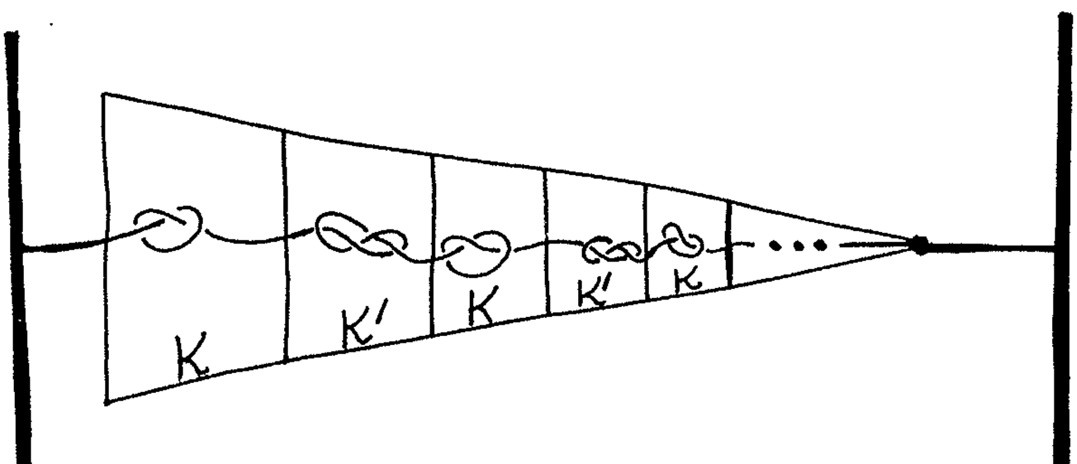
same argument transposed to higher dimensions has this limitation. For example in dimension 7, there are manifolds M and M' homeomorphic to spheres but not diffeomorphic to spheres such that M#M' is diffeomorphic to the standard 7 sphere (See [KM]).

You Can't Cancel Knots

Tie a knot in a piece of rope and then tie another knot adjacent to it. (In this picture of knots, one is *not allowed* to move any rope past the end points. Think of the end-points as attached to opposite walls of a room. With the ends attached to the wall, the rope can be moved so long as it is not removed from the wall or torn apart.)

Is it possible that the two knots taken together can undo one another even though they are individually knotted? The answer is NO. The proof is by infinite repetition [F]: Let O denote the unknot. Let K#K' denote the connected sum of knots obtained by adjacent tying. Instantiate K_∞ = K#K'#K#K'#K#... as an infinite weave in a compact space by introducing a limit point as shown below.



Then K_∞ is, by the method of infinite repetition, equal to both K and to O. Hence K must be unknotted.

This argument goes into the larger category of knots with infinite amounts of weave to make its conclusions. In order to show that the conclusion holds in the usual category of finite weaves, a topological theorem is needed stating that if finitely woven knots are equivalent in the larger category of infinite weaves then they are equivalent in the category of finite weaves. The result that supports this conclusion is found in [MO].

The Conway Proof

There is a very beautiful proof of the impossibility of knot cancellation due to John Conway (See [G].). His proof does not go off into infinite weave. Here is a sketch of Conway's proof:

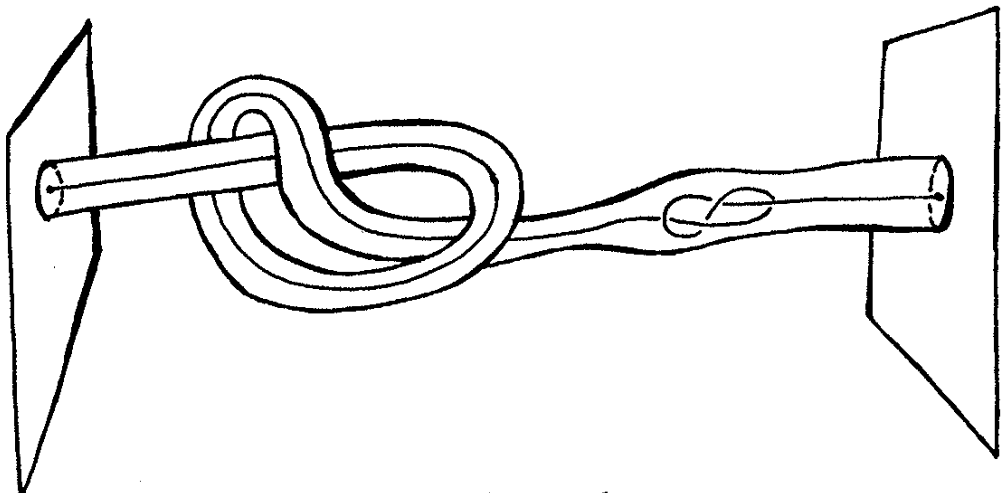


Figure 1

Put a tube T around K#K' (as shown in Figure 1 above) so that the tube is a tubular neighborhood of K and so that the tube engulfs K'. If K#K' = O, then there is a homeomorphism of the room to whose walls K#K' is attached that leaves the walls of the room fixed, and straightens K#K' to a straight line L extending from the left wall to the right wall. The tube T will be deformed by this homeomorphism to a new tube T' that does not intersect the line L. Let P be plane in the room containing L. Then P intersects the left and right walls of the room in the endpoints of L and in four points of the tube T' (two on each wall). Let a and b denote the intersection of P with T' on the left wall and let c and d denote the intersection of P with the right wall. Then P intersects T' in arcs that emanate from a,b,c,d and some closed curves in P. The arc from a cannot reach either b or d because it is separated from these points by the line L in the plane P. Therefore the arc from a must extend to c. This arc AC from a to c is necessarily unknotted in the room, since it is a non-self-intersecting arc in the plane P. However the arc AC is the image under the homeomorphism of an arc extending from one end of the tube T to the other, and by construction, this means that the arc AC must be equivalent to the knot K (since the tube is knotted in the pattern of K). Therefore we have shown that in the course of unknotting K#K' we have necessarily unknotted K itself! Therefore you cannot cancel knots.//

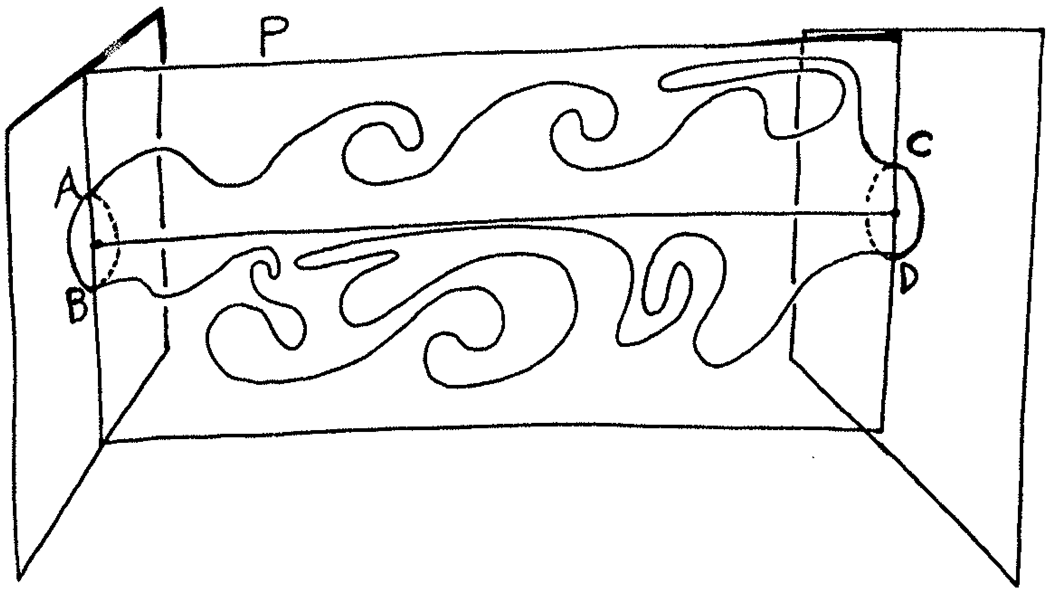
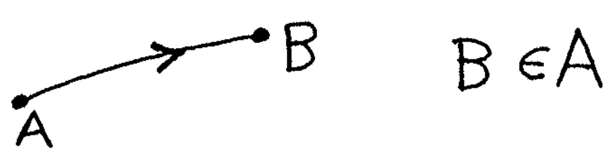


Figure 2

Graphs that Encapsulate Infinity

There is a very elegant way to represent sets in FIST that are described by systems of equations: *Any* directed graph represents such a set.

Each node in the graph represents a set. An edge directed from node A to node B encodes the relation that *B is a member of A.*



(This method of representation is used by Aczel [AC].)
A single finite set is a rooted tree where all the edges are directed away from the root as in the examples preceding this discussion. Nevertheless, any directed graph yields a set, or sets. For example,