02 INFORMATION ABOUT PRINCIPAL INVESTIGATORS/PROJECT DIRECTORS(PI/PD) and co-PRINCIPAL INVESTIGATORS/co-PROJECT DIRECTORS

Submit only ONE copy of this form **for each PI/PD and co-PI/PD** identified on the proposal. The form(s) should be attached to the original proposal as specified in GPG Section II.B. Submission of this information is voluntary and is not a precondition of award. This information will not be disclosed to external peer reviewers. ***DO NOT INCLUDE THIS FORM WITH ANY OF THE OTHER COPIES OF YOUR PROPOSAL AS THIS MAY COMPROMISE THE CONFIDENTIALITY OF THE INFORMATION.***

PI/PD Name: James G Propp

Gender: ☒ Male ☐ Female

Ethnicity: (Choose one response) ☐ Hispanic or Latino ☒ Not Hispanic or Latino

Race:
(Select one or more)
- ☐ American Indian or Alaska Native
- ☐ Asian
- ☐ Black or African American
- ☐ Native Hawaiian or Other Pacific Islander
- ☒ White

Disability Status:
(Select one or more)
- ☐ Hearing Impairment
- ☐ Visual Impairment
- ☐ Mobility/Orthopedic Impairment
- ☐ Other
- ☒ None

Citizenship: (Choose one) ☒ U.S. Citizen ☐ Permanent Resident ☐ Other non-U.S. Citizen

Check here if you do not wish to provide any or all of the above information (excluding PI/PD name): ☒

REQUIRED: Check here if you are currently serving (or have previously served) as a PI, co-PI or PD on any federally funded project ☒

Ethnicity Definition:
Hispanic or Latino. A person of Mexican, Puerto Rican, Cuban, South or Central American, or other Spanish culture or origin, regardless of race.
Race Definitions:
American Indian or Alaska Native. A person having origins in any of the original peoples of North and South America (including Central America), and who maintains tribal affiliation or community attachment.
Asian. A person having origins in any of the original peoples of the Far East, Southeast Asia, or the Indian subcontinent including, for example, Cambodia, China, India, Japan, Korea, Malaysia, Pakistan, the Philippine Islands, Thailand, and Vietnam.
Black or African American. A person having origins in any of the black racial groups of Africa.
Native Hawaiian or Other Pacific Islander. A person having origins in any of the original peoples of Hawaii, Guam, Samoa, or other Pacific Islands.
White. A person having origins in any of the original peoples of Europe, the Middle East, or North Africa.

WHY THIS INFORMATION IS BEING REQUESTED:

The Federal Government has a continuing commitment to monitor the operation of its review and award processes to identify and address any inequities based on gender, race, ethnicity, or disability of its proposed PIs/PDs. To gather information needed for this important task, the proposer should submit a single copy of this form for each identified PI/PD with each proposal. Submission of the requested information is voluntary and will not affect the organization's eligibility for an award. However, information not submitted will seriously undermine the statistical validity, and therefore the usefulness, of information recieved from others. Any individual not wishing to submit some or all the information should check the box provided for this purpose. (The exceptions are the PI/PD name and the information about prior Federal support, the last question above.)

Collection of this information is authorized by the NSF Act of 1950, as amended, 42 U.S.C. 1861, et seq. Demographic data allows NSF to gauge whether our programs and other opportunities in science and technology are fairly reaching and benefiting everyone regardless of demographic category; to ensure that those in under-represented groups have the same knowledge of and access to programs and other research and educational oppurtunities; and to assess involvement of international investigators in work supported by NSF. The information may be disclosed to government contractors, experts, volunteers and researchers to complete assigned work; and to other government agencies in order to coordinate and assess programs. The information may be added to the Reviewer file and used to select potential candidates to serve as peer reviewers or advisory committee members. See Systems of Records, NSF-50, "Principal Investigator/Proposal File and Associated Records", 63 Federal Register 267 (January 5, 1998), and NSF-51, "Reviewer/Proposal File and Associated Records", 63 Federal Register 268 (January 5, 1998).

List of Suggested Reviewers or Reviewers Not To Include (optional)

SUGGESTED REVIEWERS:

Some researchers who would be good referees for this proposal are David Aldous, Norman Biggs, Persi Diaconis, James Fill, David Griffeath, Gregory Lawler, Laszlo Lovasz, Russell Lyons, Oded Schramm, and Alistair Sinclair. Other people who might be even better referees are Joshua Cooper, Yuval Peres, Joel Spencer, and Peter Winkler; however, insofar as I have already had extensive conversations with them on the subject of quasirandomness and rotor-routers, they might be considered "collaborators" in the broadest sense of the term. I trust that that NSF will apply its standard policy (whatever it is) in these borderline cases.

REVIEWERS NOT TO INCLUDE:

The author of Review #2 (of my proposal from a year ago) seemed to have trouble understanding the proposal; he/she thought it had something to do with large deviation estimates, which is nearly as wrong as could be.

Corrected : 10/26/2005

COVER SHEET FOR PROPOSAL TO THE NATIONAL SCIENCE FOUNDATION

PROGRAM ANNOUNCEMENT/SOLICITATION NO./CLOSING DATE/if not in response to a program announcement/solicitation enter NSF 04-23 **NSF 04-23**	FOR NSF USE ONLY NSF PROPOSAL NUMBER
FOR CONSIDERATION BY NSF ORGANIZATION UNIT(S) (Indicate the most specific unit known, i.e. program, division, etc.) **DMS - PROBABILITY, DMS - ALGEBRA,NUMBER THEORY,AND COM**	**0556095**

DATE RECEIVED	NUMBER OF COPIES	DIVISION ASSIGNED	FUND CODE	DUNS# (Data Universal Numbering System)	FILE LOCATION
09/30/2005	2	03040000 DMS	1263	161202122	11/03/2005 7:03pm S

EMPLOYER IDENTIFICATION NUMBER (EIN) OR TAXPAYER IDENTIFICATION NUMBER (TIN) **396006492**	SHOW PREVIOUS AWARD NO. IF THIS IS ☐ A RENEWAL ☐ AN ACCOMPLISHMENT-BASED RENEWAL	IS THIS PROPOSAL BEING SUBMITTED TO ANOTHER FEDERAL AGENCY? YES ☒ NO ☐ IF YES, LIST ACRONYM(S) **NSA**

NAME OF ORGANIZATION TO WHICH AWARD SHOULD BE MADE **University of Wisconsin-Madison**	ADDRESS OF AWARDEE ORGANIZATION, INCLUDING 9 DIGIT ZIP CODE **750 University Ave** **Madison, WI 53706-1490**
AWARDEE ORGANIZATION CODE (IF KNOWN) **0038950000**	
NAME OF PERFORMING ORGANIZATION, IF DIFFERENT FROM ABOVE	ADDRESS OF PERFORMING ORGANIZATION, IF DIFFERENT, INCLUDING 9 DIGIT ZIP CODE
PERFORMING ORGANIZATION CODE (IF KNOWN)	

IS AWARDEE ORGANIZATION (Check All That Apply) (See GPG II.C For Definitions) ☐ SMALL BUSINESS ☐ FOR-PROFIT ORGANIZATION ☐ MINORITY BUSINESS ☐ WOMAN-OWNED BUSINESS ☐ IF THIS IS A PRELIMINARY PROPOSAL THEN CHECK HERE

TITLE OF PROPOSED PROJECT **Quasirandomness in Discrete Probability Theory**

REQUESTED AMOUNT	PROPOSED DURATION (1-60 MONTHS)	REQUESTED STARTING DATE	SHOW RELATED PRELIMINARY PROPOSAL NO. IF APPLICABLE
$ 309,445	36 months	07/01/06	

CHECK APPROPRIATE BOX(ES) IF THIS PROPOSAL INCLUDES ANY OF THE ITEMS LISTED BELOW

☐ BEGINNING INVESTIGATOR (GPG I.A)
☐ DISCLOSURE OF LOBBYING ACTIVITIES (GPG II.C)
☐ PROPRIETARY & PRIVILEGED INFORMATION (GPG I.B, II.C.1.d)
☐ HISTORIC PLACES (GPG II.C.2.j)
☐ SMALL GRANT FOR EXPLOR. RESEARCH (SGER) (GPG II.D.1)
☐ VERTEBRATE ANIMALS (GPG II.D.5) IACUC App. Date ______
☐ HUMAN SUBJECTS (GPG II.D.6)
Exemption Subsection ______ or IRB App. Date ______
☐ INTERNATIONAL COOPERATIVE ACTIVITIES: COUNTRY/COUNTRIES INVOLVED (GPG II.C.2.j)
______
☐ HIGH RESOLUTION GRAPHICS/OTHER GRAPHICS WHERE EXACT COLOR REPRESENTATION IS REQUIRED FOR PROPER INTERPRETATION (GPG I.G.1)

PI/PD DEPARTMENT **Mathematics**	PI/PD POSTAL ADDRESS **813 Van Vleck Hall** **Madison, WI 53706** **United States**
PI/PD FAX NUMBER **608-263-8891**	

NAMES (TYPED)	High Degree	Yr of Degree	Telephone Number	Electronic Mail Address
PI/PD NAME **James G Propp**	PhD	1987	608-263-5148	propp@math.wisc.edu
CO-PI/PD				
CO-PI/PD				
CO-PI/PD				
CO-PI/PD				

Electronic Signature

CERTIFICATION PAGE

Certification for Authorized Organizational Representative or Individual Applicant:

By signing and submitting this proposal, the individual applicant or the authorized official of the applicant institution is: (1) certifying that statements made herein are true and complete to the best of his/her knowledge; and (2) agreeing to accept the obligation to comply with NSF award terms and conditions if an award is made as a result of this application. Further, the applicant is hereby providing certifications regarding debarment and suspension, drug-free workplace, and lobbying activities (see below), as set forth in Grant Proposal Guide (GPG), NSF 04-23. Willful provision of false information in this application and its supporting documents or in reports required under an ensuing award is a criminal offense (U. S. Code, Title 18, Section 1001).

In addition, if the applicant institution employs more than fifty persons, the authorized official of the applicant institution is certifying that the institution has implemented a written and enforced conflict of interest policy that is consistent with the provisions of Grant Policy Manual Section 510; that to the best of his/her knowledge, all financial disclosures required by that conflict of interest policy have been made; and that all identified conflicts of interest will have been satisfactorily managed, reduced or eliminated prior to the institution's expenditure of any funds under the award, in accordance with the institution's conflict of interest policy. Conflicts which cannot be satisfactorily managed, reduced or eliminated must be disclosed to NSF.

Drug Free Work Place Certification

By electronically signing the NSF Proposal Cover Sheet, the Authorized Organizational Representative or Individual Applicant is providing the Drug Free Work Place Certification contained in Appendix C of the Grant Proposal Guide.

Debarment and Suspension Certification

(If answer "yes", please provide explanation.)

Is the organization or its principals presently debarred, suspended, proposed for debarment, declared ineligible, or voluntarily excluded from covered transactions by any Federal department or agency? Yes ☐ No ☒

By electronically signing the NSF Proposal Cover Sheet, the Authorized Organizational Representative or Individual Applicant is providing the Debarment and Suspension Certification contained in Appendix D of the Grant Proposal Guide.

Certification Regarding Lobbying

This certification is required for an award of a Federal contract, grant, or cooperative agreement exceeding $100,000 and for an award of a Federal loan or a commitment providing for the United States to insure or guarantee a loan exceeding $150,000.

Certification for Contracts, Grants, Loans and Cooperative Agreements

The undersigned certifies, to the best of his or her knowledge and belief, that:

(1) No federal appropriated funds have been paid or will be paid, by or on behalf of the undersigned, to any person for influencing or attempting to influence an officer or employee of any agency, a Member of Congress, an officer or employee of Congress, or an employee of a Member of Congress in connection with the awarding of any federal contract, the making of any Federal grant, the making of any Federal loan, the entering into of any cooperative agreement, and the extension, continuation, renewal, amendment, or modification of any Federal contract, grant, loan, or cooperative agreement.

(2) If any funds other than Federal appropriated funds have been paid or will be paid to any person for influencing or attempting to influence an officer or employee of any agency, a Member of Congress, an officer or employee of Congress, or an employee of a Member of Congress in connection with this Federal contract, grant, loan, or cooperative agreement, the undersigned shall complete and submit Standard Form-LLL, ''Disclosure of Lobbying Activities,'' in accordance with its instructions.

(3) The undersigned shall require that the language of this certification be included in the award documents for all subawards at all tiers including subcontracts, subgrants, and contracts under grants, loans, and cooperative agreements and that all subrecipients shall certify and disclose accordingly.

This certification is a material representation of fact upon which reliance was placed when this transaction was made or entered into. Submission of this certification is a prerequisite for making or entering into this transaction imposed by section 1352, Title 31, U.S. Code. Any person who fails to file the required certification shall be subject to a civil penalty of not less than $10,000 and not more than $100,000 for each such failure.

AUTHORIZED ORGANIZATIONAL REPRESENTATIVE	SIGNATURE	DATE
NAME **Linda Johnson**	**Electronic Signature**	**Sep 30 2005 2:08PM**

TELEPHONE NUMBER	ELECTRONIC MAIL ADDRESS	FAX NUMBER
608-265-9655	**johnson@ls.admin.wisc.edu**	**608-265-2275**

*SUBMISSION OF SOCIAL SECURITY NUMBERS IS VOLUNTARY AND WILL NOT AFFECT THE ORGANIZATION'S ELIGIBILITY FOR AN AWARD. HOWEVER, THEY ARE AN INTEGRAL PART OF THE INFORMATION SYSTEM AND ASSIST IN PROCESSING THE PROPOSAL. SSN SOLICITED UNDER NSF ACT OF 1950, AS AMENDED.

The PI proposes to conduct research on quasirandom analogues of discrete random systems. Many problems in discrete probability theory and its applications involve probabilities of events, or expected values of random variables, or limiting shapes, that are hard to determine analytically but can be estimated empirically using some sort of Monte Carlo simulation. In some cases, one can also get good estimates by using well-chosen deterministic simulations that mimic some but not all features of random simulation. Such quasirandom simulation schemes often are just as fast as random simulation, give smaller error, and give deterministic error-bounds rather than confidence-intervals. The PI's research will study such schemes, and will explore new ways of removing the "noise" from random systems while retaining key features of their average-case behavior, building on earlier work by the PI, Lionel Levine, Yuval Peres, Josh Cooper, and Joel Spencer, among others. The resulting non-random systems exhibit startling symmetry and unexpected structure and are worthy of study in their own right, quite aside from their applicability to answering questions about random systems.

The work will employ both computer-aided experimentation with different quasirandomization schemes and theoretical investigation of these schemes, using techniques from combinatorics, algebra, and analysis.

Part of the intellectual merit of the proposal lies in its foundational nature: the PI will be looking at some of the most basic ideas of probability theory, such as random walk, from a new perspective. This work will shed direct light on the relationship between local order and global order in spatially extended systems and may also shed indirect light on the nature of randomness itself. A further part of the intellectual merit of the proposal lies in its interdisciplinary character, with possibilities for bidirectional transfer of ideas between probability, combinatorics, computer science, and numerical analysis.

A broader impact of the proposed research is the inclusion of undergraduates as research assistants. The PI will tightly integrate education with research by training students in fundamental tools pertaining to discrete dynamical systems (both stochastic and deterministic) and then setting the trainees loose on unsolved problems. In so doing, the PI will develop the students' general skills in mathematical research, with the hope of encouraging many of them to become mathematicians or scientists (or just citizens with an appreciation of the nature of the scientific enterprise).

TABLE OF CONTENTS

For font size and page formatting specifications, see GPG section II.C.

*Proposers may select any numbering mechanism for the proposal. The entire proposal however, must be paginated. Complete both columns only if the proposal is numbered consecutively.

1 Results from Prior NSF Support

During the past five years, I have used funding from the National Science Foundation's probability program. The project was called "Research on tilings", and lasted from 1999 to 2002. The amount requested in the original budget was $370,202. That project has very little bearing on the current proposal, except insofar as my success in carrying out the project is evidence of my overall competence as a researcher. The following articles were written in pursuance of that grant:

Trees and matchings (with R. Kenyon and D. Wilson), *Electronic Journal of Combinatorics* **7**(1) (2000), R25: This article strengthened and generalized known links between enumeration of spanning trees and enumeration of (perfect) matchings, and in particular showed that many matching problems are equivalent to problems of counting matchings in *directed* graphs. (I should mention that many problems about tilings are easily rephrased as problems about matchings, and vice versa.)

A variational principle for domino tilings (with H. Cohn and R. Kenyon), *Journal of the American Mathematical Society* **14** (2001), 297–346: This article brought the fulfillment of a long-standing ambition of mine, namely, to develop a maximum entropy principle that can quantitatively account for the behavior of random tilings. Subsequent developments by Kenyon, Okounkov, and Sheffield have pushed these ideas much further.

A reciprocity theorem for domino tilings, *Electronic Journal of Combinatorics*, **8**(1) (2001), R18: This article is still a bit of an orphan, as it initiated a line of work that I have not had time to pursue very deeply (partly because my research proposal three years ago was not funded by NSF).

The many faces of alternating-sign matrices, *Discrete Mathematics and Theoretical Computer Science Proceedings* **(DM-CCG)** (2001), 43–58: This article was based on a talk I gave at a conference in Paris, growing out of work I did with the Tilings Research Group at MIT during its final year. This work helped popularize the burgeoning study of exact enumeration of states of the fully-packed loops model (starting with Razumov and Stroganov, 2000 and continuing with a half-dozen other articles by a half-dozen other researchers). Conjectures still vastly outnumber results; it is an exciting time in this sub-field, and I do not think anyone can predict when the right ideas will be found and the log-jam will break.

Generating a random sink-free orientation in quadratic time (with H. Cohn and R. Pemantle), *Electronic Journal of Combinatorics*, **9(1)** (2002), R10: I wrote this article because I wanted to get a better understanding of the power (and the limitations) of “partial rejection sampling”. Wilson’s algorithm for generating random trees via cycle-popping was the only example I had seen; the dual idea of sink-popping seemed a natural variant to explore. Nobody uses sink-popping, as far as I know. But if nothing else, this article served a purpose for the theoretical computer science community, by showing that one can sometimes do exact *sampling* in polynomial time even when the corresponding exact *counting* problem is #P-complete.

Generalized domino-shuffling, *Theoretical Computer Science* **303** (2003), 267–301: With this article I paid off an old debt, by revisiting my earliest work on tilings and making the formerly ad hoc domino-shuffling algorithm of Kuperberg and myself less “magical”. The generalized version of this algorithm linked the problems of enumeration and random sampling. I have hopes that someday analytic methods will be applied to the shuffling process itself, and that this will lead to new ways of understanding the sort of domain-boundary phenomena I wrote about in my random-tilings grant-proposal.

Additionally, my undergraduate student Hal Canary succeeded in proving one of my combinatorial conjectures in 2002; his article `math.CO/0309135` (Aztec diamonds and Baxter permutations) has been submitted for publication.

2 Proposed Research

In the past century a good deal of attention has been given to deterministic objects that “behave as if they were random”. (The most famous objects of this sort come from number theory, but the phenomenon is much more general; see e.g. the “deterministic central limit theorem” of Jozsef Beck describing the number of lattice points in a certain family of triangular regions, as described in Matousek, 1999). There is a need for a flexible kind of probability theory whose theorems will draw their inspiration from traditional probability theory but will have weaker hypotheses, replacing assumptions of randomness by assumptions that deal with individual instances rather than probability distributions.

The proposed research will lay the foundations for one such variation on probability theory, and will show that in some contexts (notably random walk and random aggregation), a notion of quasirandomness based on frequency and discrepancy suffices to ensure the occurrence of “random behavior”. Furthermore, this work will show that, by choosing methods of non-random simulation that focus on reducing discrepancy, one can sometimes get estimates of the average case behavior of a system that are much more precise than estimates obtained via ordinary (i.e., purely random) Monte Carlo simulation. Specifically, instead of giving a confidence interval of width $O(1/\sqrt{N})$ (where N is the number of trials) for some quantity of interest, quasirandom simulation, when applicable, will typically give a *certainty interval* of width $O(1/N)$ or $O((\log N)^d/N)$ for some small exponent d.

The philosophy underlying the discrepancy-based notion of quasirandomness is easiest to explain via a trivial example, namely a particle doing random walk on the graph K_3, which we think of as being imbedded in the plane. Imagine that each of the three vertices is associated with an infinite stream of bits, and that when a randomly-walking particle arrives at a particular vertex for the nth time, it decides whether to jump clockwise or counterclockwise according to whether the nth bit of the associated bit-stream is 0 or 1. It is not hard to show that as long as each infinite bit-stream contains equal numbers of 0's and 1's (in the limit), the particle will spend equal amounts of time at each of the three vertices (in the limit). Certainly the hypothesis is satisfied if the bit-streams are i.i.d., but it is also satisfied if each bit-stream goes 0,1,0,1,0,1,.... Indeed, periodic bit-streams will cause more rapid convergence to the steady-state distribution $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ than random bit-streams, with the error after N steps falling like $1/N$ rather than $1/\sqrt{N}$.

It is helpful to represent this random walk process as a black box whose inputs are the three bit-streams and whose output is a stream of 1's, 2's, and 3's (representing the three vertices of the graph). Then a quasirandom law of large numbers for this system would be a proposition asserting that if the input streams have low discrepancy from the distribution $(\frac{1}{2}, \frac{1}{2})$ (in the sense that the empirical frequencies converge rapidly), then the output stream will have low discrepancy from the distribution $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. In the i.i.d. case, the input streams and output streams all have discrepancy $O(1/\sqrt{N})$ (in the sense that, as $B \to \infty$, the probability that the discrepancy at stage N exceeds $B/\sqrt{N}$ goes to zero, uniformly in N); in the periodic case, the input streams and output streams all have discrepancy $O(1/N)$ (in the sense that there is a constant B such that the discrepancy is less than B/N for all N).

There is a spectrum of degrees of discrepancy, with $o(1)$ at one end, $O(1/N)$ at the other, and $O(1/\sqrt{N})$ in the middle. The i.i.d. input-streams case, in the middle of the spectrum, uses a very robust and general notion (i.i.d. randomness) that gives medium-low discrepancy; the periodic input-streams case uses a specific combinatorial construction (periodic non-randomness) that minimizes the discrepancy, and in so doing, speeds the convergence to the steady-state distribution.

If we were using our black box to estimate the steady-state distribution of random walk on K_3 (more specifically, the steady-state probability p_1 of being at vertex 1) — forgetting for the moment that we can solve this simple system exactly — then the i.i.d. randomness case would be ordinary Monte Carlo estimation of p_1, and in N trials we would get a confidence interval for p_1 of width $B/\sqrt{N}$, where our degree of confidence rises as B gets bigger. In contrast, the periodic non-randomness case would be quasi-Monte Carlo estimation of p_1, and in N trials we would an interval of width B/N that is certain to contain p_1. (There is a slight subtlety here: knowing that such a B exists is different from actually being able to produce one. But in this case it is not hard to make the argument fully constructive and to find a bound B such that one can be certain that p_1 lies in a particular interval of width B/N without knowing p_1 in advance.)

I have been calling low-discrepancy routing mechanisms "rotor-routers", or "rotors"; Cooper and Spencer, in building on my ideas, use the term "P-machines" to describe gadgets that are built out of rotor-routers. The rotor construction is simplest to describe and implement in situations where all transition probabilities are simple rational numbers, but a natural formulation of the concept of rotors can be given that works for any finite-state Markov chain, whether or not the transition probabilities are rational, and it can be shown in this context that quasirandom simulation with rotors satisfies a law of large numbers with discrepancy that falls like $O(1/N)$.

An earlier version of the idea of low-discrepancy deterministic quasirandom simulation of random walk was invented by Arthur Engel in his work on the "probabilistic abacus" (Engel, 1975 and 1976). Engel's mechanism was reinvented by others and studied under the heading of "the chip-firing game" (see Anderson et al., 1989; Björner et al., 1991; Björner and Lovász, 1992; Eriksson, 1996; Biggs, 1997; Biggs and Winkler, 1997; and Biggs, 1999) and "the abelian sandpile model" (Bak et al., 1988; Dhar, 1990; Creutz, 1996; and Cori and Rossin, 2000), but these reincarnations of Engel's idea tended to hide rather than clarify the link with random walk. Also, the chip-firing game effectively lumps together and entangles different sample paths; the approach to quasirandomness taken in this proposal features individual sample paths, and this makes it easier to carry over ideas from probability theory to the quasirandom context.

A version of the rotor idea can be found in the physics literature, where it is called the Eulerian walks model (Priezzhev et al., 1996), and it can also be found in the computer science literature (Rabani et al., 1998), but in both cases the point of view is very different from mine, and the link with foundations of probability theory is absent. Work on quasi-Monte Carlo methods, such as recent work of Owen and Tribble (2004), is grounded in an outlook similar to mine, but the methods and constructions are rather different. The closest link I have found between my ideas and other people's work is the "whirling tours" algorithm of Dumitriu, Tetali, and Winkler (Dumitriu et al., 2003).

The rotor scheme for quasirandom walk, when compared with the chip-firing scheme, introduces some extra structure and breaks some symmetries, but in return one gains a great deal. In particular, chip-firing can be problematic on fairly simple infinite graphs, whereas rotor-routing works extremely well even on some non-recurrent infinite graphs (though why it works as well as it does is still largely mysterious). For instance, consider random walk on Z^2. The probability that a random walker starting at $(0, 0)$ visits $(1, 1)$ before returning to $(0, 0)$ is $\pi/8$ (see e.g. Doyle and Snell, 1984). We can quasirandomize the random walk as follows: Consider a particle traveling in Z^2 (starting from $(0, 0)$) where the particle, after its first visit to a vertex v, moves to the neighbor of v that is closest to the origin (with a simple rule for breaking ties). When the particle arrives at a vertex it has visited before (other than $(1, 1)$), it exits in the direction that is 90 degrees clockwise relative to the direction it took after its most recent visit to that vertex. (We can imagine a rotor at each vertex that keeps track of which way the particle went when leaving the vertex after its most recent visit.) When the particle arrives at $(1, 1)$, it is placed back at $(0, 0)$ before continuing its walk. Let $A(N)$ be the number of times that the particle visits $(1, 1)$ before its Nth visit to $(0, 0)$. Then one finds empirically that $A(N)/N$ tends quite rapidly towards $p = \pi/8$. Specifically, $|A(N) - pN|$ stays less than 2.1 for all N up to 10,000, and for more than half of those values of N, $A(N)$ is precisely the integer closest to pN.

Ander Holroyd and I have studied phenomena like this for random walks on other graphs, and have been able to show that, for those graphs, the discrepancy $|A(N) - pN|$ stays bounded. However, our proof requires hypotheses that do not apply to Z^2, so we are still unable to say anything about

$A(N) - pN$ in this case (though a nice argument of Oded Schramm shows that $A(N)/N$ converges to p). One of our goals is to prove quasirandom laws of large numbers for occupation probabilities, hitting probabilities, and hitting times for a broad class of infinite graphs, including Z^2.

The quasirandom approach to simulating discrete random systems can be viewed as a variant of the widely-used technique of quasirandom numerical integration (see Niederreiter, 1992). (Sometimes the word "subrandom" is used as an alternative to "quasirandom".) In the context of numerical integration, lowness of discrepancy means that sub-regions get sampled nearly in proportion to their measure, with only slight deviations from true proportionality. It has been known for a long time that randomly sampling from the space over which one is integrating achieves relatively low discrepancy — this is why Monte Carlo integration works as well as it does — but it has been known for almost as long that (deterministic) quasirandom schemes can often do better. For discrete random systems, the measure space that is being sampled over may be discrete (typically a finite set or an abstract Cantor set), but the fundamental principle is the same.

Indeed, the links between low-discrepancy sampling of a continuous measure space (on the one hand) and low-discrepancy exploration of a discrete measure space (on the other) are more than mere analogies. For, the state-space of a rotor-machine for a discrete simulation often maps in a natural way to a continuous measure space, and in many such cases there is a precise way of understanding the machine's function as a process of estimating an integral $\int f \, d\mu$ by means of averages $(f(x_1) + \ldots + f(x_N))/N$, where f is a particular function on the rotor-space and where $x_1, x_2, \ldots$ is a low-discrepancy sequence in the classical sense. For instance, the P-machine for computing the golden ratio that is described in Michael Kleber's write-up of my work (Kleber, 2005) can be construed as a device that numerically integrates a two-valued function on the circle R/Z by sampling the function at the points $0, \phi, 2\phi, 3\phi, ...$ (mod 1). Additionally, the famous van der Corput sequence $0, \frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{1}{8}, \frac{5}{8}, ...$ (which may be said to have started the field of low-discrepancy sequences) corresponds to rotor-walk on an infinite directed binary tree. I expect discrepancy theory will provide many useful ideas for discrete quasirandom simulation.

There is also a link (of sorts) between the proposed work and the graph-theoretic notion of quasirandomness introduced by Fan Chung, Ronald Graham, and their collaborators. These researchers showed that several different notions of quasirandomness of a graph, some of which are based on local discrepancy, are actually equivalent, and have global consequences. However, a major theme of my work, not found in the quasirandom graph literature, is that by driving the discrepancy even lower than the discrepancy of truly random objects, one can obtain quick deterministic algorithms that reveal average-case properties of the random objects.

The published article that shows this best is Cooper and Spencer, 2004. The authors show that the same sort of boundedness-of-error described above for finite graphs also applies to multiparticle quasirandom walk on an infinite grid in any dimension. This means that the heat-equation can be solved by discrete simulation using quasirandom motion of particles.

Of course there are better ways to solve the heat-equation. But I am hopeful that variants of P-machines will apply to other partial differential equations such as reaction-diffusion equations. It is worth mentioning that the rotor-router approach to simulating heat-flow is related to the numerical method of relaxation. Specifically, in the high-density limit (many particles per site), the mechanism studied by Cooper and Spencer is nothing more than fixed-precision relaxation, with a rounding-scheme ("round up, round down, round up, round down, ...") that nicely reduces errors due to finite precision by systematically causing them to cancel. So, it is reasonable to hope that for other diffusive systems, quasirandomness will offer a way to minimize round-off error (although it is unclear how useful this would be, since rounding is usually not as important a source of error in numerical solution of PDEs as discretization).

I intend to apply quasirandom methods to the study of the Diaconis-Fulton smash product (see Diaconis and Fulton, 1991), and to related models of diffusion-limited aggregation and erosion (see Lawler et al., 1992; Lawler, 1995; and Moore and Machta, 2000). Here is a quick summary of the aggregation model called internal DLA: Start with an empty infinite square grid (the stage 0 picture). To turn the stage $N-1$ picture into the stage N picture (for $N \geq 1$), add a particle at the origin, and let the particle do a random walk, subject to two rules: (a) when the particle arrives at a site that is already occupied by a particle, the particle moves to a random neighbor; and (b) when the particle arrives at a vacant site, the particle stays there, and the Nth stage of evolution is finished. At each stage, we have a "blob" of particles in Z^2. This blob gets rounder and rounder over time. Specifically, after N particles have been added, the blob looks like a disk of radius $\sqrt{N/\pi}$, with radial fluctuations that are provably no larger than $O(N^{1/3})$ (and empirically on the order of $O(\log N)$). In a derandomized version of internal DLA each site has an associated rotor that initially points to the right, and rule (a) gets replaced by rule (a'): when the particle arrives at a site that is already occupied by a particle, the rotor at that site rotates 90 degrees clockwise and the newly-arrived particle moves to the neighboring site that the rotor now points to. Under this rule, the growing blob is remarkably close to circular; for instance, after a million particles have been added to the system, the blob sits inside a disk of radius 565.1 and contains a disk of radius 563.5. The difference between the inradius and the outradius is thus 1.6 (constituting a difference of about three tenths of one percent). See `www.math.wisc.edu/~propp/million.gif` (also on page 15 of this section of the proposal), in which four colors are used to represent the four rotor settings. Levine and Peres (2005) prove that the renormalized blob converges (in a suitable sense) to the unit disk, and while this does not explain why the convergence is so rapid, it is a good start.

Furthermore, if one looks at the states of the rotors in the nearly-circular blob, one sees fascinating patterns. These patterned disks (dubbed "Propp circles" by Ed Pegg; see Pegg, 2003) have attracted broad attention and were featured on the cover of the Winter 2005 issue of The Mathematical Intelligencer (Kleber, 2005); some sort of discrete conformal analysis seems to be needed to describe these patterns. Yuval Peres claims (in private communication) that he has come to think that this deterministic process is more conceptually central than internal DLA, and that internal DLA is "merely" a randomized version of aggregation driven by rotor-routers.

The Levine-Peres bound on discrepancy from circularity does not explain the much more rapid convergence that we see empirically, so strengthenings of the theorem are clearly desired. However, this is likely to be very hard, and I have no ideas yet for how it can be done. Instead, I intend to focus (at least initially) on extending the Levine-Peres result in two directions of increased generality: first, to replace the rotor-routing hypothesis by one that concerns local discrepancy of the driving diffusion, independent of the routing mechanism, so that asymptotic circularity for internal DLA (Lawler et al., 1992) and asymptotic circularity for rotor-router aggregation (Levine and Peres, 2005) will both follow as special cases; and second, to permit more general sorts of initial blobs and local routing-distributions, to obtain quasirandom analogues of internal DLA on more general sorts of graphs.

One compelling theoretical question arising from the earliest work on the smash product (Diaconis and Fulton, 1991) is the construction of a continuum limit for the smash product; although the smash product is a stochastic operation (the smash product of two sets is a probability distribution on sets), if the ambient graph is a grid whose grid-spacing gets sent to zero, the smash-product should converge to a deterministic, rotationally-invariant operation on sets in R^n. Experiments dramatically suggest that at least when $n = 2$, this continuum limit can be studied more effectively by way of a quasirandom version that uses rotors, rather than the smash product itself. These empirical observation call out for theoretical support.

I will also apply rotors to the study of two-sided erosion, and to the study of "ordinary" DLA (see Halsey, 2000); I had not thought the latter would be a promising direction, since rotor mechanisms seem unlikely as sources of fractal fingering, but Oded Schramm has done some simulations that suggest that there may be something here worth looking at.

Finally, in parallel with my explorations of specific topics, I will try to understand simple properties of quasirandom analogues of fundamental constructions in discrete probability theory. The governing plan is to re-read the theory from the ground up (perhaps using the book-in-progress of Aldous and Fill) and see where its formal structure requires randomness and where it does not. I also want to understand better the special algebraic properties of rotor-router mechanisms. A key concept here is the notion of a *recurrent* configuration of the rotors. For random walk on a finite graph, these configurations are in one-to-one correspondence with the spanning trees of the graph, so there is much rich combinatorics here; on the other hand, there is a connection with the chip-firing group on the graph, which is still poorly understood (see Creutz, 1996 and Cori and and Rossin, 2000).

During all three years of the funded research, I intend to give my ideas a wide audience by writing articles in various journals and by giving talks at national and international conferences. Additionally, I will travel to work with collaborators at their home-institutions and arrange to have them visit me at UW.

My work has possible pedagogical implications. Quasirandom simulation might be a useful supplementary topic for the teaching of probability theory at a pre-college level. Arthur Engel invented his probabilistic abacus for the specific purpose of helping him teach probability theory to fourth-graders, and had some success with this (although this pilot effort never underwent assessment); he has written a book-length manuscript on the subject, and other researchers (under the auspices of DIMACS) have created their own teaching materials on chip-firing in the form of an educational module (see `dimacs.rutgers.edu/Publications/Modules/Module04-1/abstract04-1.pdf`). Calculating probabilities with chip-games is much more fun for children than solving simultaneous systems of linear equations. One potential drawback is that it is not possible to give children a rigorous yet age-appropriate answer to the question of why chip-games yield correct answers, and rotor-games are subject to the same liability; yet the same might be said for many algorithms taught in pre-college math, most of which are not nearly as gripping as quasirandom simulation. Also, rotor-mechanisms lead very naturally into the topic of radix representation, including non-integer bases, which high school students find exciting.

Here are some activities related to my research that will significantly broaden its impact, above and beyond the publishing of research articles:
(1) I plan to write a survey article for the *Notices of the American Mathematical Society*, describing what is known about chip-firing and the abelian sandpile model. (As I mentioned above, there is a very strong link between chip-firing and rotor-routers.) The existing literature is spread across various disciplines, and many researchers seem to be unaware of work done by people outside of their own field (and almost nobody knows about Engel's work). Furthermore, some of the common terminology is misleading (such as the word "abelian") and other terminology is used inconsistently (such as the word "recurrent"). Finally, some beautiful and challenging problems have not gotten as much attention from mathematicians as they deserve. I hope my article will help to remedy this.
(2) I plan to rewrite the unjustly neglected article of Faltin, Metropolis, Ross and Rota (Faltin et al., 1975). This article shows how one can construct the real numbers *as decimals* (or expansions using some other base, of course). Considering that this is the most prevalent way of representing real numbers, it is in retrospect amazing that nobody thought to work out and streamline the details earlier. The key technical idea that makes the details pleasant (and not nearly as grotesque as one might predict) is an equivalence relation that turns out to be tantamount to a form of chip-firing. I plan to redo this article in a way that makes the link with chip-firing explicit, and shows how many interesting variants of the standard radix-expansion idea can be obtained by varying the chip-firing rules. I will also explain how rotor-routers give an alternate (and in some ways superior) way to "reinvent the real numbers". Finally, I will explain how these ideas put the old student-question "Why is .999... the same number as 1.000...?" into a more general setting, in which (perhaps surprisingly) .999... is actually the *superior* way

of writing the number 1. Specifically, in the equivalence class of ways of writing the number 1, .999... is the unique one that is *critical*, in Biggs' sense (suitably generalized). I would submit this article to the American Mathematical Monthly.
(3) I plan to correspond with Arthur Engel about his current efforts to create a curriculum and teacher's guide for the probabilistic abacus.
(4) I plan to hire some undergraduate programmers to improve the current version of the rotor-router applet $\texttt{http}://\texttt{www.math.wisc.edu}/\sim\texttt{propp}/\texttt{rotor}-\texttt{router}-\texttt{1.0}/$ and to contribute it to David Gale's on-line Museum of Mathematics. The final applet will deal not only with random and quasirandom walk but also with representations of numbers, computation with numbers, and the links between all these topics. Additionally, these programmers will create software that I and the undergraduate research assistants (see item (6)) will use as research tools.
(5) I plan to teach a course on discrete dynamical systems and put the course entirely on the web: not just homework problems and solutions, but videos of the lectures as well. This will become a resource for anyone with access to the Internet. Preparing this course will require a great deal of work. It will also take money, both for videotaping the course and providing for storage space and access over the web. I also am requesting money to pay for software, and in particular for Mathematica. The UW math department currently uses Maple, and there has been no interest in paying to have a Mathematica license as well. However, Mathematica is a better system to use for the study of discrete dynamical systems (which is hardly surprising, since Wolfram developed it for precisely that purpose).
(6) I plan to run a research group for two years, growing out of the aforementioned course. This group will be composed almost completely of undergraduates. Some fundamental problems related to quasirandomness do not call for great mathematical sophistication, but rather for persistence, cleverness, and creativity. I have had a great deal of success in working with students at elite colleges in the Boston area, and although my initial efforts at using UW undergraduates in the same capacity did not initially bear as much fruit as my work with Harvard and MIT undergraduates, I think I have gotten a better idea of what a UW undergraduate research team can and cannot be expected to do. My project on quasirandomness offers a range of levels of difficulty, and also mixes different styles of research (ranging from the purely theoretical to the purely experimental), so I am confident that I will be able to give all my students a satisfying research experience that will also

contribute to the growth of the field. As in the past, I will continue to aggressively recruit women participants. In response to student interest, I will offer training not just in the relevant mathematics but also in Mathematica and Maple (for running simulations and analyses) and LaTeX (for writing up results). Running this research group will take about twelve hours per week of my time for all four semesters. I am hoping that the outcome of this work will be half a dozen math arXiv preprints and two or three articles written by students or groups of students (not co-authored by me) and published in reputable journals.

Although as of this writing (the end of September, 2005) I have not written any formal preprints that outline the subject of quasirandomness and fill in the details, this situation may well have changed by the time the proposal is refereed; `http : //www.math.wisc.edu/ ∼ propp/quasirandom.html` should have up-to-date information.

I am seeking support for this work from both the probability program and the combinatorics program of NSF. Rotor-router simulation, like chip-firing simulation, can be justified as a purely combinatorial enterprise, where simple discrete rules generate patterns that are complicated enough to command interest but not so complicated as to defeat solution. Quasirandom systems like the one shown in `www.math.wisc.edu/∼propp/million.gif` have elements of beauty, surprise, and hidden structure that make them intrinsically worthy of mathematical study, and instructive examples of the way in which simple local rules can lead to complex global behavior. However, purely combinatorial considerations will not guide us to the richest examples; I am convinced that the most beautiful and challenging examples in this subject will arise from derandomization of classical probabilistic constructions. At the same time, I expect that some combinatorial processes will turn out, after the fact, to be derandomized versions of probabilistic processes. In this way, quasirandomness based on minimization of discrepancy will be a two-way bridge between probability and combinatorics.

I expect that, as far as applied probability theory is concerned, the long-term outcome of this work will be similar to the outcome of my work with David Wilson on exact sampling: a half-dozen to a dozen researchers will take up the idea of quasirandom simulation and turn it into a useful tool for people in the sciences who use discrete stochastic systems as models and who need to assess how well their models fit reality. But I am hoping that the larger impact will be on pure probability theory, and that my work of discrepancy-based quasirandomness will lead others to conduct deeper work on "random" behavior in deterministic settings from as general a point of view as possible. Lastly, I hope that many of the undergraduates who assist me with this work will be inspired to pursue research careers of their own, and that the grad students who help me supervise the research team will become enthusiastic about this model of research and spread it elsewhere.

3 PI's Qualifications

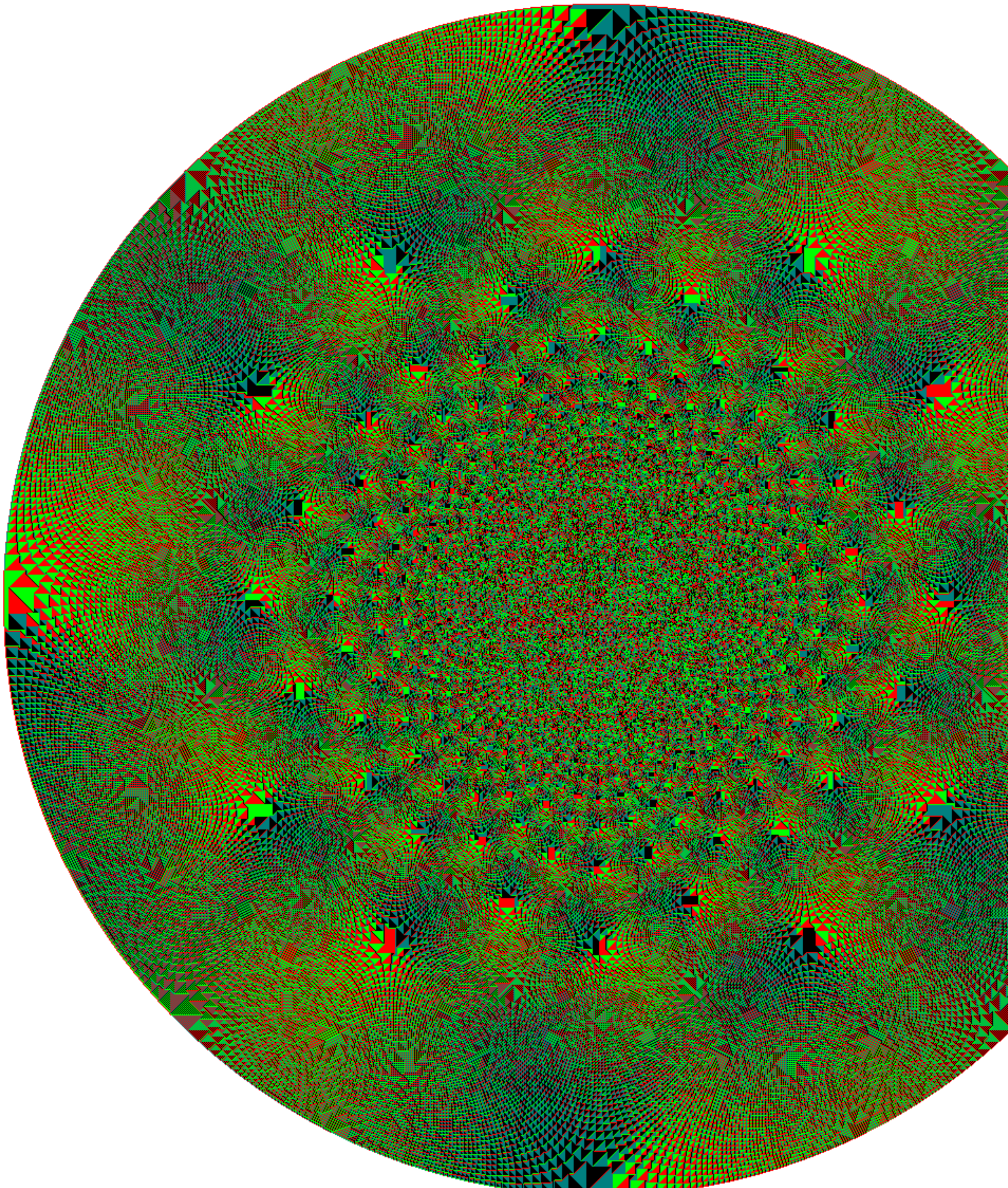
I have twenty years of experience in combinatorics and probability. The current proposal mixes several themes from my earlier work. Rotor-routers are reminiscent of my (mostly experimental) work on virtual ants (described in Propp, 1994 and Gale et al., 1995), though with some crucial differences that make the newer work more important (largely because of its direct connection with probabilistic issues). The idea of finding new ways to sample from, and obtain information about, discrete probability distributions is a key feature of the proposal, and this idea is also found in my work on coupling-from-the-past (Propp and Wilson, 1996) and partial rejection sampling (Propp and Wilson, 1998 and Cohn, Pemantle and Propp, 2002). Furthermore, the "stacks" concept (due to Diaconis and Fulton and discovered independently by Wilson) that plays a role in partial rejection sampling is a key component of the rotor-router setup. My work on random tilings (see Cohn et al., 1996 and Cohn et al., 1998) shows my command of analytic tools of the sort that may be needed in attacking asymptotic shape laws in the quasirandom context. Lastly, my work on lattice structure for orientations of graphs (arXiv: `math.CO/0209005`) relates in part to the operation of editing a directed tree by rotating one directed edge around a vertex, and I suspect that the sort of height-functions that play a role in that paper will also be relevant to the study of rotors.

I have a good record for working with undergraduates, at MIT (the Tilings Research Group), Wisconsin (the Spatial Systems Laboratory), and Harvard (Research Experiences in Algebraic Combinatorics at Harvard). One of my earliest undergraduate research assistants, Alex Ionescu, is now my colleague at Wisconsin. Other alums of my research group are Henry Cohn and David Wilson (though the latter was involved only as a graduate student), who are now both permanent members of the staff at Microsoft Research. Two Wisconsin undergraduates who did work with me in the Spatial Systems Laboratory went to an international conference in Paris (with NSF footing the bill) and presented their work there. More recently, I supervised Lionel Levine in the work that led to his undergraduate honors thesis at Harvard, and paid for him to present our work at a national conference. Articles that have grown out of work supervised by me include Wieland (2000), Kuo (2004), and Carroll and Speyer (2004).

I like to socially engineer vertically integrated mini-communities in which research takes place in a friendly and collaborative way, and I think that over the past dozen years I have learned a great deal about how to do it. One of the main things is to find a topic that has a certain kind of vertical richness. Quasirandomness is such a topic; it presents problems of many flavors and of widely varying difficulty.

I mentioned a page or two ago some of the ways in which I have stayed at the forefront of web-based dissemination of research ideas. I should also mention that I put an entire course on the web, including videos of the lectures (`http://www.math.wisc.edu/~propp/192/`); although the course was aimed at undergraduates, it presented a novel personal way of organizing standard material, as well as some material that is not even customarily taught at the graduate level, and a few ideas of my own that have not been published anywhere. Researchers and students from all over the world have told me that they found these lectures helpful. I will use the web to disseminate my course on discrete dynamical systems in the same way.

I have also had experience (and success) with taking ideas from my research that are simple and attractive and presenting them to people without much mathematical background. For instance, when I gave a talk on rotor-routers to the University of Wisconsin's Chaos Seminar (most of whose participants are non-mathematicians and many of whom are non-scientists) in Fall 2003, I spent some time on the philosophical issue of why we impose a coordinate system on a universe that does not come with a preferred frame of reference, and the ways in which this breaking of nature's symmetry comes with a price-tag. On the other hand, when I gave a talk on the same topic to some high school students who visited the UW math department in Spring 2004, I highlighted the game-like aspect of rotors, and got students involved first in estimating probabilities empirically and then in doing the calculation with rotor-routers. Part of the fun and challenge of teaching, for me, is finding the way to present a given topic to a given audience. I have a reputation as an engaging teacher of children, professors, and students at every level in between, and I would strive to live up to this reputation in my talks on quasirandom simulation.



1 Bibliography

Articles by the PI (in forward chronological order):

J. Propp and D. Wilson, Exact sampling with coupled Markov chains and applications to statistical mechanics, *Random Structures and Algorithms* **9**, 223–252 (1996); `www.math.wisc.edu/ ~ propp/sample.html`.

H. Cohn, N. Elkies and J. Propp, Local statistics for random domino tilings of the Aztec diamond, *Duke Mathematical Journal* **85**, 117–166 (1996); arXiv: `math.CO/0008243`.

J. Propp, A pedestrian approach to a method of Conway, or, a tale of two cities, *Mathematics Magazine* **70** (1997), 327–340.

J. Propp, Boundary-dependent local behavior for 2-D dimer models, *International Journal of Modern Physics B* **11**, 183–187 (1997); `www.math.wisc.edu/ ~ propp/dimer.ps.gz`.

J. Propp, Generating random elements of finite distributive lattices, *Electronic Journal of Combinatorics* **4**(2) (1997), R15; `www.combinatorics.org/Volume_4/Abstracts/v4i2r15ab.html`; arXiv: `math.CO/9801066`.

J. Propp and D. Wilson, How to get a perfectly random sample from a generic Markov chain and generate a random spanning tree of a directed graph, *Journal of Algorithms* **27**, 170–217 (1998); `www.math.wisc.edu/ ~ propp/perfectly.ps.gz`.

J. Propp and D. Wilson, Coupling from the past: a user's guide, in: *Microsurveys in Discrete Probability* (see below), 181–192 (1998); `www.math.wisc.edu/ ~ propp/user.ps.gz`.

D. Aldous and J. Propp, eds., *Microsurveys in Discrete Probability*, American Mathematical Society, 1998.

H. Cohn, M. Larsen, and J. Propp, The shape of a typical boxed plane partition New York Journal of Mathematics **4**, 137–165 (1998); arXiv: `math.CO/9801059`.

D. Bressoud and J. Propp, How the alternating-sign matrix conjecture was solved, *Notices of the American Mathematical Society* **46**, 637–646 (1999); `www.ams.org/notices/199906/fea − bressoud.pdf`.

J. Propp and R. Stanley, Domino tiling with barriers, *Journal of Combinatorial Theory, Series A* **87**, 347–356 (1999); arXiv: `math.CO/9801067`.

J. Propp, Enumeration of matchings: problems and progress, in: *New perspectives in geometric combinatorics*, L. Billera et al. eds., Mathematical Sciences Research Institute series, vol. 38, Cambridge University Press, 1999; arXiv: `math.CO/9904150`.

R. Kenyon, J. Propp, and D. Wilson, Trees and matchings, *Electronic Journal of Combinatorics*, **7**(1) (2000), R25; `www.combinatorics.org/Volume_7/Abstracts/v7i1r25.html`; arXiv: `math.CO/9903025`.

H. Cohn, R. Kenyon, and J. Propp, A variational principle for domino tilings *Journal of the American Mathematical Society* **14**, 297–346 (2001); arXiv`math.CO/0008220`.

J. Propp, The many faces of alternating-sign matrices, in: *Discrete Models: Combinatorics, Computation and Geometry*, Paris, France, 2001; arXiv: `math.CO/0208125`.

J. Propp, A reciprocity theorem for domino tilings, *Electronic Journal of Combinatorics*, **8**(1) (2001), R18; `www.combinatorics.org/Volume_8/Abstracts/v8i1r18.html`; arXiv: `math.CO/0104011`.

H. Cohn, R. Pemantle, and J. Propp, Generating a random sink-free orientation in quadratic time Electronic Journal of Combinatorics, **9**(1) (2002), R10; `www.combinatorics.org/Volume_9/Abstracts/v9i1r10.html`.

J. Propp, Generalized domino-shuffling, Theoretical Computer Science **303**, 267–301 (2003); arXiv: `math.CO/0111034`.

K. Kedlaya and J. Propp, In search of Robbins stability, accepted by Advances in Applied Mathematics; arXiv: `math.NT/0409535`.

J. Propp, Lambda-determinants and domino-tilings, accepted by Advances in Applied Mathematics; arXiv: `math.CO/0406301`.

Articles by others:

D. Aldous and J. Fill, Reversible Markov Chains and Random Walks on Graphs (book in progress).

R. Anderson, L. Lovász, P. Shor, J. Spencer, E. Tardos, and S. Winograd, Disks, Balls, and Walls: Analysis of a Combinatorial Game, *American Mathematical Monthly* **96** (1989), 481–493.

P. Bak, C. Tang, and K. Wiesenfeld, Self-organized criticality, *Phys. Rev. A* **38** (1988), 364–374.

N. Biggs, Chip-firing and the critical group of a graph, *Journal of Algebraic Combinatorics* **9** (1999), 25–45.

N. Biggs and P. Winkler, Chip firing and the chromatic polynomial, Centre for Discrete and Applicable Mathematics, technical report LSE-CDAM-97-03 (1997).

N. Biggs, Algebraic potential theory on graphs, *Bull. London Math. Soc.* **29** (1997), 641–682.

A. Björner, L. Lovász, and P. Shor, Chip-firing games on graphs, *Europ. J. Combinatorics* **12** (1991), 283–291.

A. Björner and L. Lovász, Chip-firing games on directed graphs, *Journal of Algebraic Combinatorics* **1** (1992), 304–328.

H. Canary, Aztec diamonds and Baxter permutations, preprint, 2001; arXiv: `math.CO/0309135`.

G. Carroll and D. Speyer, The cube recurrence, *Electronic Journal of Combinatorics* **11**(1) (2004), R73; `www.combinatorics.org/Volume_11/Abstracts/v11i1r73.html`.

R. Cori and and D. Rossin, On the sandpile group of a graph, *European Journal of Combinatorics* **21** (2000), 447–459. arXiv: `dept − info.labri.u − bordeaux.fr/ ∼ cori/Articles/sable.ps`.

M. Creutz, Cellular Automata and Self Organized Criticality, arXiv: `arxiv.org/abs/hep − lat/9611017`.

J. Cooper and J. Spencer, Simulating a random walk with constant error, arXiv: `front.math.ucdavis.edu/math.CO/0402323`.

D. Dhar, Self-organized critical state of sandpile automaton models, *Phys. Rev. Lett.* **64** (1990), 1613–1616.

P. Diaconis and W. Fulton, A growth model, a game, an algebra, Lagrange inversion and characteristic classes, *Recondita Math.* **49** (1991), 95–119.

P.G. Doyle and J.L. Snell, Random walks and electric networks, Mathematical Association of America, 1984; arXiv: `math.PR/0001057`.

I. Dumitriu, P. Tetali, and P. Winkler, On playing golf with two balls, *SIAM Journal on Discrete Mathematics* **16** (2003), 604–615; arXiv: `www − math.mit.edu/ ∼ dumitriu/formal/pap_05_02.ps`.

A. Engel, The probabilistic abacus, *Ed. Stud. Math.* **6** (1975), 1–22.

A. Engel, Why does the probabilistic abacus work?, *Ed. Stud. Math.* **7** (1976), 59–69.

K. Eriksson, Strong convergence and the polygon property of 1-player games, *Discrete Math.* **153** (1996), 105–122.

K. Eriksson, Chip-firing games on mutating graphs, *SIAM J. Discrete Math.* **9** (1996), 118–128.

F. Faltin, N. Metropolis, B. Ross and G.-C. Rota, The real numbers as a wreath product, *Advances in Math.* **16** (1975), 278–304.

D. Gale, J. Propp, S. Sutherland, and S. Troubetkzoy, Further travels with my ant, Mathematical Entertainments column, *Mathematical Intelligencer* **17**, #3, 48–56 (1995); republished in D. Gale, Tracking the Automatic Ant, Springer-Verlag, 1998; arXiv: `math.DS/9501233`.

T.C. Halsey, Diffusion-Limited Aggregation: a model for pattern formation, *Physics Today* **53** #11 (Nov. 2000), 36–41; `www.aip.org/pt/vol − 53/iss − 11/p36.html`.

M. Kleber, Goldbug variations, Winter 2005 issue of *Mathematical Intelligencer* (2005).

E. Kuo, Applications of graphical condensation for enumerating matchings and tilings, *Theoret. Comp. Sci.* **319** (2004), 29–57; arXiv: `math.CO/0304090`.

G.F. Lawler, Subdiffusive fluctuations for internal diffusion limited aggregation, *Annals of Probability* **23** (1995), 71–86.

G.F. Lawler, M. Bramson, and D. Griffeath, Internal diffusion limited aggregation, *Annals of Probability* **20** (1992), 2117–2140.

L. Levine, The Rotor-Router Model, Harvard College undergraduate thesis, 2002; arXiv: `math.CO/0409407`.

L. Levine and Y. Peres, Spherical asymptotics for the rotor-router model in $\mathbf{Z}^d$, arXiv: `math.PR/0503251`.

J. Matousek, Geometric Discrepancy: An Illustrated Guide, Springer, 1999.

C. Moore and J. Machta, Internal diffusion-limited aggregation: Parallel algorithms and complexity, *J. Stat. Phys.* **99** (2000) 629–660; arXiv: `cond − mat/9909233`.

H. Niederreiter, Random Number Generation and Quasi-Monte Carlo Methods, Philadelphia, PA: SIAM, 1992.

A.B. Owen and S.D. Tribble, A quasi-Monte Carlo Metropolis algorithm, *Proc. Nat. Acad. Sci.* **102** (2005) 8844–8849; `http://www.pnas.org/cgi/content/full/102/25/8844`.

E. Pegg, Propp Circles; `http : //www.mathpuzzle.com/29Jun2003.html`.

V.B. Priezzhev, D. Dhar, A. Dhar, S. Krishnamurthy, Eulerian walkers as a model of self-organised criticality, *Phys. Rev. Lett.* **77** (1996), 5079–5082; arXiv: `cond-mat/9611019`.

J. Propp, Further ant-ics, Mathematical Entertainments column, *Mathematical Intelligencer* **16**, #1, 37–42 (1994); republished in D. Gale, Tracking the Automatic Ant, Springer-Verlag, 1998.

Y. Rabani, A. Sinclair and R. Wanka, Local divergence of Markov chains and the analysis of iterative load-balancing schemes, *IEEE FOCS 1998*, 694–703.

A.V. Razumov and Yu.G. Stroganov, Spin chains and combinatorics, *J. Phys. A* **34** (2001), 3185–3190; arXiv: `cond-mat/0012141` (2000).

B. Wieland, A large dihedral symmetry of the set of alternating sign matrices, *Electronic Journal of Combinatorics*, **7**(1) (2000), R37; `www.combinatorics.org/Volume_7/Abstracts/v7i1r37.html`.

P. Winkler, Mathematical Puzzles: A Connoisseur's Collection, A.K. Peters, 2003.

1 Biography: James G. Propp

(a) Professional Preparation

Harvard College, mathematics, A.B. 1982.

University of California at Berkeley, mathematics, Ph.D. 1987.

Postdoctoral work at University of Maryland (1987–1988) and University of California at Berkeley (1988–1990), ergodic theory, funded by NSF.

(b) Appointments

University of Wisconsin at Madison, 1998 to present.

Brandeis University, Visiting Associate Professor, 2002-2003

Harvard University, Visiting Associate Professor, 2001-2002

Olin College, Visiting Associate Professor, Fall 2000

Massachusetts Institute of Technology, Assistant Professor, 1990 to 1996; Associate Professor, 1996 to 1998; Visiting Scholar, 1998 to 1999 and Fall 2000

(c) Publications

Publications most closely (i.e. least distantly) related to the proposed project:

Generating a random sink-free orientation in quadratic time by H. Cohn, R. Pemantle, and J. Propp), Electronic Journal of Combinatorics, R10, Vol. 9(1) (2002). `www.combinatorics.org/Volume_9/Abstracts/v9i1r10.html`.

Trees and matchings (by R. Kenyon, J. Propp, and D. Wilson), Electronic Journal of Combinatorics, R25, Vol. 7(1) (2000). `www.combinatorics.org/Volume_7/Abstracts/v7i1r25.html`.

How to get a perfectly random sample from a generic Markov chain and generate a random spanning tree of a directed graph (by J. Propp and D. Wilson), Journal of Algorithms **27**, 170–217 (1998). `www.dbwilson.com/ja/`.

Other significant publications:

Generalized domino-shuffling (by J. Propp), Theoretical Computer Science **303**, 267–301 (2003). `front.math.ucdavis.edu/math.CO/0111034`.

A variational principle for domino tilings (by H. Cohn, R. Kenyon and J. Propp), Journal of the American Mathematical Society **14**, 297–346 (2001). `front.math.ucdavis.edu/math.CO/0008220`.

The shape of a typical boxed plane partition (by H. Cohn, M. Larsen and J. Propp), *New York Journal of Mathematics* **4**, 137–165 (1998). `nyjm.albany.edu : 8000/j/1998/4 − 10.ps`.

Exact sampling with coupled Markov chains and applications to statistical mechanics (by J. Propp and D. Wilson), *Random Structures and Algorithms* **9**, 223–252 (1996). `www.math.wisc.edu/ ~ propp/sample.html`.

Local statistics for random domino tilings of the Aztec diamond (by H. Cohn, N. Elkies and J. Propp), Duke Mathematical Journal **85**, 117–166 (1996). `www.math.wisc.edu/ ~ propp/arctan.ps.gz`.

(d) Synergistic Activities

Speaker at Wellesley, Mt. Holyoke, and Smith Colleges, Fall 2004 (speaking about "Bugs, Blobs, and Rotor-Routers" and recruiting women for UW-Madison's Ph.D. program). Speaker to 50 students from Homestead High School (Wisconsin) about "Bugs and Rotors", Spring 2004. Speaker to 50 high school students from the greater Boston area at The Math Circle (Boston, Spring 2002 and Spring 2003) on chip-firing and Ehrhart theory.

Speaker at UW-Madison's interdisciplinary Chaos Seminar on "Randomness, Roundness, and Rules", Fall 2003.

Co-organizer (with David Aldous) of 1997 DIMACS-sponsored 5-day workshop entitled *Microsurveys in Discrete Probability*, and co-editor (with Aldous) of the workshop proceedings (published by the AMS, 1998).

Organizer of undergraduate research experiences: University of Wisconsin, Spatial Systems Laboratory (SSL), Spring 2000, Spring 2001, and Fall 2003 through Spring 2004. Harvard University, Research Experiences in Algebraic Combinatorics at Harvard (REACH), Fall 2001 through Spring 2003. MIT Tilings Research Group, 1992 through 1998. Member of advisory board for Tufts University's MCTP (Mentoring through Critical Transition Points) grant proposal, Fall 2004.

Moderator of `domino` forum (an electronic community of mathematicians, computer scientists, and physicists, as well as graduate students and undergraduates), 1995 to present.

(e) Collaborators

(i) Collaborators: Gabriel Carroll, Henry Cohn, Boris Hasselblatt, Ander Holroyd, Kiran Kedlaya, Rick Kenyon, Lionel Levine, Gregg Musiker, Robin Pemantle, Thomas Kyle Petersen, David Speyer, David Wilson.

(ii) Graduate Advisor: Jacob Feldman, University of California at Berkeley (emeritus). Principal postdoctoral sponsor: N/A.

(iii) Thesis Advisor and Postgraduate-Scholar Sponsor: N/A.

SUMMARY PROPOSAL BUDGET

YEAR 1

FOR NSF USE ONLY		
PROPOSAL NO.	DURATION (months)	
	Proposed	Granted
AWARD NO.		

ORGANIZATION
University of Wisconsin-Madison

PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR
James G Propp

A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets)	NSF Funded Person-months CAL	ACAD	SUMR	Funds Requested By proposer	Funds granted by NSF (if different)
1. James G Propp - PI	0.00	0.00	1.00	$ 9,546	$
2.					
3.					
4.					
5.					
6. (0) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE)	0.00	0.00	0.00	0	
7. (1) TOTAL SENIOR PERSONNEL (1 - 6)	0.00	0.00	1.00	9,546	
B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS)					
1. (0) POST DOCTORAL ASSOCIATES	0.00	0.00	0.00	0	
2. (0) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.)	0.00	0.00	0.00	0	
3. (1) GRADUATE STUDENTS				7,703	
4. (2) UNDERGRADUATE STUDENTS				4,000	
5. (0) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY)				0	
6. (0) OTHER				0	
TOTAL SALARIES AND WAGES (A + B)				21,249	
C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS)				5,435	
TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C)				26,684	
D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.)					
TOTAL EQUIPMENT				0	
E. TRAVEL 1. DOMESTIC (INCL. CANADA, MEXICO AND U.S. POSSESSIONS)				1,500	
2. FOREIGN				1,500	
F. PARTICIPANT SUPPORT COSTS 1. STIPENDS $ 0 2. TRAVEL 0 3. SUBSISTENCE 0 4. OTHER 0					
TOTAL NUMBER OF PARTICIPANTS (0) TOTAL PARTICIPANT COSTS				0	
G. OTHER DIRECT COSTS					
1. MATERIALS AND SUPPLIES				6,000	
2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION				0	
3. CONSULTANT SERVICES				2,000	
4. COMPUTER SERVICES				0	
5. SUBAWARDS				0	
6. OTHER				1,926	
TOTAL OTHER DIRECT COSTS				9,926	
H. TOTAL DIRECT COSTS (A THROUGH G)				39,610	
I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE) F & A cost 47% of Line H (-) Line G. 6 (Rate: 47.0000, Base: 37684)					
TOTAL INDIRECT COSTS (F&A)				17,711	
J. TOTAL DIRECT AND INDIRECT COSTS (H + I)				57,321	
K. RESIDUAL FUNDS (IF FOR FURTHER SUPPORT OF CURRENT PROJECTS SEE GPG II.C.6.j.)				0	
L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K)				$ 57,321	$

M. COST SHARING PROPOSED LEVEL $ 0 AGREED LEVEL IF DIFFERENT $

PI/PD NAME
James G Propp

ORG. REP. NAME*
Linda johnson

FOR NSF USE ONLY		
INDIRECT COST RATE VERIFICATION		
Date Checked	Date Of Rate Sheet	Initials - ORG

 ***ELECTRONIC SIGNATURES REQUIRED FOR REVISED BUDGET**

SUMMARY PROPOSAL BUDGET — YEAR 2

FOR NSF USE ONLY

PROPOSAL NO.	DURATION (months) Proposed	Granted
AWARD NO.		

ORGANIZATION
University of Wisconsin-Madison

PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR
James G Propp

A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets)	NSF Funded Person-months CAL	ACAD	SUMR	Funds Requested By proposer	Funds granted by NSF (if different)
1. James G Propp - PI	0.00	4.50	1.00	$ 33,740	$
2.					
3.					
4.					
5.					
6. (0) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE)	0.00	0.00	0.00	0	
7. (1) TOTAL SENIOR PERSONNEL (1 - 6)	0.00	4.50	1.00	33,740	
B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS)					
1. (0) POST DOCTORAL ASSOCIATES	0.00	0.00	0.00	0	
2. (0) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.)	0.00	0.00	0.00	0	
3. (1) GRADUATE STUDENTS				8,011	
4. (8) UNDERGRADUATE STUDENTS				16,000	
5. (0) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY)				0	
6. (0) OTHER				0	
TOTAL SALARIES AND WAGES (A + B)				57,751	
C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS)				14,452	
TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C)				72,203	
D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.)					
TOTAL EQUIPMENT				0	
E. TRAVEL 1. DOMESTIC (INCL. CANADA, MEXICO AND U.S. POSSESSIONS)				1,500	
2. FOREIGN				1,500	
F. PARTICIPANT SUPPORT COSTS					
1. STIPENDS $ 0					
2. TRAVEL 0					
3. SUBSISTENCE 0					
4. OTHER 0					
TOTAL NUMBER OF PARTICIPANTS (0) TOTAL PARTICIPANT COSTS				0	
G. OTHER DIRECT COSTS					
1. MATERIALS AND SUPPLIES				4,000	
2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION				0	
3. CONSULTANT SERVICES				2,000	
4. COMPUTER SERVICES				3,800	
5. SUBAWARDS				0	
6. OTHER				2,003	
TOTAL OTHER DIRECT COSTS				11,803	
H. TOTAL DIRECT COSTS (A THROUGH G)				87,006	
I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE) F & A cost 47% of Line H (-) Line G. 6 (Rate: 47.0000, Base: 85004)					
TOTAL INDIRECT COSTS (F&A)				39,952	
J. TOTAL DIRECT AND INDIRECT COSTS (H + I)				126,958	
K. RESIDUAL FUNDS (IF FOR FURTHER SUPPORT OF CURRENT PROJECTS SEE GPG II.C.6.j.)				0	
L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K)				$ 126,958	$

M. COST SHARING PROPOSED LEVEL $ 0 — AGREED LEVEL IF DIFFERENT $

PI/PD NAME
James G Propp

ORG. REP. NAME*
Linda johnson

FOR NSF USE ONLY
INDIRECT COST RATE VERIFICATION

Date Checked	Date Of Rate Sheet	Initials - ORG

 ***ELECTRONIC SIGNATURES REQUIRED FOR REVISED BUDGET**

SUMMARY PROPOSAL BUDGET

YEAR 3

FOR NSF USE ONLY		
PROPOSAL NO.	DURATION (months)	
	Proposed	Granted
AWARD NO.		

ORGANIZATION
University of Wisconsin-Madison

PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR
James G Propp

A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets)	NSF Funded Person-months CAL	ACAD	SUMR	Funds Requested By proposer	Funds granted by NSF (if different)
1. James G Propp - PI	0.00	4.50	1.00	$ 35,090	$
2.					
3.					
4.					
5.					
6. (0) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE)	0.00	0.00	0.00	0	
7. (1) TOTAL SENIOR PERSONNEL (1 - 6)	0.00	4.50	1.00	35,090	
B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS)					
1. (0) POST DOCTORAL ASSOCIATES	0.00	0.00	0.00	0	
2. (0) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.)	0.00	0.00	0.00	0	
3. (1) GRADUATE STUDENTS				8,332	
4. (8) UNDERGRADUATE STUDENTS				16,000	
5. (0) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY)				0	
6. (0) OTHER				0	
TOTAL SALARIES AND WAGES (A + B)				59,422	
C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS)				15,308	
TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C)				74,730	
D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.)					
TOTAL EQUIPMENT				0	
E. TRAVEL 1. DOMESTIC (INCL. CANADA, MEXICO AND U.S. POSSESSIONS)				1,500	
2. FOREIGN				1,500	
F. PARTICIPANT SUPPORT COSTS					
1. STIPENDS $ 0					
2. TRAVEL 0					
3. SUBSISTENCE 0					
4. OTHER 0					
TOTAL NUMBER OF PARTICIPANTS (0) TOTAL PARTICIPANT COSTS				0	
G. OTHER DIRECT COSTS					
1. MATERIALS AND SUPPLIES				4,000	
2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION				0	
3. CONSULTANT SERVICES				2,000	
4. COMPUTER SERVICES				0	
5. SUBAWARDS				0	
6. OTHER				2,083	
TOTAL OTHER DIRECT COSTS				8,083	
H. TOTAL DIRECT COSTS (A THROUGH G)				85,813	
I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE) F & A cost 47% of Line H (-) Line G. 6 (Rate: 47.0000, Base: 83730)					
TOTAL INDIRECT COSTS (F&A)				39,353	
J. TOTAL DIRECT AND INDIRECT COSTS (H + I)				125,166	
K. RESIDUAL FUNDS (IF FOR FURTHER SUPPORT OF CURRENT PROJECTS SEE GPG II.C.6.j.)				0	
L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K)				$ 125,166	$

M. COST SHARING PROPOSED LEVEL $ 0 AGREED LEVEL IF DIFFERENT $

PI/PD NAME
James G Propp

ORG. REP. NAME*
Linda johnson

FOR NSF USE ONLY		
INDIRECT COST RATE VERIFICATION		
Date Checked	Date Of Rate Sheet	Initials - ORG

 *ELECTRONIC SIGNATURES REQUIRED FOR REVISED BUDGET

SUMMARY PROPOSAL BUDGET — Cumulative

ORGANIZATION: University of Wisconsin-Madison	FOR NSF USE ONLY — PROPOSAL NO.	DURATION (months): Proposed / Granted
PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR: James G Propp	AWARD NO.	

A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets)	NSF Funded Person-months CAL	ACAD	SUMR	Funds Requested By proposer	Funds granted by NSF (if different)
1. James G Propp - PI	0.00	9.00	3.00	$ 78,376	$
2.					
3.					
4.					
5.					
6. ( ) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE)	0.00	0.00	0.00	0	
7. (1) TOTAL SENIOR PERSONNEL (1 - 6)	0.00	9.00	3.00	78,376	
B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS)					
1. (0) POST DOCTORAL ASSOCIATES	0.00	0.00	0.00	0	
2. (0) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.)	0.00	0.00	0.00	0	
3. (3) GRADUATE STUDENTS				24,046	
4. (18) UNDERGRADUATE STUDENTS				36,000	
5. (0) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY)				0	
6. (0) OTHER				0	
TOTAL SALARIES AND WAGES (A + B)				138,422	
C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS)				35,195	
TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C)				173,617	
D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.)					
TOTAL EQUIPMENT				0	
E. TRAVEL 1. DOMESTIC (INCL. CANADA, MEXICO AND U.S. POSSESSIONS)				4,500	
2. FOREIGN				4,500	
F. PARTICIPANT SUPPORT COSTS					
1. STIPENDS $ 0					
2. TRAVEL 0					
3. SUBSISTENCE 0					
4. OTHER 0					
TOTAL NUMBER OF PARTICIPANTS (0) TOTAL PARTICIPANT COSTS				0	
G. OTHER DIRECT COSTS					
1. MATERIALS AND SUPPLIES				14,000	
2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION				0	
3. CONSULTANT SERVICES				6,000	
4. COMPUTER SERVICES				3,800	
5. SUBAWARDS				0	
6. OTHER				6,012	
TOTAL OTHER DIRECT COSTS				29,812	
H. TOTAL DIRECT COSTS (A THROUGH G)				212,429	
I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE)					
TOTAL INDIRECT COSTS (F&A)				97,016	
J. TOTAL DIRECT AND INDIRECT COSTS (H + I)				309,445	
K. RESIDUAL FUNDS (IF FOR FURTHER SUPPORT OF CURRENT PROJECTS SEE GPG II.C.6.j.)				0	
L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K)				$ 309,445	$
M. COST SHARING PROPOSED LEVEL $ 0	AGREED LEVEL IF DIFFERENT $				

PI/PD NAME: James G Propp	FOR NSF USE ONLY — INDIRECT COST RATE VERIFICATION
ORG. REP. NAME*: Linda johnson	Date Checked / Date Of Rate Sheet / Initials - ORG

C *ELECTRONIC SIGNATURES REQUIRED FOR REVISED BUDGET

1 Budget Justification

A. Senior Personnel

Senior Personnel salary is for 1 month per summer at the PI's current salary with a 4% increase in each year of the proposal, and in Years 2 and 3, a teaching buyout at 53.3% of current salary for 4.5 months.

- In Summer 2007, the PI will finish putting together a new undergraduate course on Discrete Dynamical Systems, to be offered in Fall 2007; this course will serve as a feeder to the undergraduate research group that the PI will run from Fall 2007 through Spring 2009.

- In Summer 2008, the PI will help the undergraduate research assistants write up the work they did during the two preceding semesters. This part of supervising undergraduate research has proved to be very time-consuming. Ideally the results should be submitted for publication, but in cases where the paper falls short of publishability, the goal is to put the paper in the arXiv, so that it will become a permanent part of the research literature.

- In Summer 2009, the PI will help the undergraduate research assistants write up the work they did during the two preceding semesters. The PI will also do some final tidying up of the project, e.g., making sure that the web-site is complete and consistent, making sure that software created by the students is stable and well-documented, etc.

- In Years 2 and 3, the PI will put in 12 hours per week setting up and running an undergraduate research laboratory. (The University of Wisconsin has no provisions for giving faculty teaching-credit for running a laboratory in a historically non-lab-based subject like mathematics; hence a course buy-out, to partially offset this investment of time, is appropriate.) This work will result in write-ups and publications by the students themselves, not the PI.

B. Other Personnel

The PI will hire three sorts of students under the grant:

- Graduate students (all three years): One 50% appointment for one semester each year with a 4% increase in years 2 and 3 of proposal. These graduate students would pursue research on more advanced topics, and would help supervise the coders and the URAs (see below).

- Undergraduate coders: 2 students each year, paid at a student hourly rate ($12/hour), limited to $2,000 per student per year. These coders would create high-performance software to be used as research tools and easy-to-use applets to spread the group's results via the World Wide Web.

- Undergraduate research assistants (URAs): 6 students in Years 2 and 3, paid at a student hourly rate ($12/hour), limited to $2,000 per student per year. These students would engage in genuine collaborative research on problems whose solutions are not known in advance. It might be appropriate to fund these students via an REU Supplement rather than pay them as technicians, given the nature of the work.

C. Fringe Benefits

Fringe rates are as follows in year 1 with a .5year of proposal: Senior Personnel, 34.5%; Graduate Research Assistant, 26.5%; Student Hourly, 2.5%.

E. Travel

The PI requests coverage of transportation and subsistence for attendance and participation of Senior Personnel at scientific conferences in the U.S. and Canada, and collaboration with colleagues at other institutions. This request includes student travel as well. The PI also requests coverage for a small amount of foreign travel, to pay for trips to conference meetings in Europe or Asia during each year of proposal.

G. Other Direct Costs

G.1. Materials and Supplies

In Year 1, the PI will spend $2,000 on a data-projector, to enhance presentations of the PI's work to high school students.

The cost of licensing Mathematica will be roughly $2,000 per year. The UW math department currently uses Maple. However, Mathematica is a better system to use for the study of discrete dynamical systems (which is not surprising since Mathematica was designed with that purpose in mind).

The department has no interest in paying to have a Mathematica license in addition to its existing Maple license, so the PI would have to pay the licensing costs out of grant money.

Additional funds will cover the cost of photocopying, postage, reference materials, and computer supplies.

G.3. Consultant Services

In each of the three years, the PI expects to invite collaborators to visit him in Wisconsin. (The PI's two closest collaborators on this project so far, Ander Holroyd and Lionel Levine, are at the University of British Columbia and the University of California at Berkeley, respectively.)

G.4. Computer Services

It will cost an estimated $3,800 in Year 2 to put the entirety of the PI's Fall 2006 course on Discrete Dynamical Systems on the web and to make the course-videos fully available to students taking the course as well as other people with Internet access. A breakdown of this figure can be provided if NSF desires, explaining how the UW's Division of Information Technology arrived at this figure.

G.6. Other

Fee remission for B.3 is 25% (not subject to indirect cost).

I. Indirect costs

The current overhead rate is 47%. The MTDC rate in calculating the indirect cost is provisional and subject to change upon final negotiations with cognizant federal agency.

Current and Pending Support

(See GPG Section II.C.2.h for guidance on information to include on this form.)

The following information should be provided for each investigator and other senior personnel. Failure to provide this information may delay consideration of this proposal.

Investigator: **James Propp**

Other agencies (including NSF) to which this proposal has been/will be submitted. **NSA**

Support: ☒ Current ☐ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support
Project/Proposal Title: **Integrable Recurrence Relations and Combinatorics**

Source of Support: **DOD, National Security Agency**
Total Award Amount: $ **50,000** Total Award Period Covered: **02/17/04 - 02/16/06**
Location of Project: **University of Wisconsin - Madison**
Person-Months Per Year Committed to the Project. Cal: **0.00** Acad: **0.00** Sumr: **1.00**

Support: ☐ Current ☐ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support
Project/Proposal Title:

Source of Support:
Total Award Amount: $ Total Award Period Covered:
Location of Project:
Person-Months Per Year Committed to the Project. Cal: Acad: Sumr:

Support: ☐ Current ☐ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support
Project/Proposal Title:

Source of Support:
Total Award Amount: $ Total Award Period Covered:
Location of Project:
Person-Months Per Year Committed to the Project. Cal: Acad: Sumr:

Support: ☐ Current ☐ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support
Project/Proposal Title:

Source of Support:
Total Award Amount: $ Total Award Period Covered:
Location of Project:
Person-Months Per Year Committed to the Project. Cal: Acad: Sumr:

Support: ☐ Current ☐ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support
Project/Proposal Title:

Source of Support:
Total Award Amount: $ Total Award Period Covered:
Location of Project:
Person-Months Per Year Committed to the Project. Cal: Acad: Summ:

*If this project has previously been funded by another agency, please list and furnish information for immediately preceding funding period.

FACILITIES, EQUIPMENT & OTHER RESOURCES

FACILITIES: Identify the facilities to be used at each performance site listed and, as appropriate, indicate their capacities, pertinent capabilities, relative proximity, and extent of availability to the project. Use "Other" to describe the facilities at any other performance sites listed and at sites for field studies. USE additional pages as necessary.

Laboratory:

Clinical:

Animal:

Computer:

Office:

Other:

MAJOR EQUIPMENT: List the most important items available for this project and, as appropriate identifying the location and pertinent capabilities of each.

OTHER RESOURCES: Provide any information describing the other resources available for the project. Identify support services such as consultant, secretarial, machine shop, and electronics shop, and the extent to which they will be available for the project. Include an explanation of any consortium/contractual arrangements with other organizations.

In Year 2 I will put the entirety of my Fall 2006 course on Discrete Dynamical Systems on the web and make the course-videos fully available to students taking the course as well as other people with Internet access. This service will be provided by UW's Division of Information Technology.