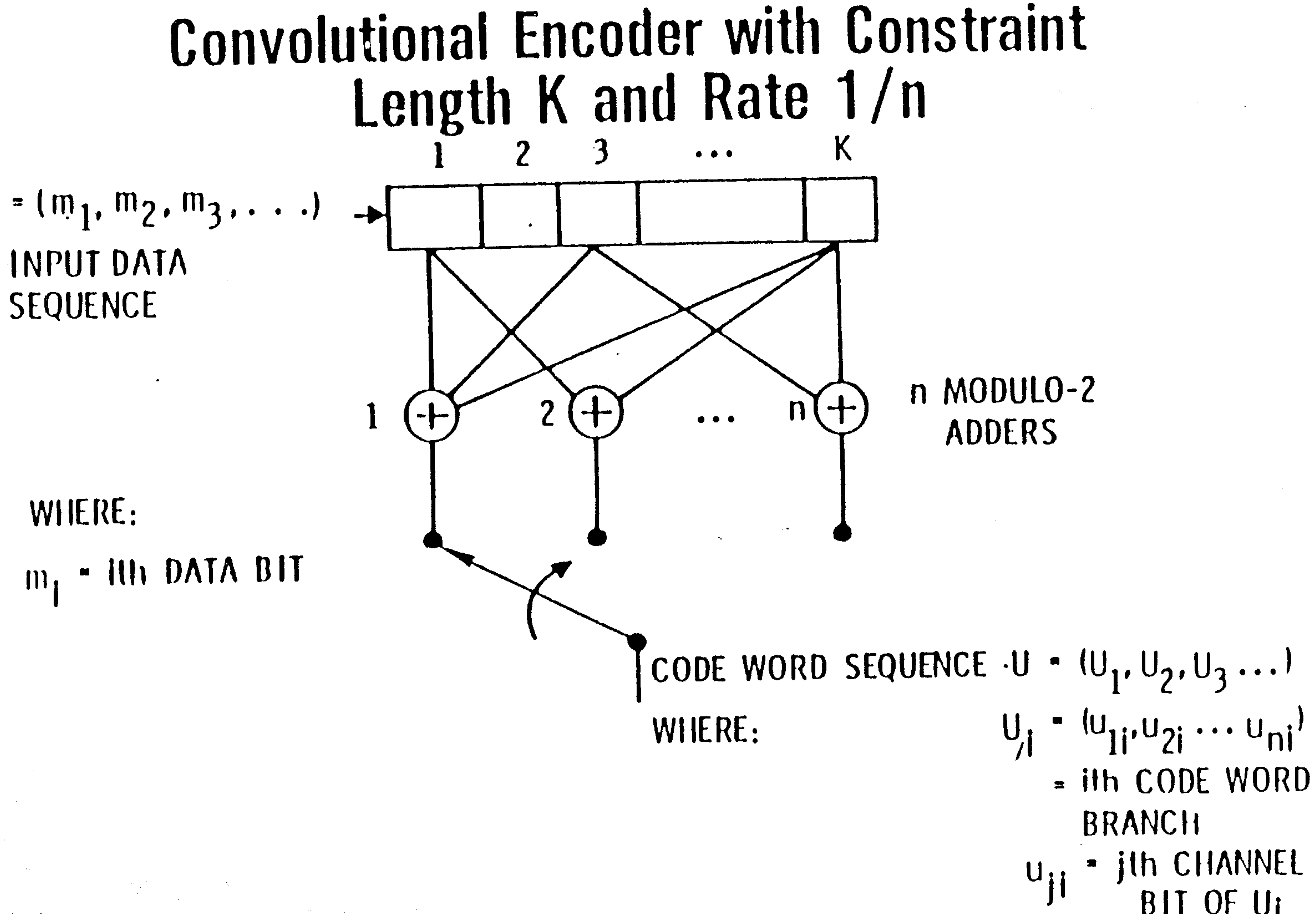
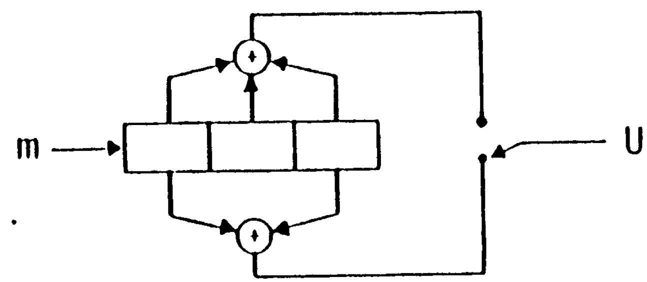
Lecture 9b Convolutional Coding/Decoding and Trellis Code modulation

Convolutional Coder Basics

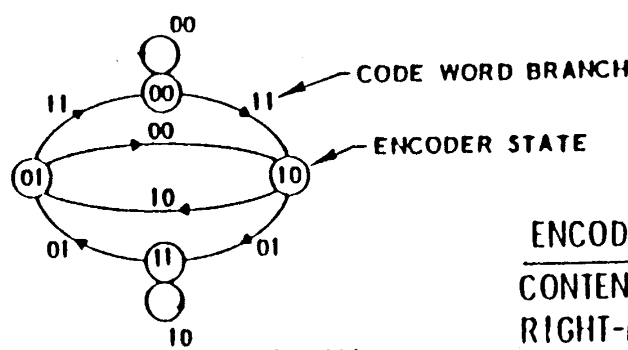


Coder State Diagram

Encoder is Characterized by State Diagram

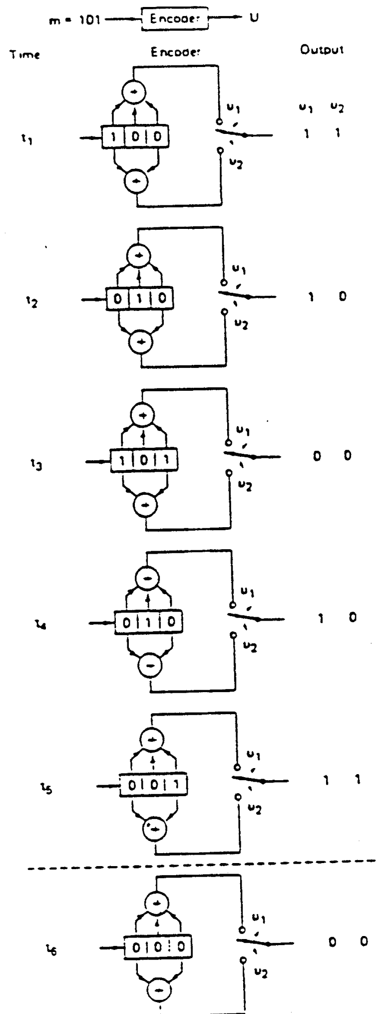


ENCODER, K = 3, 1/n = 1/2



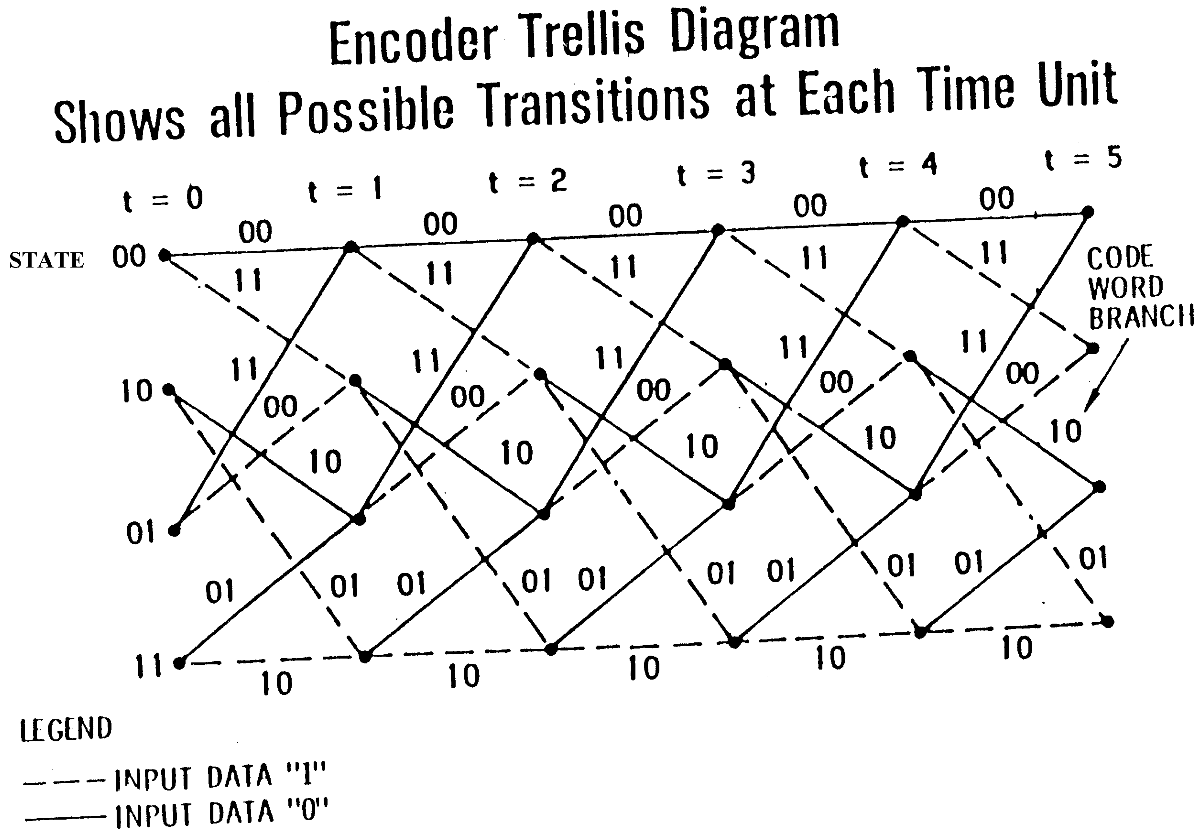
ENCODER STATE DIAGRAM

ENCODER STATE
———
CONTENTS OF THE RIGHT-MOST K-1 STAGES



Output sequence: 11 10 00 10 11

Encoder Trellis



Convolutional Encoding Example

INPUT DATA SEQUENCE = 1 1 0 1 1

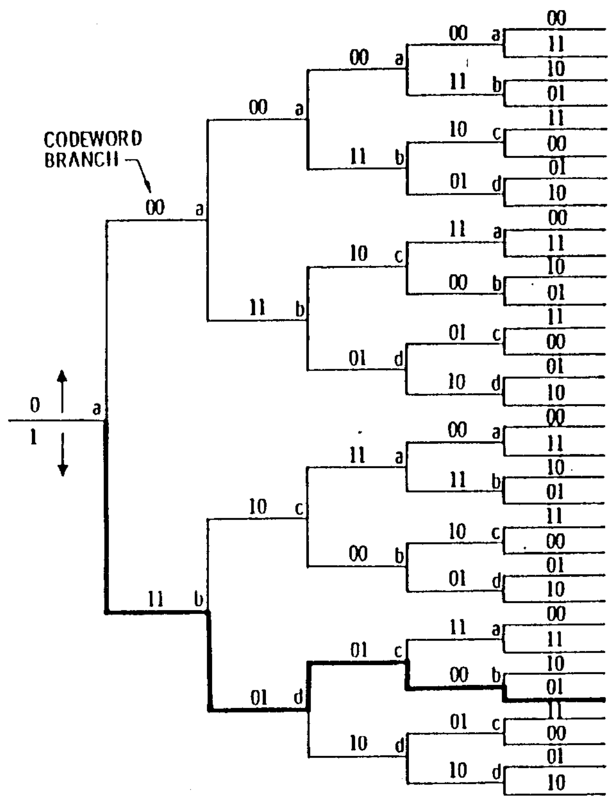
OUTPUT CODE SEQUENCE 11 01 01 00 01

INPUT	REGISTER	STATE t_i	STATE t_{i+1}	BRANCH WORD
1	1 0 0	00	10	11
1	1 1 0	10	11	01
0	0 1 1	11	01	01
1	1 0 1	01	10	00
1	1 1 0	10	11	01

state t_i

state t_{i+1}

Coder Tree



Tree Representation of Encoder

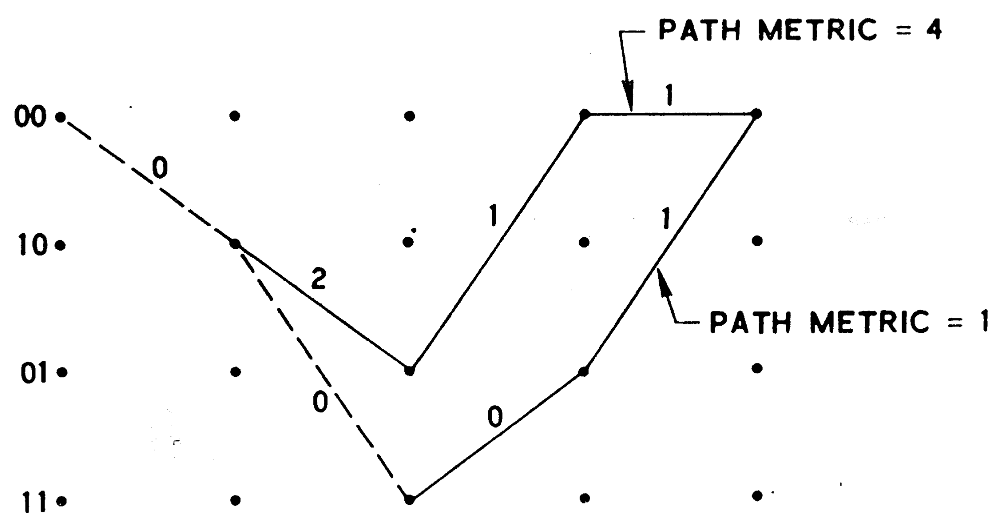
THE INPUT DATA SEQUENCE m = 11011 TRACES THE HEAVY-LINE PATH, HAVING THE CODEWORD SEQUENCE

U = 11 01 01 00 01

Viterbi Decoding

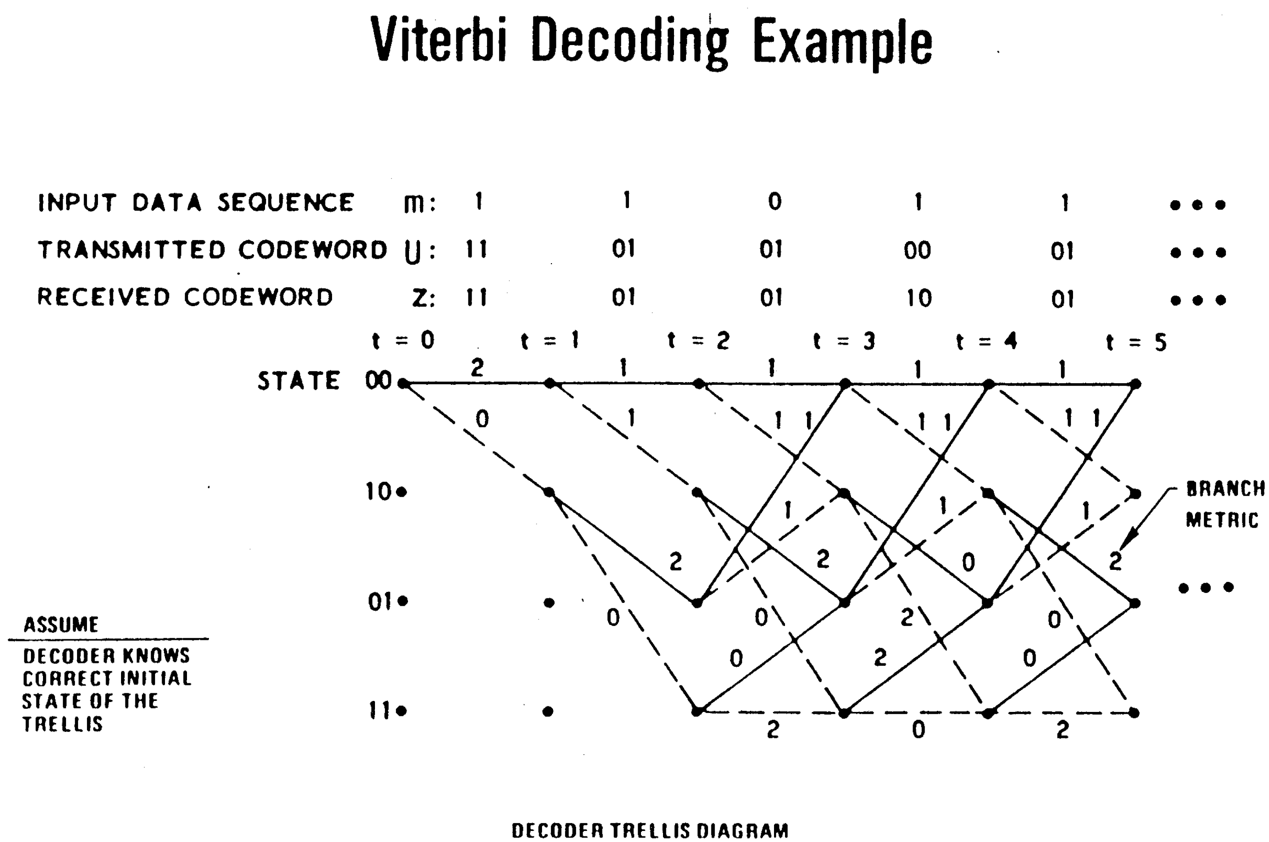
- For Simplicity assume Binary Sym.Channel
- Encoder has Constraint length 3, Rate ½
- A trellis represents the decoder
- Trellis transitions are labeled with branch metrics (hamming distance between branch code word and received codeword
- If two paths merge the path with larger metric is eliminated

If Two Paths Merge, One of Them Can be Eliminated

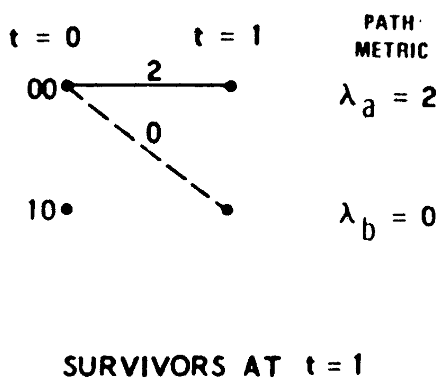


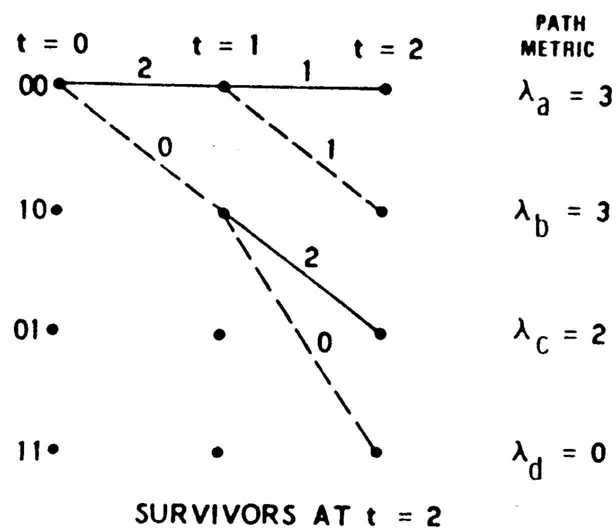
PATH METRICS FOR TWO REMERGING PATHS
AT t = 4

Decoder Trellis

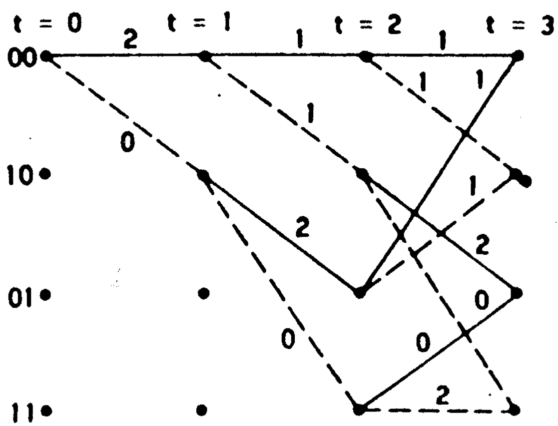


Selection of Survivor Paths

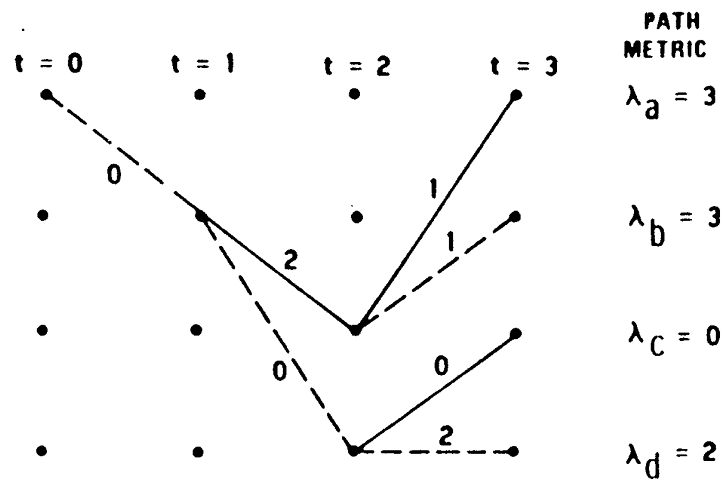




Selection of Survivor Paths (cont'd)



METRIC COMPARISONS AT t = 3

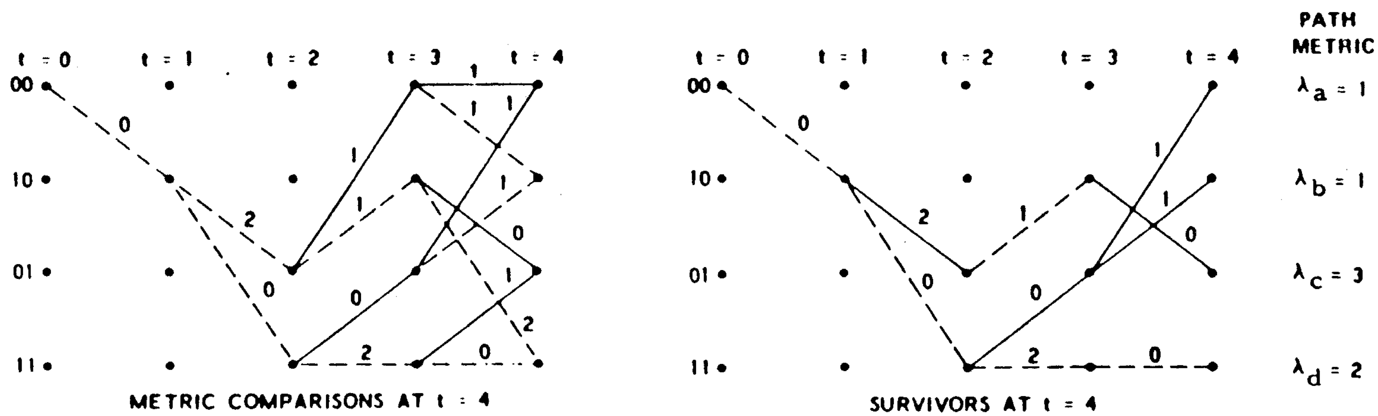


SURVIVORS AT t = 3

ONE PATH ENTERING EACH STATE CAN BE ELIMINATED

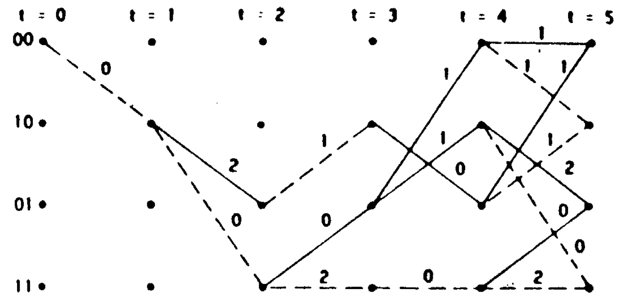
DECODER DECIDES THAT TRANSITION FROM t = 0 TO t = 1, WAS PRODUCED BY DATA BIT "1"

Selection of Survivor Paths (cont'd)

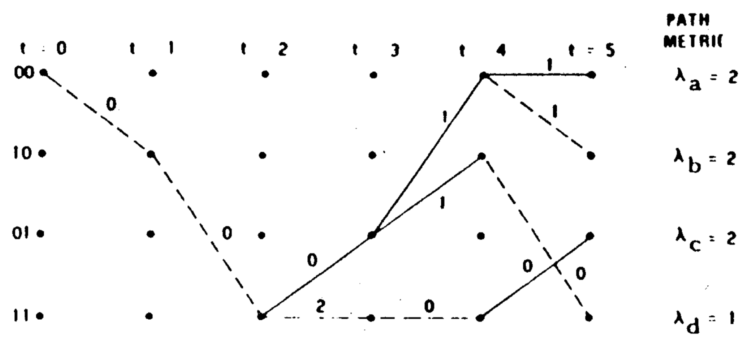


AGAIN, ONE OF TWO PATHS ENTERING
SAME STATE CAN BE ELIMINATED

Selection of Survivor Paths (concluded)



METRIC COMPARISONS AT t = 5



SURVIVORS AT t = 5

DECODER DECIDES, THE SECOND DATA BIT = "1"

Free Distance

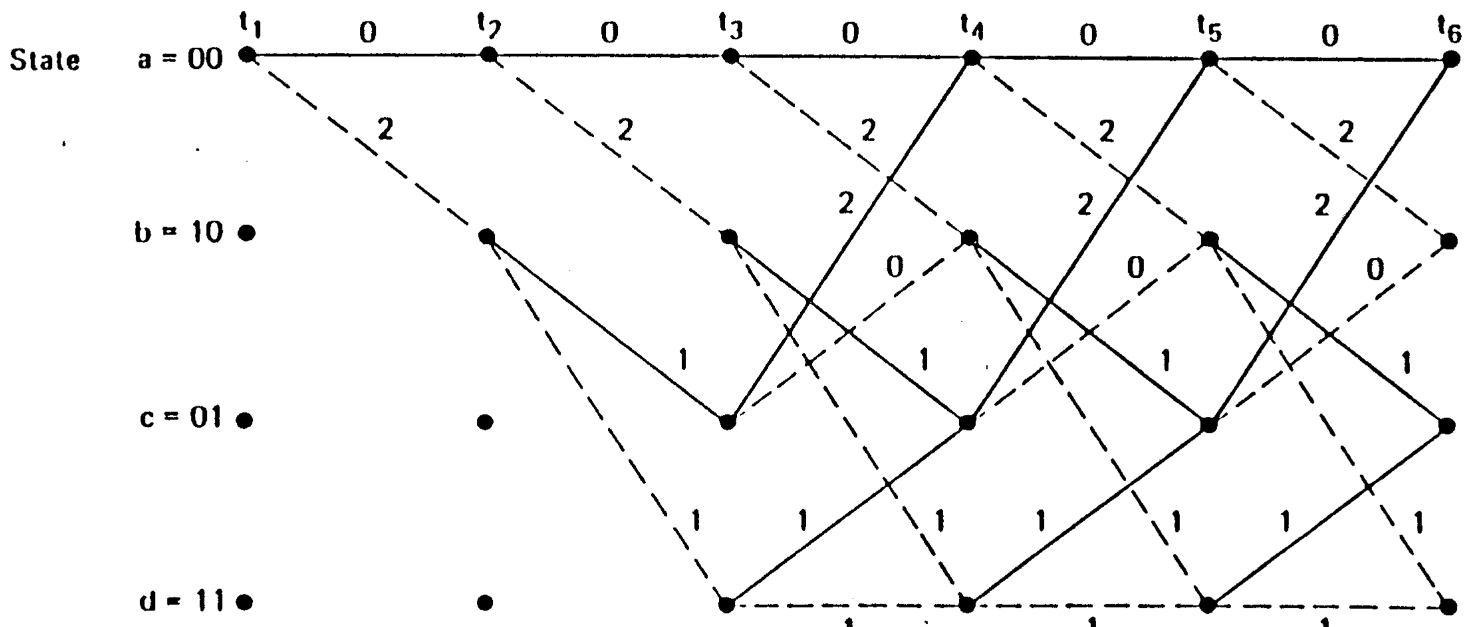
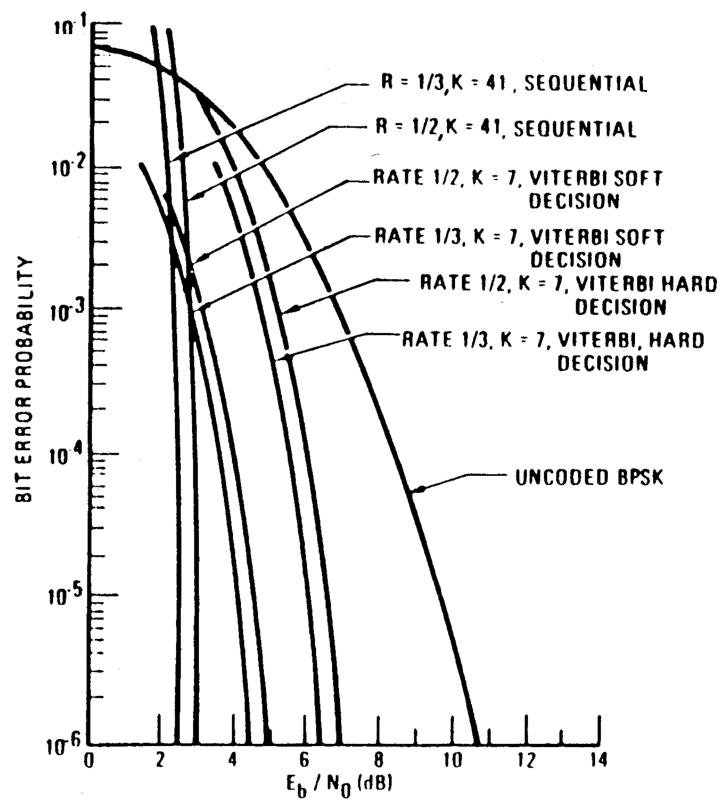


Figure 6.13 Trellis diagram, labeled with distances from the all-zeros path.

free distance (5)

Viterbi Decoder Performance Rate 1/2 vs Rate 1/3 K = 7 Hard vs Soft



Free Distance and Coding Gain

- FREE EUCLIDEAN DISTANCE, d_f, IS THE MINIMUM DISTANCE, IN EUCLIDEAN UNITS, BETWEEN A SELECTED CODE SEQUENCE AND EACH OF THE POSSIBLE ERROR-EVENT PATHS

- A LOWER BOUND ON ERROR-EVENT PROBABILITY IS GIVEN BY

$$P_E \geq Q(d_f/2\sigma)$$

WHICH IS ASYMPTOTICALLY EXACT AT HIGH SNR

- THE ASYMPTOTIC CODING GAIN IS THEREFORE DEFINED AS

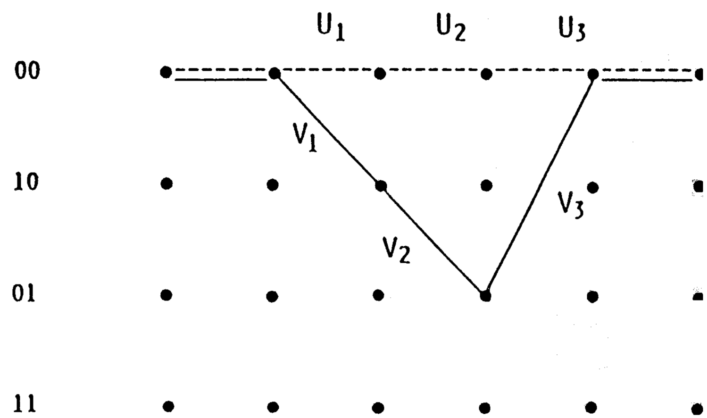
$$G = 20 \log_{10} (d_f/d_{ref})$$

- FOR HIGH SNR AND A GIVEN ERROR PROBABILITY, THIS YIELDS THE SAME RESULT AS

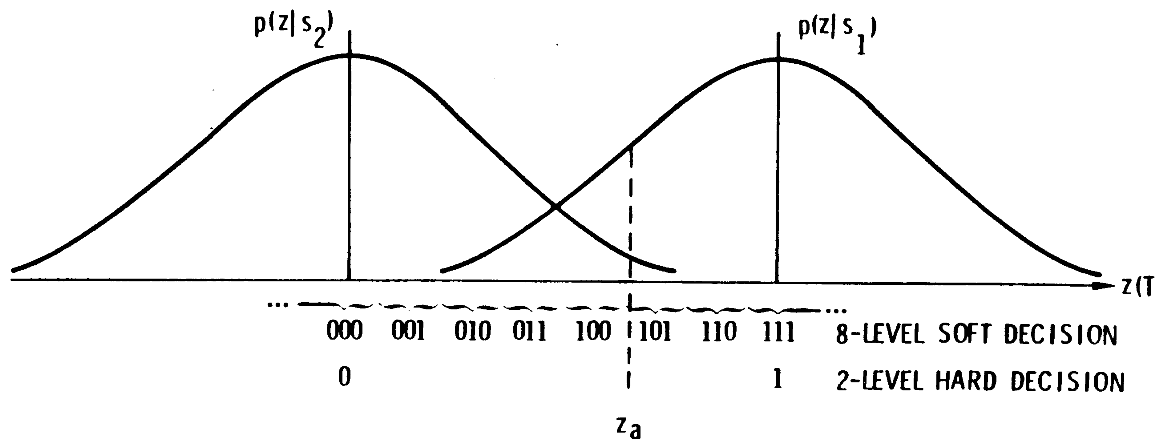
$$G = (E_B/N_0)_{coded} - (E_B/N_0)_{uncoded}$$

WHERE E_B/N_0 IS EXPRESSED IN DECIBELS

ILLUSTRATION OF AN ERROR EVENT



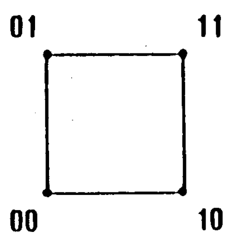
Hard and Soft Decoding Decisions



- EVENTUALLY, ALL DATA DECISIONS MUST BE HARD DECISIONS (binary)
- SOFT DECISIONS CONTAIN CONFIDENCE MEASURES REGARDING THE BINARY DECISIONS
- CONVOLUTIONAL DECODING USING SOFT DECISIONS, WITH 3 bits OF QUANTIZATION, PERFORMS APPROXIMATELY 2 dB BETTER THAN HARD DECISIONS

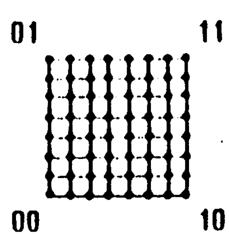
● CONSIDER A RATE 1/2 BINARY CODE, SUCH THAT EACH INPUT BIT YIELDS A PAIR OF CODE SYMBOLS

HARD DECISION (2-level quantization)



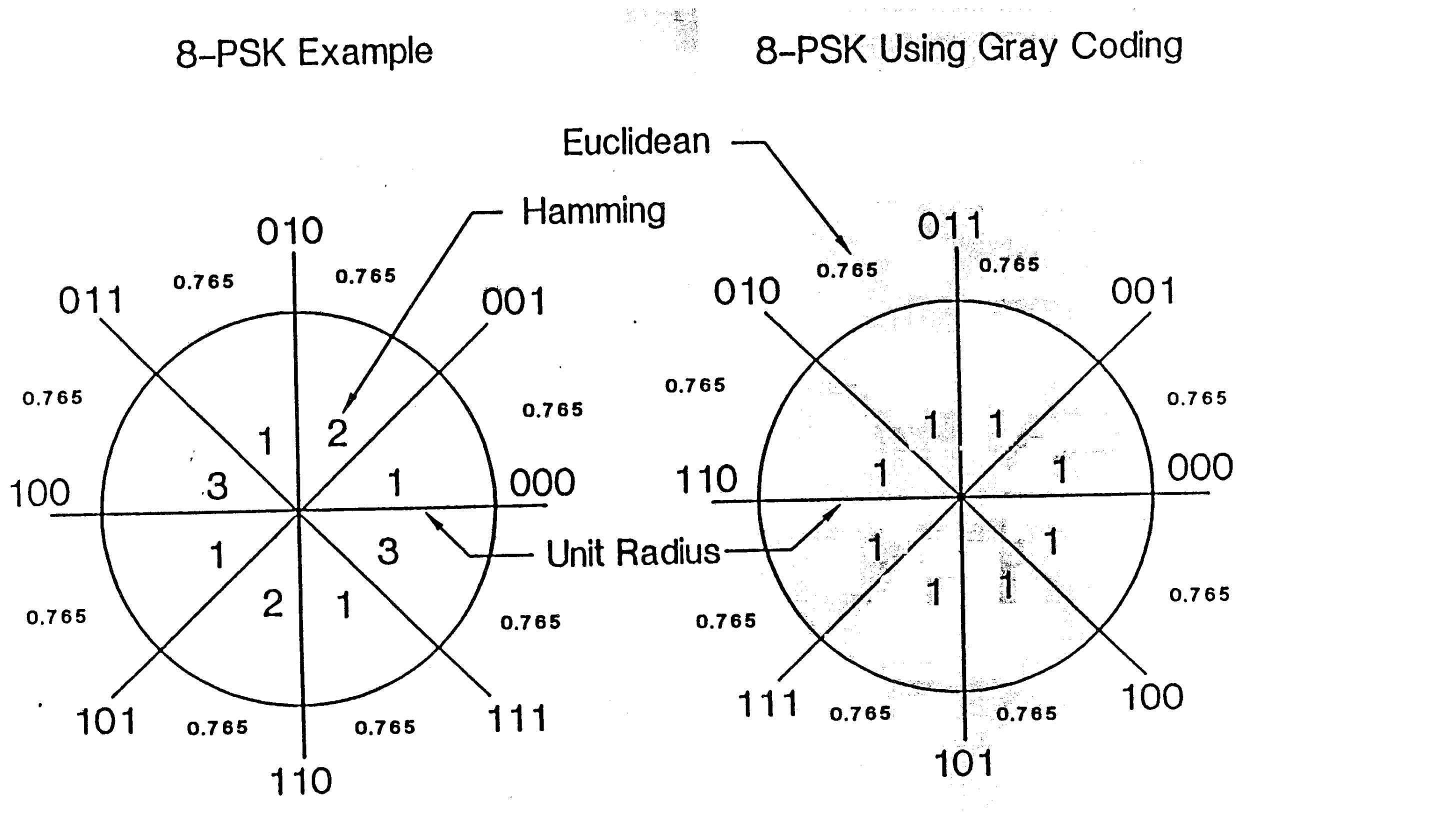
● WE CAN REPRESENT EACH PAIR OF RECEIVED CODE SYMBOLS AS ONE OF THE CORNERS OF A SQUARE

SOFT DECISION (8-level quantization)



● WE CAN REPRESENT EACH PAIR OF RECEIVED CODE SYMBOLS AS ONE POINT OUT OF THE SET OF ALLOWABLE DISCRETE POINTS IN THE PLANE

No relationship between Hamming and Euclidean Distance

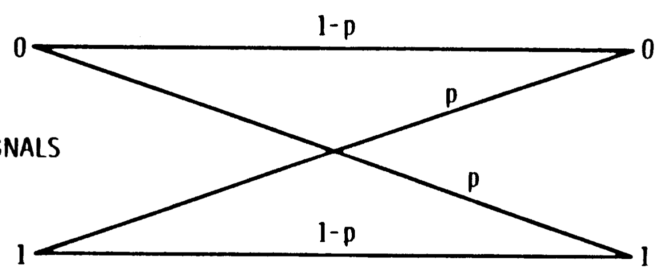


Channel Models

BINARY SYMMETRIC CHANNEL

(HARD DECISION CHANNEL)

XMTD SIGNALS

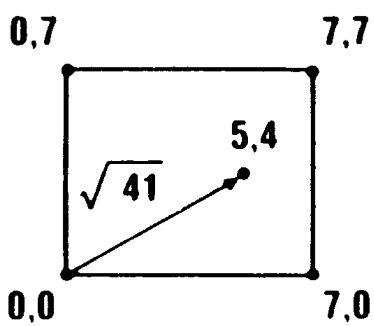


RCVD SIGNALS

ONE OF TWO DISCRETE SYMBOLS RATHER THAN A CONTINUOUS AMPLITUDE SIGNAL

- HAMMING DISTANCE IS APPROPRIATE METRIC TO DESCRIBE THE DISTANCE BETWEEN U AND Z
- U IS CHOSEN TO MINIMIZE HAMMING DISTANCE

Euclidean Distance



- ASSUME 8-LEVEL QUANTIZATION, 0 TO 7 (allowing only positive integers)
- IF A PAIR OF CODED BITS, 1 1, ARE TRANSMITTED, THEN THE TWO RECEIVED CODE SYMBOLS ARE 7,7 (in the absence of noise)
- THE EUCLIDEAN DISTANCE OF THE RECEIVED SYMBOLS FROM THE 0,0 SYMBOLS IS $7\sqrt{2}$
- IF THE TWO RECEIVED CODE SYMBOLS ARE 5,4, THEN THE EUCLIDEAN DISTANCE FROM THE 0,0 SYMBOLS IS $\sqrt{41}$

A PROCEDURE FOR SOFT DECISION DECODING OF M-ARY SIGNALS

- SOFT DECISION DECODING OF BINARY SIGNALS USING A EUCLIDEAN METRIC IS WELL KNOWN

- HOW CAN SOFT DECISION DECODING OF M-ARY SIGNALS BE ACCOMPLISHED?

- THE MATCHED FILTER OUTPUT AMPLITUDES, A_i, WHERE $i = 0, \ldots, M-1$, ARE FIRST NORMALIZED, SO THAT

$$a_i \qquad \frac{A_i}{\sum_{i=0}^{M-1} A_i}$$

- FOR A BINARY CODE, EACH RECEIVED SYMBOL MUST BE CONVERTED TO k BITS, WHERE $k = \log_2 M$, BEFORE DECODING

- THE SYMBOL-TO-BIT CONVERSION YIELDS SOFT DECISION SYMBOLS b_i WHERE $i = 0, \ldots k-1$

- AT EACH DETECTION INTERVAL A SEQUENCE OF k-DIGIT SOFT DECISION SYMBOLS ARE SENT TO THE DECODER

Introduction to TCM

What is TCM?

- TCM schemes achieve coding gains without using additional bandwidth
- The technique employs non-binary modulation in conjunction with a finite state encoder
- The encoder dictates the selection of modulation waveforms for generating a sequence of coded waveforms
- At the receiver the noisy signals are detected/decoded by a soft decision maximum-likelihood decoder
- TCM is another technique in the evolution of coding methods that contributes toward the fulfillment of shannon;s coding gain prediction

- Today TCM can be used to privde a coding of 3 dB with relative ease
- 6 dB coding gain can be provided with additional complexity

Evolution of TCM

- First Proposed by Ungerboeck and Csajka in 1976 (IBM Research in Swiss)
- A more detailed publication in 1982 received the information theory best paper award
- In 1984 TCM with 4 dB coding gain was adopted by the CCITT for use in high speed voice band modem

Structured Sequence Coding

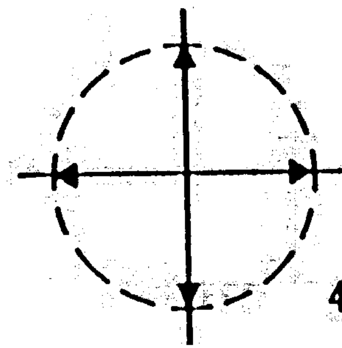
- Channel coding can be partitioned into two study areas: Structured sequence coding (parity bit design) and Waveform coding
- Waveform coding attempts to find better waveforms which provide improved distance properties
- Waveform coding can include redundant waveforms as in the case of TCM

Reasons for Disappointing results of conventional coding methods when the channel is band-limited

- With hard-decision decoding, irreversible errors can be made by the demodulator prior to decoding
- With soft decision decoding (using Euclidean distance), the following problem becomes apparent
- For a code, the optimized for hamming distance, the mapping of code symbols into non-binary modulation waveforms doe not guarantee a good Euclidean distance structure
- Generally one cannot find a monotonic relationship between hamming and Euclidean distances
- Squared Euclidean and hamming distances are equivalent only in the case of binary modulation or 4 phase modulation

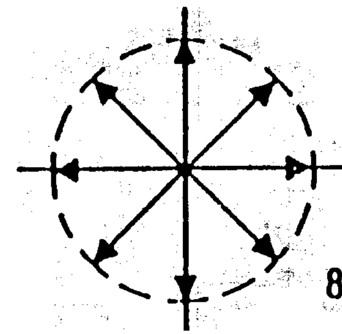
FOR BANDLIMITED CHANNELS, THE CONVENTIONAL CODING METHODS ARE DISAPPOINTING

- CONSIDER UNCODED 4-PSK WITH $P_B = 10^{-5}$



4-ary PSK

- FOR 8-PSK WITH THE SAME SNR, $P_B > 10^{-2}$ BECAUSE OF THE DECREASED DISTANCE BETWEEN THE 8-PSK SIGNAL VECTORS



8-ary PSK

- A RATE 2/3 BINARY CONVOLUTIONAL CODE WITH CONSTRAINT LENGTH 7, CAN REDUCE P_B TO 10^{-5}

- THUS, AFTER USING A FAIRLY COMPLEX (64 STATE) VITERBI DECODER, P_B ONLY BREAKS EVEN WITH UNCODED 4-PSK

Coding gain when bandwdith expansion is allowed

- WHEN COMPARED TO UNCODED BPSK AT A BIT ERROR PROBABILITY OF 10^{-5}, SUCH CODES OFFER THE FOLLOWING CODING GAINS

CODE RATE	HARD VS SOFT DECISIONS	CODING GAIN (dB)
1/2	HARD	3
1/3	HARD	3.5
1/2	SOFT	5
1/3	SOFT	5.5

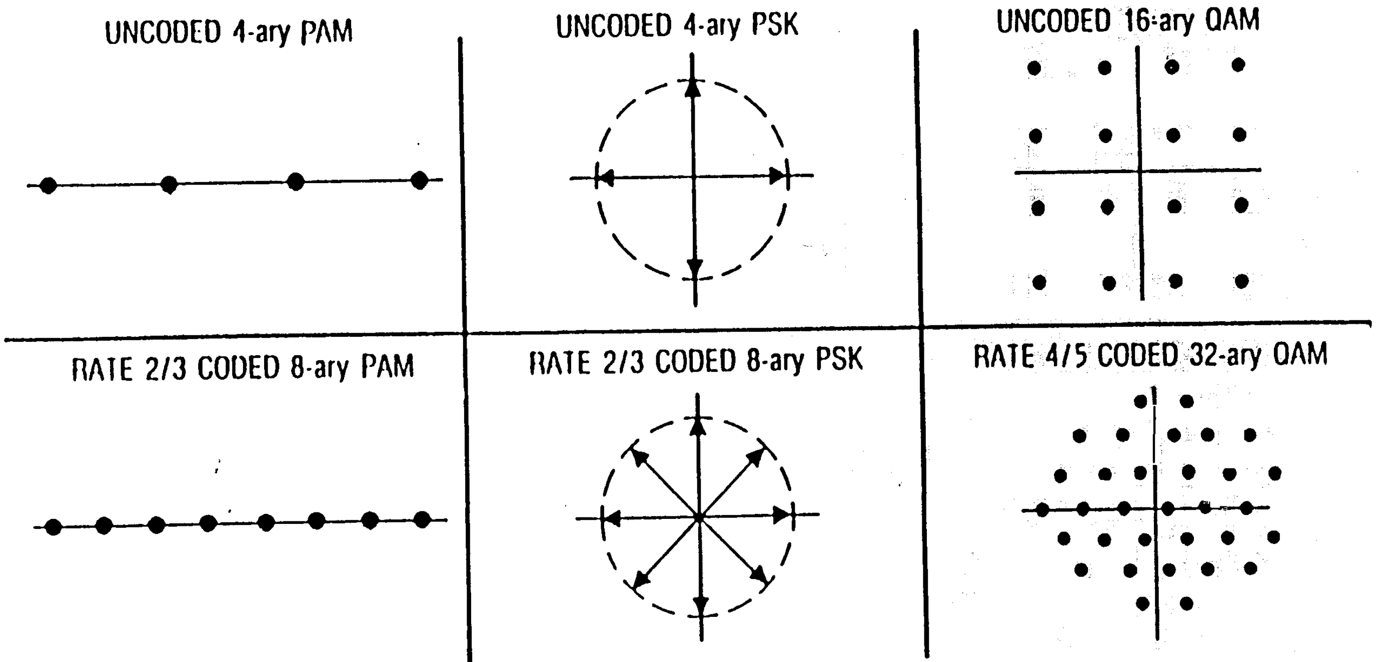
- THE RATE 1/2 CODE REQUIRES AN INCREASE OF BANDWIDTH BY A FACTOR OF 2 OVER UNCODED TRANSMISSION

Theory of TCM

- SIGNAL-SET EXPANSION PROVIDES REDUNDANCY FOR CODING
- MOST OF THE ACHIEVABLE CODING GAIN CAN BE OBTAINED BY EXPANDING THE SIGNAL SET BY A FACTOR OF 2 OVER THAT USED FOR UNCODED MODULATION. THUS THE SIGNAL SET SIZE $M = 2^{k+1}$ FOR THE TRANSMISSION OF k BITS PER MODULATION INTERVAL
- CODING AND SIGNAL-MAPPING FUNCTIONS ARE DESIGNED JOINTLY SO AS TO MAXIMIZE THE FREE EUCLIDEAN DISTANCE
- THIS ALLOWS FOR THE CONSTRUCTION OF MODULATION CODES SUCH THAT THE FREE DISTANCE SIGNIFICANTLY EXCEEDS THE MINIMUM DISTANCE BETWEEN UNCODED MODULATION SIGNALS (AT THE SAME INFORMATION RATE, BANDWIDTH, AND SIGNAL POWER)
- TCM EXTENDS THE PRINCIPLES OF CONVOLUTIONAL CODING FROM THE DOMAIN OF BINARY BASEBAND SIGNALS TO THE DOMAIN OF NONBINARY MODULATION WAVEFORMS

Theory of TCM

- CHOOSE A SIGNALING ALPHABET, LARGER THAN THE BASIC DATA ALPHABET BY USING MULTILEVEL SIGNALING
- FOR EXAMPLE:



- ALPHABET SIZE IS INCREASED TO PROVIDE THE REDUNDANCY NEEDED FOR CODING. NEITHER BANDWIDTH NOR AVERAGE POWER IS INCREASED
- THE CODED POINTS ARE CLOSER TO EACH OTHER THAN THE UNCODED POINTS. HOWEVER d_{min} OF A TRELLIS CODE IS GOVERNED BY DEPENDENCIES INTRODUCED BY THE CONVOLUTIONAL ENCODER, RATHER THAN THE DISTANCE BETWEEN POINTS IN THE SIGNAL SPACE

Set Partitioning Rules for 8-PSK

- All signals should occur with equal frequency and with a fair amount of regularity and symmetry
- Transitions originating from the same state are assigned signals either from subset B0 or B1
- Transitions joining in the same state are assigned signals either from subset B0 or B1
- Parallel transitions receive signals either from subset C0 or C1 or C2 or C3

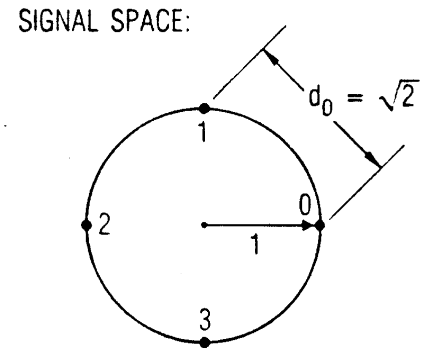
Partitioning (cont'd)

- Rule 1 follows our intuition that good codes should have a regular structure
- Rules 2,3,4 guarantee that bit sequences are assigned to waveforms so that the free distance will exceed the free distance of the uncoded 4 PSK reference modulation by at least 3 dB
- Parallel transitions refer to the branch words resulting from the transmission of uncoded bits along with coded bits.

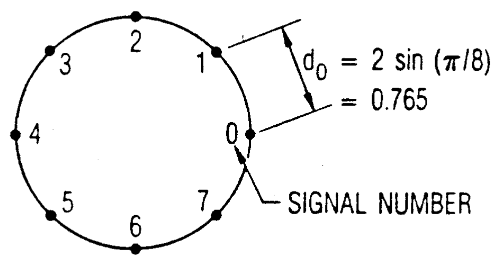
TCM Code Construction

- First a suitable trellis structure is selected. This can be done without any particular encoder in mind
- If k bits are to be encode per modulation interval then there must be 2^k possible transitions from each state to a successor state
- More than one transition (Parallel transitions) may occur between pairs of states
- Next from an extended set of 2^k+1 modulation signals, assignments of signals to trellis transitions will be made so as to maximize the free Euclidean distance

Modulation Signal Sets



(a) UNCODED 4-PSK



(b) RATE 2/3 CODED 8-PSK

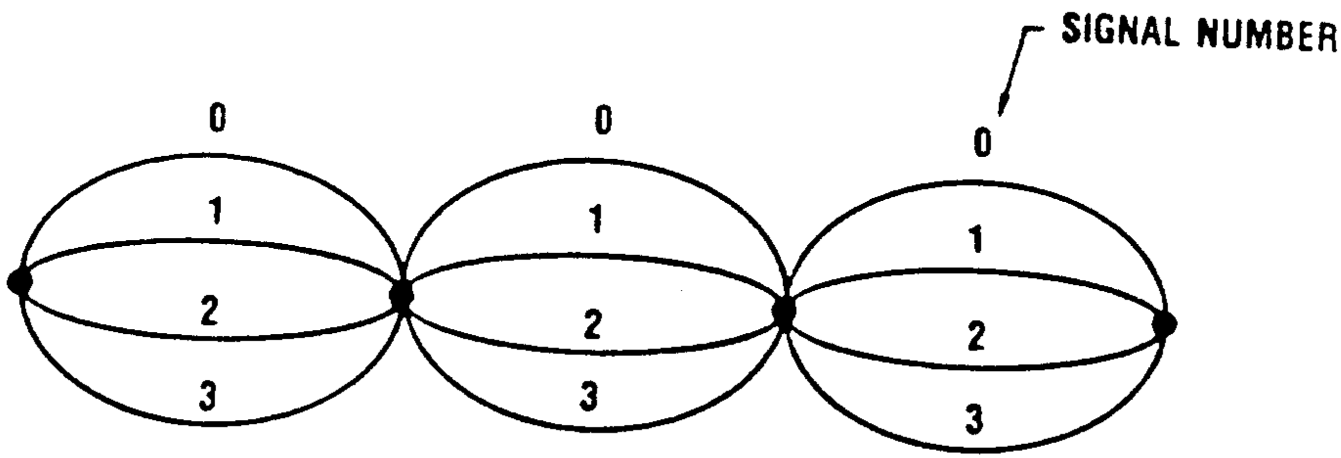


Figure 5. One-state trellis diagram for uncoded 4-PSK.

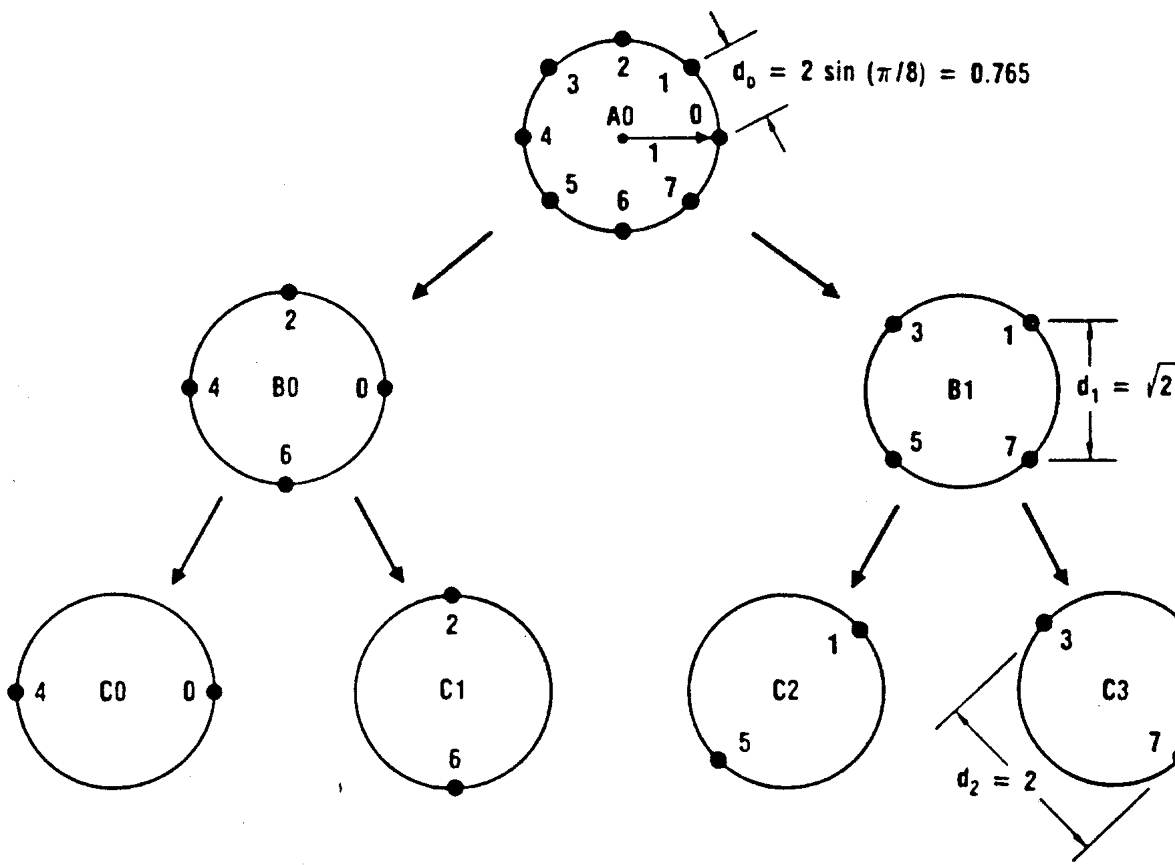
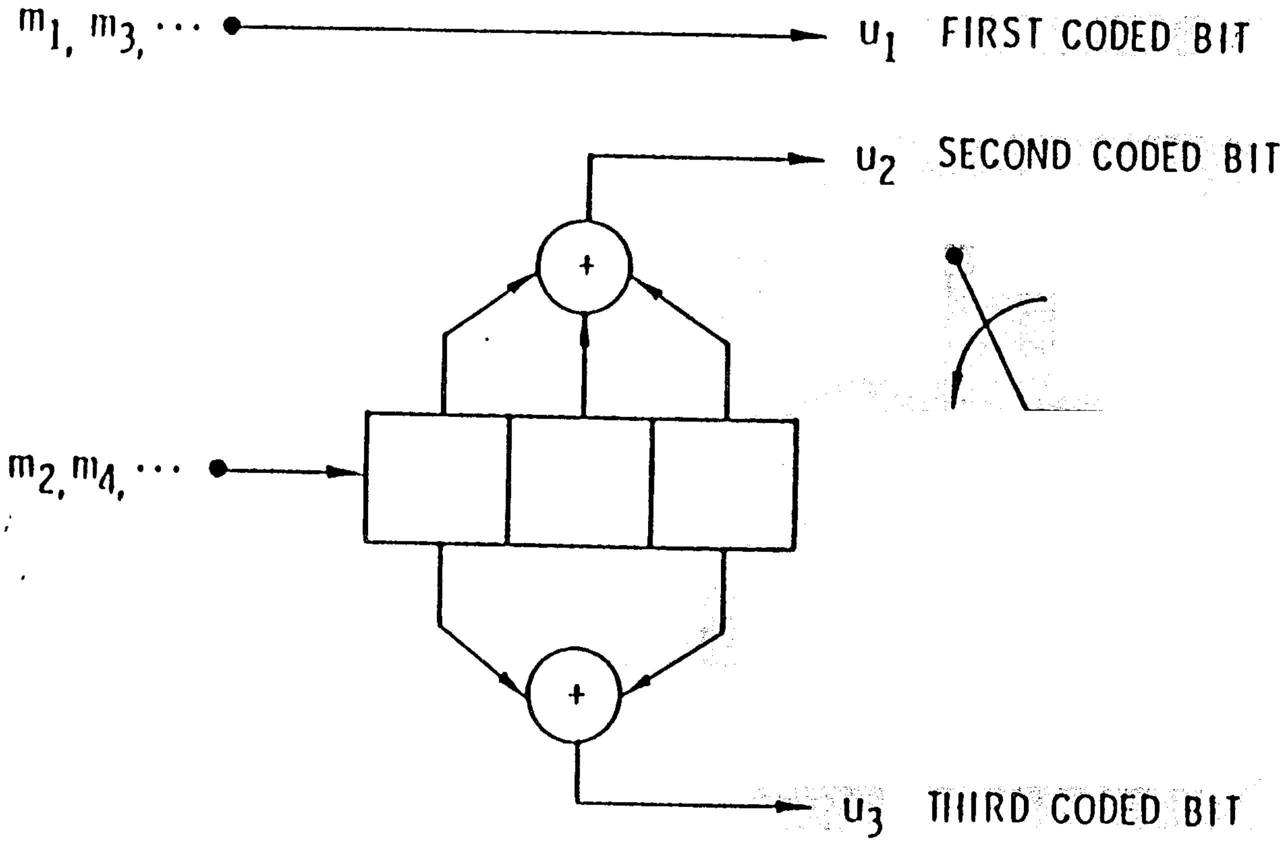


Figure 4. Ungerboeck partitioning of 8-PSK signal set.

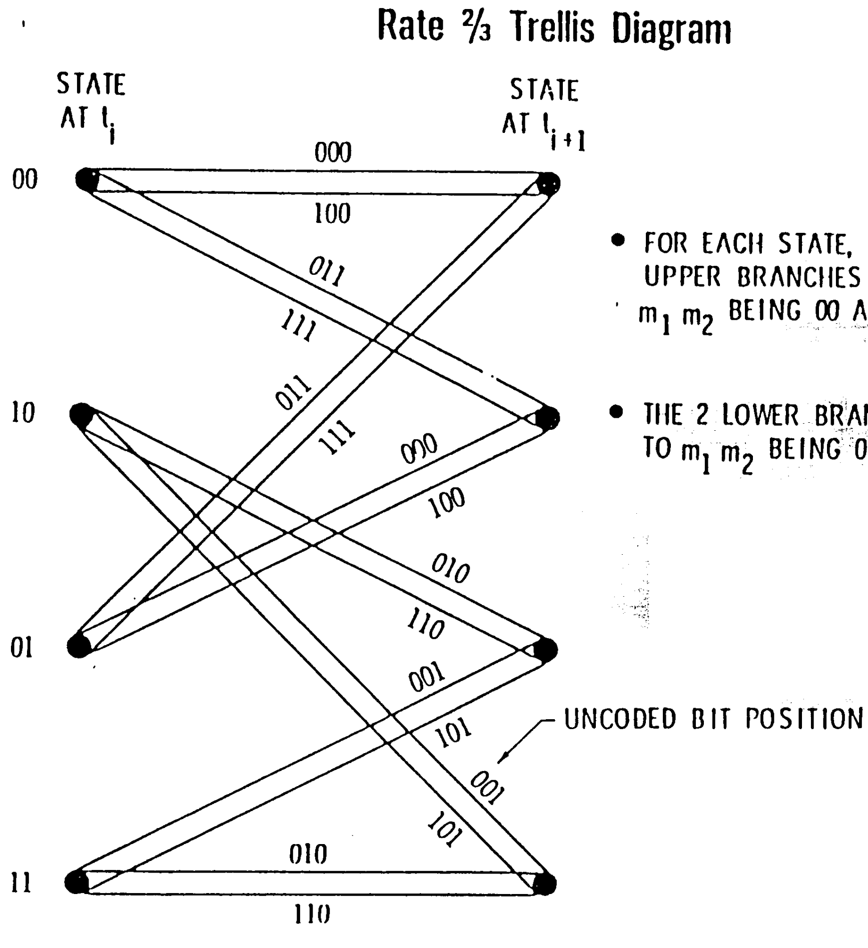
4-TCM Encoder



EACH ODD NUMBERED INPUT BIT REMAINS UNCODED

EACH EVEN NUMBERED INPUT BIT IS ENCODED BY THE RATE 1/2 ENCODER

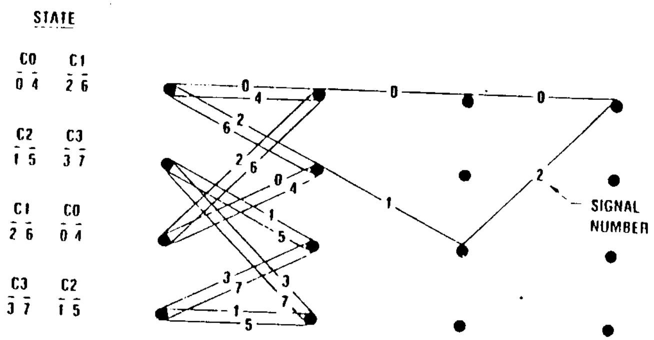
TCM Trellis



- FOR EACH STATE, THE 2 EMERGING UPPER BRANCHES CORRESPOND TO $m_1 m_2$ BEING 00 AND 10
- THE 2 LOWER BRANCHES ARE DUE TO $m_1 m_2$ BEING 01 AND 11

4 State Trellis with parallel Paths

Four-State Trellis (with Parallel Paths) for Coded 8-PSK



ERROR-EVENT PATH 2, 1, 2:

$$d = \sqrt{d_1^2 + d_0^2 + d_1^2} = \sqrt{2 + 0.585 + 2} = 2.2$$

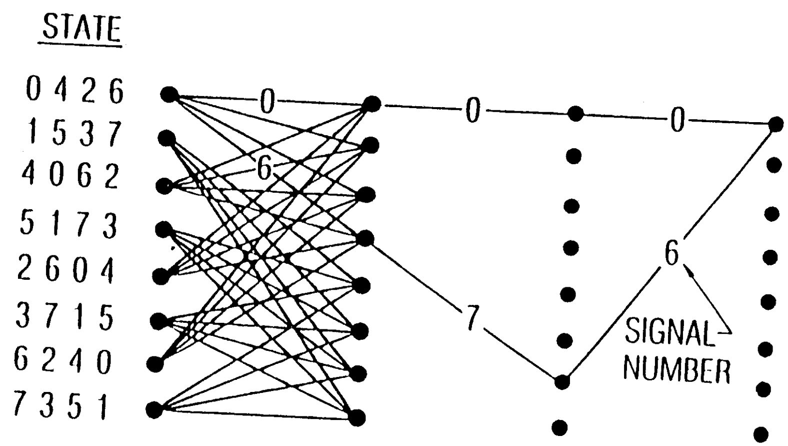
ERROR-EVENT PATH 4 (the parallel transition):

$$d = d_2 = 2$$

- THUS, FOR THIS EXAMPLE, THE FREE EUCLIDEAN DISTANCE IS 2
- ASYMPTOTIC GAIN $= 10 \log_{10} \dfrac{(d_2^2)_{\text{CODED 8 PSK}}}{(d_{rel}^2)_{\text{UNCODED 4 PSK}}} = 10 \log_{10}\left(\dfrac{4}{2}\right) = 3 \text{ dB}$

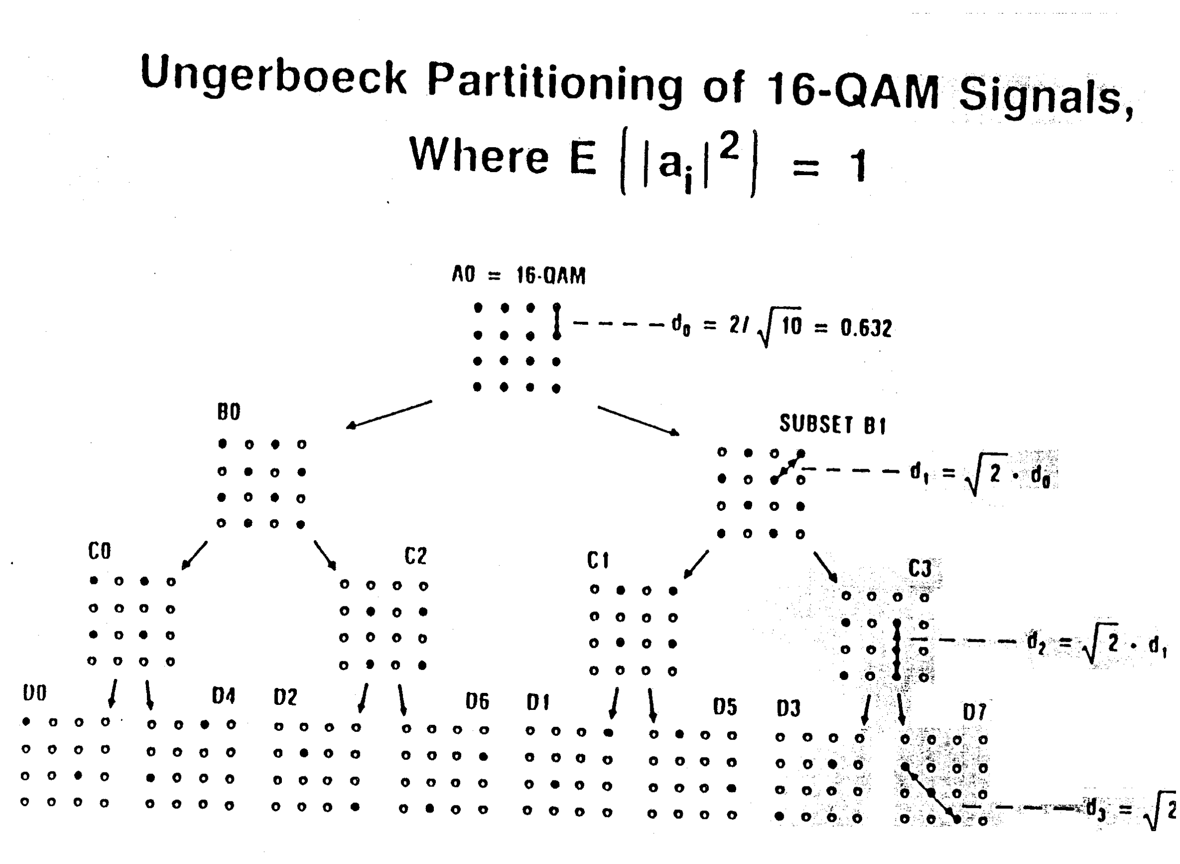
8 state trellis with partitioning

Eight-State Trellis Diagram for Coded 8-PSK

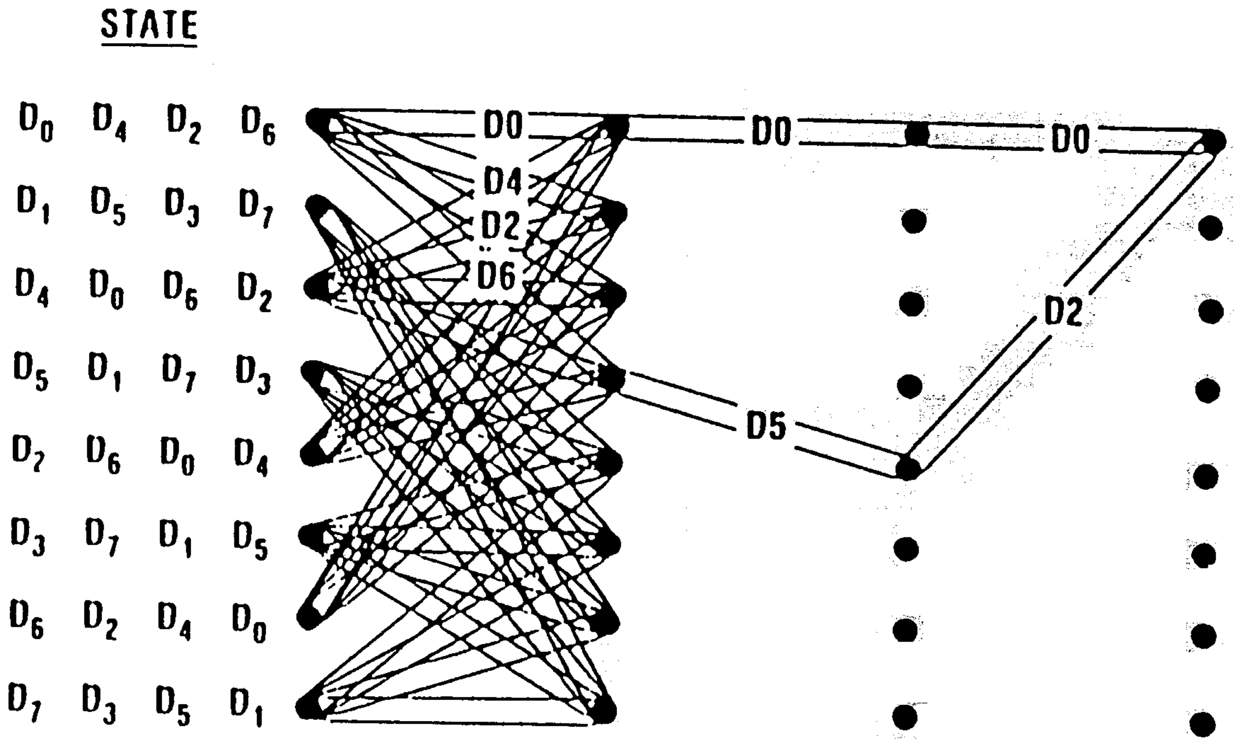


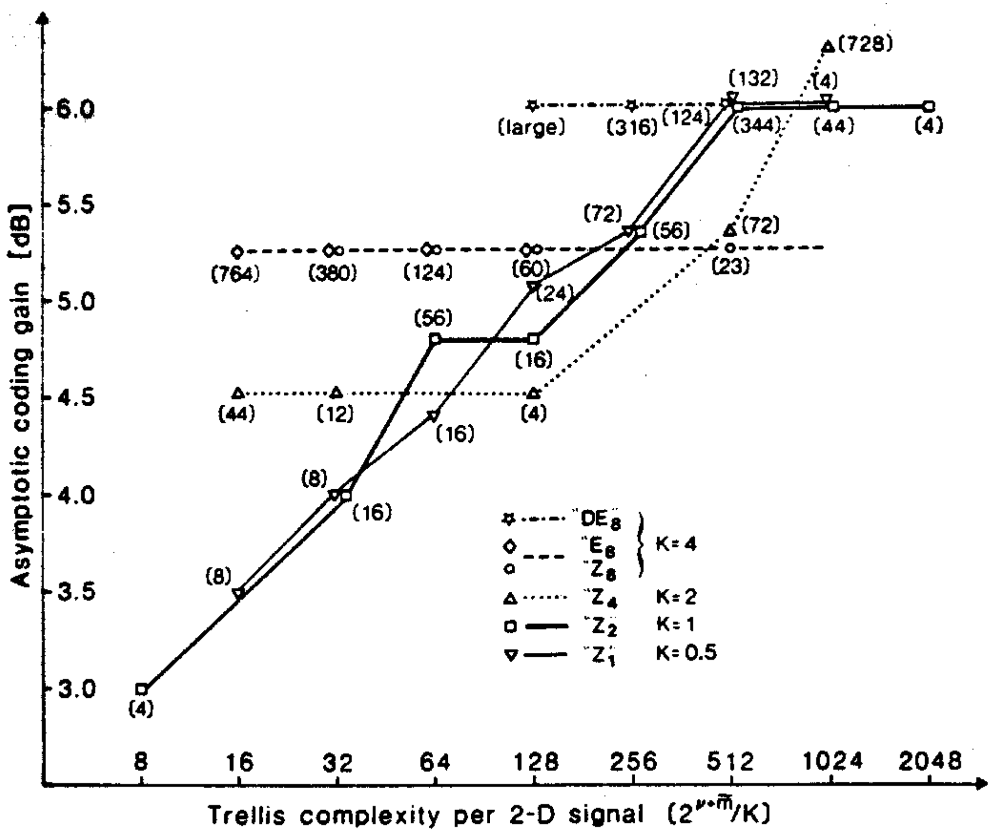
$$\text{ASYMPTOTIC CODING GAIN} = 10 \log_{10} \frac{(d_1^2 + d_0^2 + d_1^2)_{\text{CODED 8 PSK}}}{(d_{\text{ref}}^2)_{\text{UNCODED 4 PSK}}} = 10 \log_{10} \left(\frac{4.585}{2} \right) = 3.6 \text{ dB}$$

16 QAM Partitioning



16-QAM Trellis





Asymptotic coding gain [dB]
6.0
5.5
5.0
4.5
4.0
3.5
3.0
8
16
32
64
128
256
512
1024
2048
Trellis complexity per 2-D signal $(2^{\nu+\bar{m}}/K)$
(728)
(132)
(4)
(large)
(316)
(124)
(344)
(44)
(4)
(72)
(56)
(72)
(764)
(380)
(124)
(60)
(23)
(24)
(56)
(16)
(44)
(12)
(4)
(16)
(8)
(16)
(8)
(4)
"DE8"
"E8"
"Z8"
K=4
"Z4" K=2
"Z2" K=1
"Z1" K=0.5