Station to hold another conference on this theme in 2017.

One side project that we plan to tackle with the benefit of our computational infrastructure for doing linear algebra in feature spaces (as defined above) is a problem about alternating sign matrices (ASMs) raised by Striker and Williams (see section 8.3 of [SW]). It has been known for over a decade that n-by-n ASMs admit a representation as states of the O(1) loop model, and these states in turn map to states of a simpler combinatorial model (namely, the link-patterns model) that naturally admits an action of the cyclic group of order $2n$. (See [Pr02] for some of the many existing avatars of ASMs.) Numerical clues suggest that ASMs admit a representation quite different from any studied hitherto, in which the cyclic group of order $2n$ is replaced by a cyclic group of order $3n-2$. Unfortunately the clues are extremely indirect. How can we get information about the nature of this new, unknown combinatorial representation ϕ of ASMs, mapping ASMs to some class Y of combinatorial objects? We propose to make use of dynamics and linear algebra, and some computer time, to make progress on the problem.

Here is a sketch of the idea. Let X be the set of n-by-n ASMs. Striker and Williams devised a map $T : X \to X$ called superpromotion [SW] whose action on X appears to be "trying" (but failing) to be of order $3n-2$, and we suspect that this indicates the existence of a picture that looks like this:

$$\begin{array}{ccc} X & \xrightarrow{T} & X \\ {\scriptstyle\phi}\big\downarrow & & \big\downarrow{\scriptstyle\phi} \\ Y & \xrightarrow[U]{} & Y \end{array} \quad .$$

Here Y is an unknown set, $U : Y \to Y$ is an unknown map of order $3n-2$, and ϕ is a surjection from X to Y that makes the diagram commute. (Cf. the relationship between gyration of ASMs and rotation of the associated link-patterns [W].) Some information about Y is encoded in the equivalence relation $\equiv_\phi$ on X induced by ϕ. We do not have access to this relation directly, but we know that for each ASM A in X, $T^{3n-2}(A) \equiv_\phi A$. If we furthermore assume that $\equiv_\phi$ is the transitive closure of a collection of relations given by "local moves", then we should find that, for a suitable feature space V and a suitable linearization map $L : X \to V$, the vector $L(T^{3n-2}(A)) - L(A)$ should be a sum of vectors that correspond to the local moves needed to change A into $T^{3n-2}(A)$ — moves that the map ϕ "doesn't see". By looking at enough vectors of the form $L(T^{3n-2}(A)) - L(A)$ we may begin to sense what those local moves are, and thus reconstruct the equivalence relation $\equiv_\phi$, bringing us a step closer to figuring out what ϕ is (and what Y and $U : Y \to Y$ are).

Gyration and superpromotion of ASMs are examples of the more general phenomenon of "resonance". This somewhat vague concept is best explained in terms of an example: when n is large, and one looks at how the action of gyration splits up the set of n-by-n ASMs into orbits, nearly all of the orbits have size divisible by $2n$; and for those rare orbits whose size is not divisible by $2n$, one finds that in the majority of cases, the orbit-size divided by $2n$ is a fraction with small denominator.

We believe that when resonance appears (and researchers have discovered several new examples in the past few years), it indicates that there is a cyclic group action — not an action on the original objects, but on factor-objects of an appropriate kind (in the sense that link-patterns can be viewed as "factors" of ASMs).

If we succeed in finding a map from ASMs onto a combinatorial model admitting a cyclic action of order $3n - 2$, we can expect (based on the history of the study of link-patterns of ASMs) that our work will provide a large amount of fodder for enumerative combinatorialists, and may be of interest to physicists (cf. the line of work that recently culminated in the proof of the Razumov-Stroganov conjecture by Cantini and Sportiello [CS10]). Note that (as was pointed out by Striker [Str]) Lemma 3.1 of [CS10] can be paraphrased as a homomesy result.

An important phenomenon in the piecewise-linear realm of dynamics is that, for certain piecewise-linear maps T taking some polytope X to itself, orbits of finite cardinality coexist with orbits of infinite cardinality. For instance, it appears that in the case of the lift of Wieland's gyration operation on order-n ASMs from the combinatorial realm to the piecewise-linear realm, for $n \geq 4$, the points in the polytope belonging to orbits of size $2n$ have measure strictly between 0 and 1; this is probably related in some way to the resonance phenomenon. We hope we can interest the dynamical systems community in developing piecewise-linear dynamical systems theory, using some of our combinatorially-derived examples as catalysts for progress.

It is worth mentioning that the notion of homomesy as an organizing principle of combinatorial dynamics arose from Propp's work on chip-firing (a.k.a. sandpile dynamics) and rotor-routing (a.k.a. Eulerian walker dynamics). (Most of Propp's articles on these dynamics, such as [HL+], [HP], and [Pr10], do not explicitly comment on homomesy; for a clarification of the link, see section 2.7 of [PR].) These systems, which Propp studied in his two previous NSF grants and which he views as prototypes of a theory of derandomized stochastic processes, both exhibit homomesy for quantities that are derandomized analogues of fundamental quantities from probability theory (mean hitting time, stationary measure, escape probability, etc.). We expect to continue to draw inspiration from probabilistic models, and thus will continue to study approaches to derandomization. However, our focus in this three-year project will be on the properties of finite processes, since the infinite processes Propp studied appear to require tools beyond the ones currently at our disposal. Also, our emphasis will be on the dynamical properties of the derandomized models, rather than on quantification of the extent to which the derandomized models shadow their random counterparts.

Lastly, we mention the reciprocal relationship between phenomenon (2) (when two dynamical systems have the same orbit-structure) and phenomenon (5) (when a single dynamical system has homomesies). In many cases, one explains homomesies by finding a suitable equivariant bijection (also called a conjugacy or an isomorphism in dynamical contexts) that respects the statistic being studied, and such bijections also clarify the orbit-structure of a system. So we expect an intimate interaction between these two sorts of questions in the course of the research.

2 Broader Impacts of Proposed Work

Combinatorial, piecewise-linear, and birational dynamical systems are extremely well-suited to exploration in the sort of vertically-integrated research groups that Propp has been running at various schools for over two decades. One of the main challenges of running a group like this is finding a topic whose topsoil has not already been strip-mined by earlier researchers and that has the right mix of breadth and depth. Dynamical algebraic combinatorics is such a topic; it is a fairly new subject, and it presents problems of many flavors and of widely varying difficulty. It helps that many combinatorial problems can be stated in an accessible and appealing way. We expect that it will be possible for students with varying degrees of exposure to higher mathematics to conduct experiments, find patterns, formulate conjectures, and devise rigorous proofs.

Over the course of the proposed grant, it is expected that half a dozen undergraduates will have the benefit of participating in genuine mathematical research, working on problems whose outcome is not known in advance. All of the participants will get direct experience doing work in this vein, as part of a research community combining Ph.D.'s, grad students, and undergraduates. Those participants who find this style of collaboration enjoyable may start up similar REU efforts themselves as they pursue their own mathematical careers. This promulgation of Propp's style of cultivating young talent could in the long run prove to be the greatest impact of all.

3 Qualifications of the PIs

Propp has 30 years of experience in combinatorics and dynamics. The current proposal mixes several themes from his past research. His graduate work in ergodic theory gave him a dynamical perspective that most algebraic combinatorialists do not have, and his more recent work with Hasselblatt [HP] gave him some familiarity with continuous piecewise-linear dynamics. Gyration — the prototypical example of what we here dub "resonance" — was invented by Ben Wieland [W] when he was an undergraduate working in Propp's undergraduate research group at MIT. Propp's knowledge of probabilistic combinatorics makes him familiar with the sort of situations in which the smallness of denominators in the expected values of certain combinatorial quantities makes it plausible that some sort of group action is at work. He is very experienced in conducting experimental mathematics research with the aid of computers and with the more specific task of reverse-engineering combinatorial structures from clues provided by algebra. (Carroll-Speyer groves [CS04], discovered under Propp's supervision, are one example of this; so are the graphs associated with Markoff numbers [BPW] and Somos sequences [P05].) This background will be useful in studies of resonance phenomena.

Propp has a 25-year history of working with undergraduates and graduate students in vertically integrated mini-communities in which research takes place in a friendly and collaborative way. He has designed and managed such communities at

MIT (the Tilings Research Group and later the Chips And Rotor-Routers Initiative), the University of Wisconsin (the Spatial Systems Lab), Harvard University (Research Experiences in Algebraic Combinatorics at Harvard), UMass Lowell (Experiences in Quasirandomness at Lowell), and Tufts University (Research Experiences in Dynamics, Combinatorics, and Algebra at Tufts). Alums of Propp's research teams include Henry Cohn, Harald Helfgott, Lionel Levine, David Speyer, Lauren Williams, and David Wilson.

Roby has been studying the inner workings of various combinatorial processes since graduate school. His dissertation [R91] was among the first works to harness Fomin's then-new technology of "growth diagrams" to unify the treatment of and better understand various versions of the fundamental Robinson-Schensted-Knuth (RSK) correspondence. In [RSSW] he delved deeply into the inner workings of the combinatorics of Young tableaux, relating Schützenberger's *jeu de taquin* to other operations on tableaux.

Roby has a long history of outreach to K–12 education and work in pedagogy at the college level, encouraging exploratory, hands-on learning and problem solving. He has extensive experience teaching at (a) the Ross Program at Ohio State University (5 summers as student, counselor, and head counselor), (b) Hampshire College Summer Studies in Mathematics (2 summers as senior staff), (c) the PROMYS program at Boston University (2 summers as founding head counselor, 1 as a recitation instructor, 1 guiding problem-solving teams), (d) MathPath (2 summers as faculty), and (e) Canada/USA Mathcamp (1 summer as a visitor). At PROMYS Roby helped guide teams made of up of high-school students and teachers to work collaboratively on open-ended research problems. The nature of the Ross and PROMYS programs is much like research: guiding students to learn by solving a coordinated series of problems, rather than reading or viewing a polished presentation of the material.

In addition to working with talented students in the STEM pipeline, Roby has spent much time working to improve K–12 mathematics education for all students. He directed a large grant-funded professional development program in the San Francisco Bay Area (ACCLAIM) for six years which served over 2000 teachers. He is on the author team of a number of standard elementary, middle, and high-school textbooks published by Harcourt and Holt that are in use all over the country. He was the founding "mathlets" editor for the MAA's *Journal of Online Math and its Applications.* He served as a member of the California Mathematics Project Advisory Board 1998–2005, chairing it from 2000 to 2005, a position of considerable statewide leadership in mathematics education in California. All of this work is informed by his research; his track record indicates a high probability of any NSF-funded research having a much broader impact in the long term than just on the math research community.

As director of UConn's Quantitative Learning Center for 8 years, Roby worked hard to create a team of faculty, graduate students, and undergraduates, building esprit de corps and a productive working environment. Such human factors are crucial to the success of a research group. He worked with Propp's REDCAT undergraduate research group at Tufts in 2014–15 to gain greater experience working effectively with undergraduates on mathematics research.

Over the years many of Roby's contributions have been focused more on pedagogy and outreach, reflecting both his interest in these arenas and the types of positions he held. Since stepping down from his directorship of UConn's Q Center in 2012 and returning to full-time faculty in the math department, he has returned to focussing most of his attention on mathematical research, becoming one of the leaders in the emerging area of dynamical algebraic combinatorics and homomesy.

Propp and Roby introduced the notion of homomesy four years ago [PR]. Propp went on to work with Einstein [EP13] [EP14], but the two were unable to fully lift their results (such as periodicity) to the birational setting without appealing to an unproved reciprocity formula; Roby, in collaboration with Grinberg [GR2], proved the formula. This work was described in Propp's 2014 FPSAC talk [P14a]. Propp and Roby continue to be active leaders in the effort to understand dynamical algebraic combinatorics, e.g., they were two of the four organizers of the weeklong workshop devoted to the topic held at the American Institute of Mathematics in March 2015. Roby was invited to write an exposition of the area by the editors of a forthcoming IMA volume to appear sometime in 2015 [Ro].

4 Results from Prior NSF Support

Since 2010, Propp has been funded by NSF's combinatorics program, through a project called "Deterministic analogues of random processes" (NSF award number 1001905). This project was originally scheduled to last until 2013; with three no-cost extensions, it has been prolonged until 2016, with a total budget of $200,000.

4.1 Articles

The articles that Propp has published during his current grant fall into two categories: those directly pursuant to the stated aims of the grant proposal (written in 2009) and others with varying degrees of connection to the topic of quasirandom simulation of random processes. As of this writing, the articles in the first category are the research articles [P10a] and [GLPZ] and the expository article [LP]. They deal with the two main examples of derandomized random systems that Propp has studied: chip-firing models and rotor-router models. Propp has also written an unpublished expository/educational article [P10b] applying chip-firing ideas to elementary mathematics, somewhat in the spirit of [FMRR] but in a much more elementary vein. In the second category are the research articles [FPR], [LPRW], and [PR], the expository article [P12], and the semi-expository article [P13]. Additionally, Propp completed a survey article on enumeration of tilings [P15] that appeared as a book chapter in 2015, and is at work on several other articles, including one with David Einstein and Alexander Holroyd (not available as a preprint at the time of this writing) and another with Shirshendu Ganguly, Lionel Levine, and Yuval Peres [GLPP]; these last two are directly relevant to the theme of the current grant.

4.2 Intellectual merit

Propp succeeded in finding a satisfactory derandomized analogue of finite-state Markov chain theory using rotor-routing, similar to existing chip-firing models but more broadly applicable since transition probabilities need not be rational. This will serve in the future as a launching point for study of infinite-state Markov chains and more structured stochastic systems, such as the competitive erosion model that Propp invented and jointly explored with Shirshendu Ganguly, Lionel Levine, and Yuval Peres. (Indeed, Propp invented competitive erosion in tandem with a derandomized version of it that appears to exhibit the same conformal invariance properties; once the article with Ganguly et al. is published, Propp will write an experimental article supporting the conjecture that the deterministic model also obeys a form of conformal invariance.)

Propp's work also led him down two avenues that he was not expecting but which might prove to be more important, and of broader applicability, than the work described in his original proposal.

One outgrowth has to do with the way in which one uses finite data to estimate asymptotic behavior. More precisely, if one is trying to determine the average long-term behavior of a system that has been observed within a finite time-window, then simply averaging the behavior that one has seen within that time-window may not be the best way to make use of the data, *if* (big if!) there are signs of periodicity in the data. In that case, one can often do qualitatively better using *tapered* averaging [Pr14a, Pr14b] that puts less weight on information near the edges of the window. To be more specific, if n observations have been made in the window, then an ordinary average has error on the order of $1/n$, while tapering gives error on the order of $1/n^{3/2}$ or even better. The form of tapering that achieves this bound (using a quadratic kernel) does not depend on the details of the periodicities in the data; a one-size-fits-all approach works. These results elicited interest at the workshop on discrepancy theory held at ICERM in October 2014, and led Josef Dick and Roswitha Hofer to prove Propp's conjecture [P14c] that tapered averaging permits one to circumvent the Schmidt discrepancy bound. See also [P14b].

The other outgrowth is the notion of homomesy (the subject of the current proposal). Both the sandpile model and the rotor-router model exhibit homomesy, and once Propp set aside the probabilistic interpretation of the homomesic quantities and focused on their homomesic nature, it was natural to explore other manifestations of this phenomenon. Over the past four years, Propp has given a half-dozen seminar talks on preliminary findings associated with ideas described in the Proposed Research section of this document. This extremely general idea (whose potential breadth of application should be clear from section 2 of [PR]) might turn out to be more important than either quasirandomness or tapered averaging.

4.3 Broader impacts

Propp was one of the four co-organizers of the American Institute of Mathematics workshop "Generalizations of chip-firing and the critical group" [AIM1] held July 8–12, 2013. Propp used his NSF funds to support the participation of two graduate students who might not otherwise have been able to attend. One of them solved a problem posed at the workshop and now is poised to do doctoral work in this area. Propp and Roby were two of the four co-organizers of the American Institute of Mathematics workshop "Dynamical algebraic combinatorics" [AIM2] held March 23–27, 2015. This workshop launched half a dozen ongoing collaborative ventures bringing senior and junior researchers together (e.g., [CHHM], [EFGJMPR], and [STWW]).

One problem that arose from Propp's work on rotor-routers seemed sufficiently interesting, and sufficiently self-contained, that Propp thought it would be an ideal problem to assign to a talented and ambitious high school student. Xiaoyu He, who participated in the PRIMES program at MIT under the supervision of Tanya Khovanova, did not fully solve the problem (which remains open), but the partial results described in his article "On the classification of universal rotor-routers" [He] were of sufficiently high quality to earn him $10,000 Davidson Fellowship in 2012.

Although Propp has not written up his ideas relating chip-firing with the teaching of arithmetic (other than the unpublished article [P10b]), one of the talks Propp gave turned out to have quite a large indirect impact. A few years back Propp gave a Boston Math Circle talk on these ideas, included a peek into a topic he calls "Base One and a Half" [T1]. Propp's host for the visit, Jim Tanton, found the approach very inspiring, and expanded it into an entire pedagogy via the device of "Exploding Dots". Here we see, over the course of decades, a beautiful interaction between research and pedagogy: chip-firing was invented by Arthur Engel to explain probability to 4th graders; Propp and others studied it as a research topic; and now Tanton is using it to teach a host of topics from high school mathematics. (See [T2] in which Tanton acknowledges the role Propp played in closing this circle.)

Propp's recent work on homomesy has also had some educational impact. Two high school students have already coauthored articles on homomesy through MIT's RSI program: Kelvin Wang (working with math grad student David Rush; see [RW]) and Ingrid Zhang (working with math grad student Sam Hopkins; see [HZ]). Additionally, Propp thought that one of the easier homomesy results would make a good (and novel) contest problem, so he sent it to the organizers of the Bay Area Mathematics Olympiad, who used it for the Spring 2013 contest (see problem 6 in [BAMO]).

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Biography: James G. Propp

(a) Professional Preparation

Harvard College (Cambridge, MA), mathematics, A.B. 1982.

U.C. Berkeley (Berkeley, CA), mathematics, Ph.D. 1987.

Postdoctoral work at University of Maryland (College Park, MD; 1987–1988) and University of California at Berkeley (Berkeley, CA; 1988–1990) in ergodic theory, funded by NSF.

(b) Non-visiting Appointments

University of Massachusetts Lowell, 2006 to present.

University of Wisconsin at Madison, 1998 to 2006.

Massachusetts Institute of Technology, Assistant Professor, 1990 to 1996; Associate Professor, 1996 to 1998; Visiting Scholar, 1998 to 2000.

(c) Publications

Five publications most closely related to the proposal:

D. Einstein, M. Farber, E. Gunawan, M. Joseph, M. Macauley, J. Propp and S. Rubinstein-Salzedo, Noncrossing partitions, toggles, and homomesies. Submitted to *Electron. J. Combin.*, October, 2015.

D. Einstein and J. Propp, Piecewise-linear and birational toggling, *Discrete Mathematics and Theoretical Computer Science*, DMTCS Proceedings, 26th International Conference on Formal Power Series and Algebraic Combinatorics (FPSAC 2014), 513–524; `http://www.dmtcs.org/dmtcs-ojs/index.php/proceedings/article/view/dmAT0145/4518`.

B. Hasselblatt and J. Propp, Degree-growth of monomial maps. *Ergodic Theory Dynam. Systems* **27** (2007), no. 5, 1375–1397 (see section 8 in particular).

A. E. Holroyd, L. Levine, K. Meszaros, Y. Peres, J. Propp, and D. B. Wilson, Chip-firing and rotor-routing on directed graphs. In and out of equilibrium 2, 331–364, *Progr. Probab.* **60**, Birkhäuser, Basel, 2008.

J. Propp and T. Roby, Homomesy in products of two chains. *Electron. J. Combin.* **22** (2015), no. 3, Research Paper P3.4; `http://www.combinatorics.org/ojs/index.php/eljc/article/view/v22i3p4`.

Five other publications by the proposer:

G.P. Giacaglia, L. Levine, J. Propp and L. Zayas-Palmer, Local-to-global principles for rotor walk. *Electron. J. Combin.* **19**(1) (2012), Article P5.

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(d) Synergistic Activities

1. Propp has organized vertically-integrated research communities of undergraduates and graduate students at MIT, the University of Wisconsin, Harvard University, and Tufts University.

2. In 2008, David Wilson and Propp co-wrote a chapter for the book *Markov Chains and Mixing Times* by Levin, Peres, and Wilmer; it brings their "coupling from the past" simulation method to a broader audience.

3. In 2012, Propp curated a collection of images related to the subject of random spatial processes and gave a public presentation entitled "Wild Beauty: Postcards from Mathematical Worlds", displaying these images to the interested public and explaining some of the mathematical background behind them. He has given the talk twice and made it available on the web.

4. Since its launch in summer 2015, Propp's Mathematical Enchantments site site has provided hundreds of readers with essays on a variety of mathematical topics (including chip-firing: see the October 2015 installment).

5. Propp is on the Advisory Council of the Museum of Mathematics, and has done work on explanatory text for existing exhibits as well as design work on an exhibit-in-progress involving combinatorics and probability.

(e) Collaborators and Other Affiliations

Collaborators (16): Henry Cohn (Microsoft), Josef Dick (U. New South Wales), David Einstein (UML), Miriam Farber (MIT), Shirshendu Ganguly (U. Washington), Emily Gunawan (U. Minnesota), Roswitha Hofer (Johannes Kepler), Alexander Holroyd (Microsoft), Michael Joseph (UConn), Lionel Levine (Cornell), Stephen Linton (U. St. Andrews), Matthew Macauley (Clemson), Yuval Peres (Microsoft), Tom Roby (UConn), Simon Rubinstein-Salzedo (Stanford), and Julian West (St. Andrews).

Graduate Advisor: Jacob Feldman, University of California at Berkeley (emeritus). Principal postdoctoral sponsors: N/A.

Thesis Advisees (3): David Gupta (MIT, 1998; no longer in mathematics), William Jockusch (MIT, 1992; no longer in mathematics), David Wilson (MIT, 1996; now at Microsoft Research).

Biographical Sketch

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(a) Professional Preparation

Swarthmore College; Mathematics; B.A. (with High Honors), 1985;
Massachusetts Institute of Technology; Mathematics; Ph.D., 1991;
University of Tokyo, Japan *Japan Society for the Promotion of Science Fellow*, 1991–1993;

(b) Appointments

2005–present: **Associate Professor**, University of Connecticut, Storrs, CT

2005–2012: **Director**, Quantitative Learning Center, University of Connecticut, Storrs, CT

2011–2012: **Visiting Professor**, Massachusetts Institute of Technology, Cambridge, MA

2002–2005: **Associate Professor**, California State University East Bay, Hayward, CA

1997–2002: **Assistant Professor**, California State University East Bay, Hayward, CA

1995–1997: **Research Associate**, University of Wisconsin, Madison, WI

1993–1995: **Visiting Assistant Professor**, Reed College, Portland, OR

(c) Publications

(i) Five publications most closely related to the proposal:

1. Grinberg, D. & **Roby, T.**, Iterative properties of birational rowmotion I: generalities and skeletal posets. *Elec. J. Combin.*, in secondary revisions. `http://web.mit.edu/~darij/www/algebra/skeletal.pdf`

2. Grinberg, D. & **Roby, T.**, Iterative properties of birational rowmotion II: rectangles and triangles. *Elec. J. Combin.* **22**(3), #P3.40, 2015. `http://www.combinatorics.org/ojs/index.php/eljc/article/view/v22i3p40`

3. Linton, S., Propp, J., **Roby, T.** and West, J., Equivalence classes of permutations under various relations generated by constrained transpositions. *J. of Integer Sequences* **15** (2012), Article 12.9.1.

4. Propp, J. & **Roby, T.**, Homomesy in products of two chains. *Elec. J. Combin.* **22**(3), #P3.4, 2015. `http://www.combinatorics.org/ojs/index.php/eljc/article/view/v22i3p4`

5. **Roby, T.**, Dynamical Algebraic Combinatorics and the Homomesy Phenomenon. To appear in A. Beveridge, J. Griggs, L. Hogben, G. Musiker, P. Tetali, eds., *Recent Trends in Combinatorics (IMA Volume in Mathematics and its Applications)*, Springer, 2015. `http://www.math.uconn.edu/~troby/homomesyIMA2015final.pdf`

(ii) Five other significant publications:

6. Benkart, G., and **Roby, T.**, Down-Up algebras. *J. of Algebra* **209**, 305–344 (1998).

7. Merris, R., and **Roby, T.**, The lattice of threshold graphs. *J. of Ineq. in Pure and Appl. Math*, **6**, #1 (2005).

8. **Roby, T.**, and Terada, I., A two-dimensional pictorial presentation of Berele's insertion algorithm for symplectic tableaux. *The Elec. J. of Combin.*, **12**(1), 2005, R4. `http://www.combinatorics.org/ojs/index.php/eljc/article/view/v12i1r4`

9. **Roby, T.**, Sottile, F., Stroomer, J., and West, J., Complementary algorithms for tableaux. *J. of Combinatorial Theory A* **96**, 127–161 (2001).
10. **Roby, T.**, The JOMA mathlets project. *J. of Online Math and its Appl.* **1** (Jan, 2001).

(d) Synergistic Activities

1. **Conference & Workshop Organization:** (1) Co-organizer for AIM workshop on "Dynamical Algebraic Combinatorics" to be held 23–27 March 2015. (2) Organized a special session on "Algebraic Combinatorics" at 2012 SIAM Conference on Discrete Math, Halifax, Nova Scotia, Canada, 18–21 June 2012. (3) Received grants of $15,400 from the National Science Foundation and $15,200 from the National Security Agency to support the 16th Annual Conf. on Formal Power Series and Alg. Comb. (FPSAC), held June 28–July 2, 2004 at the University of British Columbia.
2. **Director, Quantitative Learning Center at UConn:** Created from scratch a thriving center for students to get support with their learning in quantitatively intensive courses at UConn. Built logistical and IT infrastructure. Supervised mentored and trained: associate director, assistant director, program assistant, 5–12 graduate assistants, and about 20–100 peer tutors who handled from 600 to 11,000 student visits per semester as the center grew. Oversaw annual budgets of up to about $500,000. `http://qcenter.uconn.edu`
3. **Director, ACCLAIM Professional Development Institutes for K–12 Teachers:** Founded, directed and taught in ACCLAIM (Alameda County Collaborative for Learning and Instruction in Mathematics), a large, professional development program for K–12 teachers, in cooperation with Alameda County Office of Education and local school districts. ACCLAIM served more than 2000 teachers from 2000–2005, providing in-depth content-based training in math and pedagogy, receiving state and local grants totalling $1.8 million, and overseeing teacher stipends worth $2.4 million.
4. **K–12 Textbook Author:** Served on the textbook author team for the following series of elementary, middle, and high-school textbooks:
   A. (with Maletsky, E. and seven other authors), Harcourt, Inc., Orlando: (1) *Harcourt Math (Grades K–6)*, 2004; (2) *California HSP Math (Grades K–6)*, 2007. (3) *HSP Math (Grades K–6)*, 2009.

   B. (with Burger, E. and seven other authors) Holt, Rinehart and Winston, Austin: (1) *Holt California Algebra 1*, 2007. (2) *Holt California Geometry*, 2007. (3) *Holt Mathematics Course 1–3 [Grades 6–8]*, 2008.
5. **Editor of JOMA:** Served as the Founding Mathlets Editor for the MAA's *J. of Online Math and its Appl.*, 2001–2003. `http://www.maa.org/publications/periodicals/loci/joma/about-joma`

(e) Collaborators & Other Affiliations

Collaborators and Co-Editors: Darij Grinberg, MIT; Steven Linton, University of St. Andrew; Gregg Musiker, University of Minnesota; James Propp, UMass Lowell; Julian West, British government; total: 5.

Graduate Advisers and Postdoctoral Sponsors: Richard P. Stanley, MIT; Itaru Terada, University of Tokyo; Robin Pemantle, University of Wisconsin; total: 3.

Thesis Adviser and Postgraduate-Scholar Sponsor: *Thesis students:* David Goggin, CSU Hayward (M.S., 1999, now in industry); Terence C. Buencamino, CSU Hayward (M.S., 2000, now in industry); Michael Joseph, UConn (Ph.D., current); Elizabeth Sheridan-Rossi, UConn (Ph.D., current); total graduate students: 4; total postgraduate scholars: 0.

SUMMARY PROPOSAL BUDGET

YEAR 1

FOR NSF USE ONLY		
PROPOSAL NO.	DURATION (months) Proposed	Granted
AWARD NO.		

ORGANIZATION
University of Massachusetts Lowell

PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR
James Propp

A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets)	NSF Funded Person-months CAL	ACAD	SUMR	Funds Requested By proposer	Funds granted by NSF (if different)
1. James G Propp - PI	0.00	0.00	2.00	28,640	
2.					
3.					
4.					
5.					
6. (0) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE)	0.00	0.00	0.00	0	
7. (1) TOTAL SENIOR PERSONNEL (1 - 6)	0.00	0.00	2.00	28,640	
B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS)					
1. (0) POST DOCTORAL SCHOLARS	0.00	0.00	0.00	0	
2. (0) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.)	0.00	0.00	0.00	0	
3. (1) GRADUATE STUDENTS				21,000	
4. (2) UNDERGRADUATE STUDENTS				4,800	
5. (0) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY)				0	
6. (0) OTHER				0	
TOTAL SALARIES AND WAGES (A + B)				54,440	
C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS)				1,839	
TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C)				56,279	
D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.)					
TOTAL EQUIPMENT				0	
E. TRAVEL 1. DOMESTIC (INCL. U.S. POSSESSIONS)				1,500	
2. FOREIGN				1,500	
F. PARTICIPANT SUPPORT COSTS					
1. STIPENDS $ 0					
2. TRAVEL 0					
3. SUBSISTENCE 0					
4. OTHER 0					
TOTAL NUMBER OF PARTICIPANTS (0) TOTAL PARTICIPANT COSTS				0	
G. OTHER DIRECT COSTS					
1. MATERIALS AND SUPPLIES				300	
2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION				0	
3. CONSULTANT SERVICES				1,000	
4. COMPUTER SERVICES				0	
5. SUBAWARDS				0	
6. OTHER				5,000	
TOTAL OTHER DIRECT COSTS				6,300	
H. TOTAL DIRECT COSTS (A THROUGH G)				65,579	
I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE) MTDC (Rate: 53.0000, Base: 60579)					
TOTAL INDIRECT COSTS (F&A)				32,107	
J. TOTAL DIRECT AND INDIRECT COSTS (H + I)				97,686	
K. SMALL BUSINESS FEE				0	
L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K)				97,686	

M. COST SHARING PROPOSED LEVEL $ 0 AGREED LEVEL IF DIFFERENT $

PI/PD NAME
James Propp

ORG. REP. NAME*
Linda Concino

FOR NSF USE ONLY		
INDIRECT COST RATE VERIFICATION		
Date Checked	Date Of Rate Sheet	Initials - ORG

1 ***ELECTRONIC SIGNATURES REQUIRED FOR REVISED BUDGET**

1600651

SUMMARY PROPOSAL BUDGET

YEAR 2

FOR NSF USE ONLY		
PROPOSAL NO.	DURATION (months)	
	Proposed	Granted
AWARD NO.		

ORGANIZATION: **University of Massachusetts Lowell**

PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR: **James Propp**

A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets)	NSF Funded Person-months CAL	ACAD	SUMR	Funds Requested By proposer	Funds granted by NSF (if different)
1. James G Propp - PI	0.00	0.00	2.00	29,499	
2.					
3.					
4.					
5.					
6. (0) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE)	0.00	0.00	0.00	0	
7. (1) TOTAL SENIOR PERSONNEL (1 - 6)	0.00	0.00	2.00	29,499	
B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS)					
1. (0) POST DOCTORAL SCHOLARS	0.00	0.00	0.00	0	
2. (0) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.)	0.00	0.00	0.00	0	
3. (1) GRADUATE STUDENTS				21,630	
4. (2) UNDERGRADUATE STUDENTS				4,944	
5. (0) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY)				0	
6. (0) OTHER				0	
TOTAL SALARIES AND WAGES (A + B)				56,073	
C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS)				1,894	
TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C)				57,967	
D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.)					
TOTAL EQUIPMENT				0	
E. TRAVEL 1. DOMESTIC (INCL. U.S. POSSESSIONS)				1,500	
2. FOREIGN				1,500	
F. PARTICIPANT SUPPORT COSTS 1. STIPENDS $ 0 2. TRAVEL 0 3. SUBSISTENCE 0 4. OTHER 0					
TOTAL NUMBER OF PARTICIPANTS (0) TOTAL PARTICIPANT COSTS				0	
G. OTHER DIRECT COSTS					
1. MATERIALS AND SUPPLIES				300	
2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION				0	
3. CONSULTANT SERVICES				1,000	
4. COMPUTER SERVICES				0	
5. SUBAWARDS				0	
6. OTHER				5,000	
TOTAL OTHER DIRECT COSTS				6,300	
H. TOTAL DIRECT COSTS (A THROUGH G)				67,267	
I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE) MTDC (Rate: 53.0000, Base: 62267)					
TOTAL INDIRECT COSTS (F&A)				33,002	
J. TOTAL DIRECT AND INDIRECT COSTS (H + I)				100,269	
K. SMALL BUSINESS FEE				0	
L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K)				100,269	

M. COST SHARING PROPOSED LEVEL $ 0 | AGREED LEVEL IF DIFFERENT $

PI/PD NAME: **James Propp**

ORG. REP. NAME*: **Linda Concino**

FOR NSF USE ONLY		
INDIRECT COST RATE VERIFICATION		
Date Checked	Date Of Rate Sheet	Initials - ORG

2 *ELECTRONIC SIGNATURES REQUIRED FOR REVISED BUDGET

SUMMARY PROPOSAL BUDGET

YEAR 3

ORGANIZATION **University of Massachusetts Lowell**		FOR NSF USE ONLY			
		PROPOSAL NO.	DURATION (months)		
			Proposed	Granted	
PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR **James Propp**		AWARD NO.			

A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets)	NSF Funded Person-months CAL	ACAD	SUMR	Funds Requested By proposer	Funds granted by NSF (if different)
1. **James G Propp - PI**	0.00	0.00	2.00	**30,384**	
2.					
3.					
4.					
5.					
6. (**0**) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE)	0.00	0.00	0.00	**0**	
7. (**1**) TOTAL SENIOR PERSONNEL (1 - 6)	0.00	0.00	2.00	**30,384**	
B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS)					
1. (**0**) POST DOCTORAL SCHOLARS	0.00	0.00	0.00	**0**	
2. (**0**) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.)	0.00	0.00	0.00	**0**	
3. (**1**) GRADUATE STUDENTS				**22,279**	
4. (**2**) UNDERGRADUATE STUDENTS				**5,092**	
5. (**0**) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY)				**0**	
6. (**0**) OTHER				**0**	
TOTAL SALARIES AND WAGES (A + B)				**57,755**	
C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS)				**1,951**	
TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C)				**59,706**	
D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.)					
TOTAL EQUIPMENT				**0**	
E. TRAVEL 1. DOMESTIC (INCL. U.S. POSSESSIONS)				**1,500**	
2. FOREIGN				**1,500**	
F. PARTICIPANT SUPPORT COSTS 1. STIPENDS $ **0** 2. TRAVEL **0** 3. SUBSISTENCE **0** 4. OTHER **0**					
TOTAL NUMBER OF PARTICIPANTS (**0**) TOTAL PARTICIPANT COSTS				**0**	
G. OTHER DIRECT COSTS					
1. MATERIALS AND SUPPLIES				**300**	
2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION				**0**	
3. CONSULTANT SERVICES				**1,000**	
4. COMPUTER SERVICES				**0**	
5. SUBAWARDS				**0**	
6. OTHER				**5,000**	
TOTAL OTHER DIRECT COSTS				**6,300**	
H. TOTAL DIRECT COSTS (A THROUGH G)				**69,006**	
I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE) **MTDC (Rate: 53.0000, Base: 64006)**					
TOTAL INDIRECT COSTS (F&A)				**33,923**	
J. TOTAL DIRECT AND INDIRECT COSTS (H + I)				**102,929**	
K. SMALL BUSINESS FEE				**0**	
L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K)				**102,929**	

M. COST SHARING PROPOSED LEVEL $ **0**	AGREED LEVEL IF DIFFERENT $

PI/PD NAME **James Propp**	FOR NSF USE ONLY		
	INDIRECT COST RATE VERIFICATION		
ORG. REP. NAME* **Linda Concino**	Date Checked	Date Of Rate Sheet	Initials - ORG

3 ***ELECTRONIC SIGNATURES REQUIRED FOR REVISED BUDGET**

SUMMARY PROPOSAL BUDGET

Cumulative

ORGANIZATION University of Massachusetts Lowell				FOR NSF USE ONLY	
				PROPOSAL NO.	DURATION (months) Proposed / Granted
PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR James Propp				AWARD NO.	

A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets)	NSF Funded Person-months CAL	ACAD	SUMR	Funds Requested By proposer	Funds granted by NSF (if different)
1. James G Propp - PI	0.00	0.00	6.00	88,523	
2.					
3.					
4.					
5.					
6. ( ) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE)	0.00	0.00	0.00	0	
7. (1) TOTAL SENIOR PERSONNEL (1 - 6)	0.00	0.00	6.00	88,523	
B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS)					
1. (0) POST DOCTORAL SCHOLARS	0.00	0.00	0.00	0	
2. (0) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.)	0.00	0.00	0.00	0	
3. (3) GRADUATE STUDENTS				64,909	
4. (6) UNDERGRADUATE STUDENTS				14,836	
5. (0) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY)				0	
6. (0) OTHER				0	
TOTAL SALARIES AND WAGES (A + B)				168,268	
C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS)				5,684	
TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C)				173,952	
D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.)					
TOTAL EQUIPMENT				0	
E. TRAVEL 1. DOMESTIC (INCL. U.S. POSSESSIONS)				4,500	
2. FOREIGN				4,500	
F. PARTICIPANT SUPPORT COSTS					
1. STIPENDS $ 0					
2. TRAVEL 0					
3. SUBSISTENCE 0					
4. OTHER 0					
TOTAL NUMBER OF PARTICIPANTS (0) TOTAL PARTICIPANT COSTS				0	
G. OTHER DIRECT COSTS					
1. MATERIALS AND SUPPLIES				900	
2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION				0	
3. CONSULTANT SERVICES				3,000	
4. COMPUTER SERVICES				0	
5. SUBAWARDS				0	
6. OTHER				15,000	
TOTAL OTHER DIRECT COSTS				18,900	
H. TOTAL DIRECT COSTS (A THROUGH G)				201,852	
I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE)					
TOTAL INDIRECT COSTS (F&A)				99,032	
J. TOTAL DIRECT AND INDIRECT COSTS (H + I)				300,884	
K. SMALL BUSINESS FEE				0	
L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K)				300,884	

M. COST SHARING PROPOSED LEVEL $ 0	AGREED LEVEL IF DIFFERENT $		
PI/PD NAME James Propp	FOR NSF USE ONLY		
	INDIRECT COST RATE VERIFICATION		
ORG. REP. NAME* Linda Concino	Date Checked	Date Of Rate Sheet	Initials - ORG

C *ELECTRONIC SIGNATURES REQUIRED FOR REVISED BUDGET

Budget Justification for UMass Lowell

Senior Personnel

Senior Personnel salary is for 2 months per summer each year. Salary in Year 1 is assumed to be 3% above the Lead PI's 2015–2016 salary, with a subsequent estimated increase of 3% from Year 1 to Year 2 and from Year 2 to Year 3.

B. Other Personnel

The Lead PI will hire two sorts of students under the grant:

- Graduate students (all three years): One RA appointment for two semesters each year with a 3% increase in years 2 and 3 of the proposal. These graduate students (possibly including doctoral students from the department of computer science pursuing the computational mathematics option) would conduct research on more advanced topics, and would help supervise the URAs (see below).
- Undergraduate research assistants (URAs): 2 students in each year, paid at a student hourly rate ($12/hour), limited to $2,400 per student per year. These students would engage in collaborative research on problems whose solutions are not known in advance and which are germane to the Lead PI's research on dynamical algebraic combinatorics. It might be appropriate to fund these students via an REU Supplement rather than pay them as technicians, given the nature of the work. Other students would focus on coding and would create high-performance software to be used as research tools and easy-to-use applets to spread the group's results via the World Wide Web.

Fringe Benefits

Fringe rates are as follows in year 1. Faculty, Undergraduate Students, and Graduate Students during the summer: 1.65%. Graduate Students during the academic year: 7.65%.

Fringe rates in years 2 and 3 are based on a projected .5% increase in each year of proposal.

Travel

The Lead PI requests coverage of transportation and subsistence for attendance and participation of Senior Personnel at scientific conferences in the U.S. and Canada, and collaboration with colleagues at other institutions. This request includes student travel as well. The Lead PI also requests coverage for a small amount of foreign travel, to pay for trips to conference meetings in Europe or Asia during each year of proposal. For example, it is likely that the conference series "Algebra, Combinatorics, Dynamics and Applications" that has been held in Europe on and off for several years will resume again at some point; it would be beneficial for the Lead PI and other participants in the project to attend.

Other Direct Costs

Materials and Supplies

Major expenses are not anticipated, but minor expenses seem likely, so a small allocation has been made.

Consultant Services

In each of the three years, the Lead PI expects to invite collaborators to visit him in Massachusetts. The Lead PI's closest collaborators on this project (leaving aside Roby, who is local) are likely to be Lionel Levine (Cornell), Jessica Striker (NDSU), and Nathan Williams (UQAM). Additionally, consultations regarding software may be necessary.

Other Funds in the amount of $5,000 per year are requested to cover a portion of the tuition/fees for the graduate student. The remainder is paid for by the University.

Indirect costs

The federally negotiated indirect cost rate is 53% of modified total direct costs excluding equipment and tuition.

SUMMARY PROPOSAL BUDGET

YEAR 1

ORGANIZATION **University of Connecticut**				FOR NSF USE ONLY PROPOSAL NO.	DURATION (months) Proposed	Granted
PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR **Thomas Roby**				AWARD NO.		
A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets)	NSF Funded Person-months CAL	ACAD	SUMR	Funds Requested By proposer	Funds granted by NSF (if different)	
1. **Thomas Roby - PI**	0.00	0.00	1.00	15,670		
2.						
3.						
4.						
5.						
6. (0) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE)	0.00	0.00	0.00	0		
7. (1) TOTAL SENIOR PERSONNEL (1 - 6)	0.00	0.00	1.00	15,670		
B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS)						
1. (0) POST DOCTORAL SCHOLARS	0.00	0.00	0.00	0		
2. (0) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.)	0.00	0.00	0.00	0		
3. (1) GRADUATE STUDENTS				31,205		
4. (2) UNDERGRADUATE STUDENTS				4,800		
5. (0) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY)				0		
6. (0) OTHER				0		
TOTAL SALARIES AND WAGES (A + B)				51,675		
C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS)				10,498		
TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C)				62,173		
D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.)						
TOTAL EQUIPMENT				0		
E. TRAVEL 1. DOMESTIC (INCL. U.S. POSSESSIONS)				5,000		
2. FOREIGN				4,500		
F. PARTICIPANT SUPPORT COSTS 1. STIPENDS $ 0 2. TRAVEL 0 3. SUBSISTENCE 0 4. OTHER 0						
TOTAL NUMBER OF PARTICIPANTS (0) TOTAL PARTICIPANT COSTS				0		
G. OTHER DIRECT COSTS						
1. MATERIALS AND SUPPLIES				7,000		
2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION				0		
3. CONSULTANT SERVICES				0		
4. COMPUTER SERVICES				0		
5. SUBAWARDS				0		
6. OTHER				8,285		
TOTAL OTHER DIRECT COSTS				15,285		
H. TOTAL DIRECT COSTS (A THROUGH G)				86,958		
I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE) **Modified Total Direct Costs (MTDC) (Rate: 58.0000, Base: 78673)**						
TOTAL INDIRECT COSTS (F&A)				45,630		
J. TOTAL DIRECT AND INDIRECT COSTS (H + I)				132,588		
K. SMALL BUSINESS FEE				0		
L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K)				132,588		
M. COST SHARING PROPOSED LEVEL $ 0	AGREED LEVEL IF DIFFERENT $					

PI/PD NAME **Thomas Roby**	FOR NSF USE ONLY INDIRECT COST RATE VERIFICATION		
ORG. REP. NAME* **Joni Gould**	Date Checked	Date Of Rate Sheet	Initials - ORG

1 *ELECTRONIC SIGNATURES REQUIRED FOR REVISED BUDGET

1600661

SUMMARY PROPOSAL BUDGET

YEAR 2

FOR NSF USE ONLY		
PROPOSAL NO.	DURATION (months)	
	Proposed	Granted
AWARD NO.		

ORGANIZATION
University of Connecticut

PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR
Thomas Roby

A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets)	NSF Funded Person-months CAL	ACAD	SUMR	Funds Requested By proposer	Funds granted by NSF (if different)
1. Thomas Roby - PI	0.00	0.00	1.00	16,454	
2.					
3.					
4.					
5.					
6. (0) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE)	0.00	0.00	0.00	0	
7. (1) TOTAL SENIOR PERSONNEL (1 - 6)	0.00	0.00	1.00	16,454	
B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS)					
1. (0) POST DOCTORAL SCHOLARS	0.00	0.00	0.00	0	
2. (0) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.)	0.00	0.00	0.00	0	
3. (1) GRADUATE STUDENTS				32,141	
4. (2) UNDERGRADUATE STUDENTS				4,800	
5. (0) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY)				0	
6. (0) OTHER				0	
TOTAL SALARIES AND WAGES (A + B)				53,395	
C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS)				11,568	
TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C)				64,963	
D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.)					
TOTAL EQUIPMENT				0	
E. TRAVEL 1. DOMESTIC (INCL. U.S. POSSESSIONS)				5,000	
2. FOREIGN				4,500	
F. PARTICIPANT SUPPORT COSTS 1. STIPENDS $ 0 2. TRAVEL 0 3. SUBSISTENCE 0 4. OTHER 0					
TOTAL NUMBER OF PARTICIPANTS (0) TOTAL PARTICIPANT COSTS				0	
G. OTHER DIRECT COSTS					
1. MATERIALS AND SUPPLIES				6,500	
2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION				0	
3. CONSULTANT SERVICES				0	
4. COMPUTER SERVICES				0	
5. SUBAWARDS				0	
6. OTHER				8,782	
TOTAL OTHER DIRECT COSTS				15,282	
H. TOTAL DIRECT COSTS (A THROUGH G)				89,745	
I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE) **Modified Total Direct Costs (MTDC) (Rate: 58.0000, Base: 80963)**					
TOTAL INDIRECT COSTS (F&A)				46,959	
J. TOTAL DIRECT AND INDIRECT COSTS (H + I)				136,704	
K. SMALL BUSINESS FEE				0	
L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K)				136,704	

M. COST SHARING PROPOSED LEVEL $ 0 AGREED LEVEL IF DIFFERENT $

PI/PD NAME
Thomas Roby

ORG. REP. NAME*
Joni Gould

FOR NSF USE ONLY		
INDIRECT COST RATE VERIFICATION		
Date Checked	Date Of Rate Sheet	Initials - ORG

 *ELECTRONIC SIGNATURES REQUIRED FOR REVISED BUDGET

SUMMARY PROPOSAL BUDGET

YEAR 3

ORGANIZATION **University of Connecticut**				FOR NSF USE ONLY PROPOSAL NO.	DURATION (months) Proposed	Granted
PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR **Thomas Roby**				AWARD NO.		
A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets)	NSF Funded Person-months CAL	ACAD	SUMR	Funds Requested By proposer	Funds granted by NSF (if different)	
1. **Thomas Roby - PI**	0.00	0.00	1.00	17,277		
2.						
3.						
4.						
5.						
6. (0) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE)	0.00	0.00	0.00	0		
7. (1) TOTAL SENIOR PERSONNEL (1 - 6)	0.00	0.00	1.00	17,277		
B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS)						
1. (0) POST DOCTORAL SCHOLARS	0.00	0.00	0.00	0		
2. (0) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.)	0.00	0.00	0.00	0		
3. (1) GRADUATE STUDENTS				33,105		
4. (2) UNDERGRADUATE STUDENTS				4,800		
5. (0) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY)				0		
6. (0) OTHER				0		
TOTAL SALARIES AND WAGES (A + B)				55,182		
C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS)				12,554		
TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C)				67,736		
D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.)						
TOTAL EQUIPMENT				0		
E. TRAVEL 1. DOMESTIC (INCL. U.S. POSSESSIONS)				5,000		
2. FOREIGN				4,500		
F. PARTICIPANT SUPPORT COSTS 1. STIPENDS $ 0 2. TRAVEL 0 3. SUBSISTENCE 0 4. OTHER 0						
TOTAL NUMBER OF PARTICIPANTS (0) TOTAL PARTICIPANT COSTS				0		
G. OTHER DIRECT COSTS						
1. MATERIALS AND SUPPLIES				6,500		
2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION				0		
3. CONSULTANT SERVICES				0		
4. COMPUTER SERVICES				0		
5. SUBAWARDS				0		
6. OTHER				9,309		
TOTAL OTHER DIRECT COSTS				15,809		
H. TOTAL DIRECT COSTS (A THROUGH G)				93,045		
I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE) **Modified Total Direct Costs (MTDC) (Rate: 58.0000, Base: 83736)**						
TOTAL INDIRECT COSTS (F&A)				48,567		
J. TOTAL DIRECT AND INDIRECT COSTS (H + I)				141,612		
K. SMALL BUSINESS FEE				0		
L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K)				141,612		
M. COST SHARING PROPOSED LEVEL $ 0	AGREED LEVEL IF DIFFERENT $					
PI/PD NAME **Thomas Roby**				FOR NSF USE ONLY INDIRECT COST RATE VERIFICATION		
ORG. REP. NAME* **Joni Gould**				Date Checked	Date Of Rate Sheet	Initials - ORG

3 ***ELECTRONIC SIGNATURES REQUIRED FOR REVISED BUDGET**

1600661

SUMMARY PROPOSAL BUDGET

Cumulative

ORGANIZATION University of Connecticut	FOR NSF USE ONLY PROPOSAL NO.	DURATION (months) Proposed	Granted
PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR Thomas Roby	AWARD NO.		

A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets)	NSF Funded Person-months CAL	ACAD	SUMR	Funds Requested By proposer	Funds granted by NSF (if different)
1. Thomas Roby - PI	0.00	0.00	3.00	49,401	
2.					
3.					
4.					
5.					
6. ( ) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE)	0.00	0.00	0.00	0	
7. (1) TOTAL SENIOR PERSONNEL (1 - 6)	0.00	0.00	3.00	49,401	
B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS)					
1. (0) POST DOCTORAL SCHOLARS	0.00	0.00	0.00	0	
2. (0) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.)	0.00	0.00	0.00	0	
3. (3) GRADUATE STUDENTS				96,451	
4. (6) UNDERGRADUATE STUDENTS				14,400	
5. (0) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY)				0	
6. (0) OTHER				0	
TOTAL SALARIES AND WAGES (A + B)				160,252	
C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS)				34,620	
TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C)				194,872	
D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.)					
TOTAL EQUIPMENT				0	
E. TRAVEL 1. DOMESTIC (INCL. U.S. POSSESSIONS)				15,000	
2. FOREIGN				13,500	
F. PARTICIPANT SUPPORT COSTS 1. STIPENDS $ 0 2. TRAVEL 0 3. SUBSISTENCE 0 4. OTHER 0					
TOTAL NUMBER OF PARTICIPANTS (0) TOTAL PARTICIPANT COSTS				0	
G. OTHER DIRECT COSTS					
1. MATERIALS AND SUPPLIES				20,000	
2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION				0	
3. CONSULTANT SERVICES				0	
4. COMPUTER SERVICES				0	
5. SUBAWARDS				0	
6. OTHER				26,376	
TOTAL OTHER DIRECT COSTS				46,376	
H. TOTAL DIRECT COSTS (A THROUGH G)				269,748	
I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE)					
TOTAL INDIRECT COSTS (F&A)				141,156	
J. TOTAL DIRECT AND INDIRECT COSTS (H + I)				410,904	
K. SMALL BUSINESS FEE				0	
L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K)				410,904	

M. COST SHARING PROPOSED LEVEL $ 0 AGREED LEVEL IF DIFFERENT $

PI/PD NAME Thomas Roby	FOR NSF USE ONLY INDIRECT COST RATE VERIFICATION		
ORG. REP. NAME* Joni Gould	Date Checked	Date Of Rate Sheet	Initials - ORG

C *ELECTRONIC SIGNATURES REQUIRED FOR REVISED BUDGET

A. Salaries and Wages -- Senior Personnel

The Principal Investigator, Dr. Thomas Roby, will devote 1 summer month per year throughout the three-year project period. This will involve working on important open research problems, and supervising the work of graduate students and other collaborators involved with the project.

B. Salaries and Wages-Other Personnel

Graduate Assistant academic and summer support is requested for all three years. Supported graduate students will be expected to devote 20 hours per week to working on research problems. This will include meeting regularly with the PI and other collaborators, writing computer programs, gathering data, proving results, and disseminating them through scholarly publications and presentations. Talented undergraduates will be brought into the research group as computer programmers and transitioned into larger research roles as their mathematical maturity and background knowledge increase.

NOTE: A 20-hour, 9-month UCONN graduate assistant is considered full-time for the academic year.

C. Fringe Benefits

Fringe Benefits are calculated as direct costs in accordance with University of Connecticut's indirect cost rate agreement. Current rates are for senior personnel 27.9% for year 1, 29% for year 2 and 30% for year 3. For the Graduate Assistant 22.9% for year 1, 24% for year 2 and 25% for year 3 during academic year; and 7.3% for first summer, 9% and 10% for second and third summer; 4.1%, 6.0% and 7.0% for student labor.

D. Equipment: Not Applicable.

E. Travel

E1. Domestic Travel: A total of $15,000 is requested for the PI and graduate students to travel to partner institutions to work with collaborators and travel to domestic conferences. These include AMS conferences held both regionally and nationally, and workshops held at various mathematics institutes and departments. (Specific locations and times vary from year to year.) Likely domestic trips would include visits to the University of Minnesota, UC Davis, the University of Michigan, and the University of Kentucky, all of which have several researchers whose collaboration would be invaluable to furthering the project. All travel costs will be consistent with all state and federal guidelines in effect at the time the trips are taken.

E2. Foreign Travel: A total of $13,500 is requested for the PI and graduate students to travel to foreign institutions to work with collaborators and to international conferences, such as the annual conferences in Formal Power Series and Algebraic Combinatorics (held in a different international location each year). A number of the researchers working in this area are at foreign institutions, where in-person collaborations would be the most effective means of furthering the research goals. Examples include, Soichi Okada in Nagoya, Roger Behrend in Cardiff, Christian Krattenthaler in Vienna, among many others. All travel costs will be consistent with state and federal guidelines in effect at the time the trips are taken.

F. Participant Support Costs: Not Applicable.

G. Other Direct Costs

1. Materials and Supplies: Materials needed are primarily books on specialized research topics. Funds are requested for a laptop computer, sufficient to handle the kinds of large computations that normally arise in problems in this area. Since this grant's proposed research involved significant computations with large discrete dynamical systems, having an up-to-date portable computer is mission-critical. Often when running computations, the computer will be slow or unavailable for the PIs' routine work, which will be done on other computers. Extreme care will be taken to insure that the computer remains with the PIs, and is safeguarded against theft. Other equipment needs include backup and storage media, which are important for insuring that data is preserved even in the event of hardware failure.

2. Publication Costs/Documentation/Dissemination: Not Applicable

3. Consultant Services: Not Applicable

4. Computer Services: Not Applicable

5. Subawards: Not Applicable

6. Other: Tuition

As per the University of Connecticut's policy, 60% of in-state tuition fees is requested for the 20-hour Graduate Assistant, with a 6% increase for subsequent years.

I. Indirect Costs

Indirect Costs are calculated in accordance with University of Connecticut's federally negotiated indirect cost rate agreement, which is currently 58% of modified total direct costs (MTDC), as per the DHHS letter dated February 6, 2015.

Current and Pending Support

(See GPG Section II.C.2.h for guidance on information to include on this form.)

The following information should be provided for each investigator and other senior personnel. Failure to provide this information may delay consideration of this proposal.

Investigator: James Propp

Other agencies (including NSF) to which this proposal has been/will be submitted.

Support: ☒ Current ☐ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support
Project/Proposal Title: Deterministic analogues of random processes

Source of Support: National Science Foundation
Total Award Amount: $ 200,000 Total Award Period Covered: 07/01/10 - 06/30/16
Location of Project: University of Massachusetts Lowell
Person-Months Per Year Committed to the Project. Cal:0.00 Acad: 0.00 Sumr: 1.00

Support: ☐ Current ☒ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support
Project/Proposal Title: Collaborative Research: Dynamical Algebraic Combinatorics (This Proposal)

Source of Support: National Science Foundation
Total Award Amount: $ 300,884 Total Award Period Covered: 07/01/16 - 06/30/19
Location of Project: University of Massachusetts Lowell
Person-Months Per Year Committed to the Project. Cal:0.00 Acad: 0.00 Sumr: 2.00

Support: ☐ Current ☐ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support
Project/Proposal Title:

Source of Support:
Total Award Amount: $ Total Award Period Covered:
Location of Project:
Person-Months Per Year Committed to the Project. Cal: Acad: Sumr:

Support: ☐ Current ☐ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support
Project/Proposal Title:

Source of Support:
Total Award Amount: $ Total Award Period Covered:
Location of Project:
Person-Months Per Year Committed to the Project. Cal: Acad: Sumr:

Support: ☐ Current ☐ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support
Project/Proposal Title:

Source of Support:
Total Award Amount: $ Total Award Period Covered:
Location of Project:
Person-Months Per Year Committed to the Project. Cal: Acad: Summ:

*If this project has previously been funded by another agency, please list and furnish information for immediately preceding funding period.

Current and Pending Support

(See GPG Section II.C.2.h for guidance on information to include on this form.)

The following information should be provided for each investigator and other senior personnel. Failure to provide this information may delay consideration of this proposal.

Investigator: Thomas Roby

Other agencies (including NSF) to which this proposal has been/will be submitted.

Support: ☐ Current ☒ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support

Project/Proposal Title: This Proposal: Collaborative Research: Dynamical Algebraic Combinatorics

Source of Support: NSF: Combinatorics

Total Award Amount: $ 410,904 Total Award Period Covered: 07/01/16 - 06/30/19

Location of Project: University of Connecticut

Person-Months Per Year Committed to the Project. Cal:0.00 Acad: 0.00 Sumr: 1.00

Support: ☐ Current ☒ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support

Project/Proposal Title: Collaborative Research: Statistical mechanics in algebraic combinatorics and representation theory using Sage

Source of Support: NSF: Combinatorics

Total Award Amount: $ 726,406 Total Award Period Covered: 07/01/16 - 06/30/19

Location of Project: University of Connecticut

Person-Months Per Year Committed to the Project. Cal:0.00 Acad: 0.00 Sumr: 1.00

Support: ☐ Current ☐ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support

Project/Proposal Title:

Source of Support:

Total Award Amount: $ Total Award Period Covered:

Location of Project:

Person-Months Per Year Committed to the Project. Cal: Acad: Sumr:

Support: ☐ Current ☐ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support

Project/Proposal Title:

Source of Support:

Total Award Amount: $ Total Award Period Covered:

Location of Project:

Person-Months Per Year Committed to the Project. Cal: Acad: Sumr:

Support: ☐ Current ☐ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support

Project/Proposal Title:

Source of Support:

Total Award Amount: $ Total Award Period Covered:

Location of Project:

Person-Months Per Year Committed to the Project. Cal: Acad: Summ:

*If this project has previously been funded by another agency, please list and furnish information for immediately preceding funding period.

FACILITIES

FACILITIES: Identify the facilities to be used at each performance site listed and, as appropriate, indicate their capacities, pertinent capabilities, relative proximity, and extent of availability to the project. Use "Other" to describe the facilities at any other performance sites listed and at sites for field studies. Use additional pages if necessary.

Laboratory: **N/A**

Clinical: **N/A**

Animal: **N/A**

Computer: **MacBook Pro**

Office: **N/A**

Other: **N/A**

MAJOR EQUIPMENT: List the most important items available for this project and, as appropriate, identify the location and pertinent capabilities of each.

N/A

OTHER RESOURCES: Provide any information describing the other resources available for the project. Identify support services such as consultant, secretarial, machine shop, and electronics shop, and the extent to which they will be available for the project. Include an explanation of any consortium/contractual/subaward arrangements with other organizations.

N/A

Facilities, Equipment and other Resources

Not Applicable

DATA MANAGEMENT PLAN

I. Data Description

This project in pure mathematics will not generate scientific data. The product will consist of research articles to which the PI and his students will contribute (PDFs) and a small website describing the group's work. These will be hosted on the faculty server.

II. Access and Sharing

All articles will be posted on the arXiv, and those that are published in electronic journals will also be available from the websites of those journals. In addition, the PI will keep copies of the articles on his individual website.

III. Metadata

There will be no metadata associated with this project.

IV. Intellectual Property Rights

The data will be freely provided to the scientific community for non-commercial purposes so long as credit is given; the PIs will retain the rights to the data when used for commercial purposes, although it is not anticipated that any of the ideas that the PI will explore could have any commercial application.

V. Preservation Infrastructure

Given the minimal storage requirements of the grant, no special provisions for preservation infrastructure need be made.