CHAPTER 3

3.1 Generally, for $N = i^2 + i \cdot j + j^2$, we can do the followings to find the nearest co-channel neighbors of a particular cell:

(1) move i cells along any chain of hexagons and then

(2) turn 60 degree counter-clockwise and move j cells

From the following figure, using the cosine law, we have

$$D^2 = [i \cdot (2R')]^2 + [j \cdot (2R')]^2 - 2 i \cdot (2R') \cdot j \cdot (2R') \cdot \cos 120^\circ$$

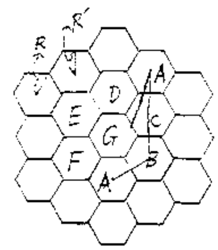
Where $R' = \frac{\sqrt{3}}{2} R$, therefore

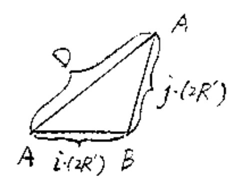
$$D = \sqrt{3 i^2 \cdot R^2 + 3 j^2 \cdot R^2 + i \cdot j \cdot 3R^2}$$

$$= \sqrt{3(i^2 + i \cdot j + j^2)} \cdot R$$

$$= \sqrt{3N} \cdot R$$

Hence, $Q = \frac{D}{R} = \sqrt{3N}$





3.2

Example 1:

In general, the average power of $v_1(t)$ is $P_1 = \lim_{T \to \infty} \int_{-T/2}^{T/2} |v_1(t)|^2 dt = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} v_1(t) v_1{}^*(t) dt$

or, $P_1 = \underbrace{\langle |v_1(t)|^2 \rangle}_{\substack{\text{mean value of the} \\ \text{scalar product}}} = \|v_1(t)\|^2 = \underbrace{\langle v_1(t), v_1(t) \rangle}_{\text{scalar product}}$

if the above is periodic over the interval T_0,
then

$$P_1 = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |v_1(t)|^2 dt = \frac{1}{T_0} \int_{T_0/2}^{T_0/2} v_1(t) v_1{}^*(t) dt$$

now,

$$P_{1+2} = \|v_1(t) + v_2(t)\|^2 = \|v_1(t)\|^2 + \langle v_1(t), v_2(t) \rangle + \langle v_2(t), v_1(t) \rangle + \|v_2(t)\|^2$$

and

$$\langle v_2(t), v_1(t) \rangle = \langle v_1(t), v_2(t) \rangle^*$$

and

$$\langle v_1(t), v_2(t) \rangle = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} v_1(t) v_2{}^*(t) dt$$

Similarly, if we can assume $v(t)$ and $v_2(t)$ are both periodic with period T_0, then

$$\langle v_1(t), v_2(t) \rangle = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} v_1(t) v_2{}^*(t) dt$$

If the signals are uncorrelated, then $v_1(t) + v_2(t)$ has a correlation of

$$R_{total} = R_1(\tau) + R_{1,2}(\tau) + R_{2,1}(\tau) + R_2(\tau)$$

If uncorrelated for all τ, then

$$R_{1,2}(\tau) = E[v_1(t) v_2(t+\tau)] = E[v_1(t)] E[v_2(t+\tau)]$$

If the signals are uncorrelated then $P_{total} = P_1 + P_2$ and at least one signal must have a zero mean. There are no other special conditions, since it was stated in the problem that the signals are statistically independent. A gaussian process is also assumed.

This homework problem submitted by Mark Glasgow, Northern Virginia Site, Commonwealth Graduate Engineering Program, Spring 1997.

3-2 Cont'd

Example 2

Given two independent voltages $v_1(t)$ and $V_2(t)$ that are added together, determine the normalized average power

$$P_{AV} = \overline{(v_1(t) + v_2(t))^2} = E[v_1^2] + E[v_2^2] + 2E[v_1]E[v_2]$$

If $E[v_1] = 0$ or $E[v_2] = 0$ or $E[v_1] = E[v_2] = 0$

then the resulting power is

$$P_{AV} = \overline{v_1^2} + \overline{v_2^2}$$

which is equal to the sum of the individual powers.

If $v_1(t)$ and $V_2(t)$ are uncorrelated (but not necessarily independent) the normalized average power of the voltage sum is

$$P_{AV} = \overline{\left(v_1(t) + v_2(t)\right)^2} = E[v_1^2] + E[v_2^2] + 2E[v_1 v_2]$$

From the correlation property, if the two signals are uncorrelated

$$E[v_1 v_2] = E[v_1]E[v_2]$$

so two signals that are uncorrelated will have a combined average power equal to the sum of the individual powers if either signal is mean zero.

It should be noted that statistically independentr.v.s are uncorrelated, but uncorrelated r.v.s may or may not be statistically independent.

Also, if the signals are orthogonal,

$$E[v_1 v_2] = 0$$

This homework problem submitted by John B. Call, Commonwealth Graduate Engineering Program, Spring 1997.

$\boxed{3\text{-}3}$ Since $S = kN$, where N is the cluster size, we have

$$N = \frac{S}{k}.$$

By the definition of frequency reuse factor, we have

$$\text{frequency reuse factor} = \frac{1}{N} = \frac{k}{S}.$$

3-5 (a) Let i_0 be the number of co-channel interfering cells, for omni-derectional antennas, $i_0=6$. Assume $n=4$, we have $\frac{S}{I}=\frac{(\sqrt{3N})^n}{i_0}>15dB=31.623 \Rightarrow N>4.59$

$\Rightarrow \underline{\underline{N=7}}$

(b) For 120° sectoring, $i_0=2$.

$$\frac{S}{I}=\frac{(\sqrt{3N})^n}{i_0}>31.623 \Rightarrow N>2.65 \Rightarrow \underline{\underline{N=3}}$$

(c) For 60° sectoring, $i_0=1$.

$$\frac{S}{I}=\frac{(\sqrt{3N})^n}{i_0}>31.623 \Rightarrow N>1.87 \Rightarrow \underline{\underline{N=3}}$$

From (a), (b) and (c) we can see that using 120° sectoring can increase the capacity by a factor of 7/3, or 2.333. Although using 60° sectoring can also increase the capacity by the same factor, it will decrease the trunking efficiency therefore we choose the 120° sectoring.

3.6 solution not available

be lost due to weak signal condition.

$\boxed{3\text{-}8}$ For $n=3$

(a) $i_0=6$, $\frac{S}{I}=\frac{(\sqrt{3N})^n}{i_0}>31.623 \Rightarrow N>11 \Rightarrow \underline{\underline{N=12}}$

(b) $i_0=2$, $\frac{S}{I}=\frac{(\sqrt{3N})^n}{i_0}>31.623 \Rightarrow N>5.29 \Rightarrow \underline{\underline{N=7}}$

(c) $i_0=1$, $\frac{S}{I}=\frac{(\sqrt{3N})^n}{i_0}>31.623 \Rightarrow N>3.33 \Rightarrow \underline{\underline{N=4}}$

From (a), (b) and (c), we can see that using 60° sectoring can increase the capacity by a factor of 12/3, or 4. For 120° sectoring, this factor is only 12/7, or 1.714. Therefore, we choose the 60° sectoring.

$\underline{3\cdot 11}$ By the same method used in example 3-9, when going from omni-directional antennas to 60° sectored antennas, the number of channels per sector $=\frac{57}{6}=9.5$. Given $P_r[\text{blocking}]=1\%$, from the Erlang B distribution we have the total offered traffic intensity per sector $A \doteq 4.1$ Erlangs. For $\mu=1$ call/hour, $H=2$ minute/call, the number of calls that each seach sector can handle per hour is

$U=\frac{A}{\mu H}=\frac{4.1}{\frac{1}{60}\cdot 2}=123$ users

$\Rightarrow$ cell capacity $=6\times 123=738$ users, from example 2.9,

$\Rightarrow$ loss in trunking efficiency $=1-\frac{738}{1326}\doteq 0.44=\underline{\underline{44\%}}$