the *Notices* articles are intended as an introduction to a topic rather than as reviews, and editorial policy is that references should be designed as pointers to the immediately relevant literature and not as exhaustive lists. We mention this explicitly in the text. Our theme is the unified view deriving from the conditioning property and the logarithmic asymptotics; Vershik's approach is quite different and as such is not central to our theme.

To Arnold's specific points. The Poisson–Dirichlet approximation for the large prime factors we attribute, correctly, to Billingsley (1972). For combinatorial structures, our interest is precisely in the fact that the Poisson–Dirichlet approximation holds for a huge class of structures and not just for isolated instances, for which we cite the appropriate reference, Hansen (1994). It would have been polite to have mentioned Vershik's theorem for uniformly distributed random permutations at this juncture, but hardly essential. The fact that this limiting structure is common to primes and combinatorial structures we attribute not to ourselves, but to Knuth and Trabb Pardo (1976), a reference ten years earlier than that quoted by Arnold.

There seems little point in discussing the remainder of Arnold's letter. We should, however, mention that Vershik has been an honoured guest, not only in Zürich, but also at the University of Southern California, as have innumerable other Russian mathematicians. Also, that incorrect attribution does not work exclusively against Russians, as evidenced by continuing reference to that most useful and widely quoted probability inequality as Chebyshev's.

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(Received February 11, 1998)

How to Teach Limits and Continuity

In recent issues of the *Notices* (May 1997, pp. 559–563; September 1997, pp. 893 and 932–934; and January 1998, p. 6) there are interesting letters of David Mumford, Saunders Mac Lane, Leonard Gillman, and Peter D. Lax about ways of teaching limits, continuity, and uniform continuity. But I believe that the authors do not point their fingers at the root of the difficulty and how to overcome it.

The difficulty, I believe, comes from the fact that quantifiers constitute an advanced linguistic tool whose goal is to avoid the introduction of ad hoc names and symbols. The students whom we teach are not sufficiently used to this tool, and hence they find it difficult to overcome this linguistic barrier.

As always, the solution is to explain to the students what we have in mind. Thus when we claim that $a_n \to a$, we mean that we have a function $N(\epsilon)$ from positive reals to positive integers, such that

$$(*) \quad n > N(\epsilon) \text{ implies } |a_n - a| < \epsilon.$$

When we say that f is continuous, we mean that we have a function $\delta(x, \epsilon)$, where x is any element in the domain of f and ϵ is any positive real, such that

$$(**) \quad |x - y| < \delta(x, \epsilon) \text{ implies } |f(x) - f(y)| < \epsilon.$$

After all, the only way to prove $a_n \to a$ or to prove that f is continuous is to build the appropriate functions $N(\epsilon)$ or $\delta(x, \epsilon)$ respectively and to prove (*) or (**) for those functions. And I believe that a student who is not able to prove that, say, $1/n^{1/2} \to 0$ or that $x^{1/2}$ is continuous does not understand convergence or continuity. On the other hand, a student who is able to prove a few theorems of that kind does understand those concepts.

One final remark: A naive interpretation of quantifiers accepts Platonism or physicalism, namely, a Platonic or a physical existence of all the elements of the domains to which the quantifiers are referring. Operations such as $N(\epsilon)$ and $\delta(x, \epsilon)$, which are called Skolem functions, justify the use of quantifiers without forcing us to accept Platonism or physicalism. Indeed, we construct those operations in our brains, but we do not (and cannot) build all the elements of their domains or ranges, since, as a rule, those sets are too large. Thus quantifiers are mere abbreviations by means of which we can avoid (but not always) naming or denoting some Skolem functions.

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(Received February 2, 1998)

Keep Young Scholars Programs Running

The NSF's decision to cut the funding for the Young Scholars program ("The Demise of the Young Scholars Program", *Notices*, March 1998) is a tragic mistake. The total cost of these programs, \$10 million a year, is relatively small, yet the potential benefit to society is enormous. The argument that "these students are already highly talented and motivated and such programs simply add to their advantages" badly misses the mark. In fact, many of these youngsters are intellectually and socially isolated in their school environments, and it is by no means certain that they will, without help, fulfill their potential. These programs affirm that what they are interested in is worthwhile and valued and not merely "weird". My daughter, Lenore, who is a mathematician, was in David Kelly's fine program at Hampshire, and I am by no means certain that she would be in our profession if it weren't for that experience.

If NSF funding cannot be restored, other sources should be pursued. (Bill Gates, are you listening?) Also, those of us who can afford to do so should contemplate making donations to those programs which are still in existence. It is money well spent!

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(Received February 13, 1998)