DIVERSITY TECHNIQUES IN WIRELESS COMMUNICATION SYSTEMS

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Abstract- This report provides a brief introduction to the various types of Diversity systems in wireless systems.

INTRODUCTION TO DIVERSITY

Diversity is a powerful communication receiver technique that provides wireless link improvement at a relatively low cost. Diversity techniques are used in wireless communications systems to primarily to improve performance over a fading radio channel. In such a system, the receiver is provided with multiple copies of the same information signal which are transmitted over two or more real or virtual communication channels. Thus the basic idea of diversity is repetition or redundancy of information. In virtually all the applications, the diversity decisions are made by the receiver and are unknown to the transmitter.

TYPES OF DIVERSITY

We know from previous reports that fading can be classified into small scale and large scale fading. Small-scale fades are characterized by deep and rapid amplitude fluctuations which occur as the mobile moves over distances of just a few wavelengths. For narrow-band signals, this typically results in a Rayleigh faded envelope. In order to prevent deep fades from occurring, ***microscopic diversity techniques*** can exploit the rapidly changing signal. If the antenna elements of the receiver are separated by a fraction of the transmitted wavelength, then the various copies of the information signal or generically termed as **branches**, can be combined suitably or the strongest of them can be chosen as the received signal. Such a diversity technique is termed as Antenna or Space diversity.

Large scale fading, caused due to shadowing, can be combated using macroscopic diversity wherein the distances of consideration are of the order of the distances between two base stations.

Diversity techniques are effective when the branches considered are assumed to be independently faded or the envelopes are uncorrelated.

PRACTICAL TECHNIQUES OF DIVERSITY

There are mainly five techniques of diversity practically used:

1. **Frequency Diversity**:

The same information signal is transmitted on different carriers, the frequency separation between them being at least the coherence bandwidth.

2. **Time Diversity**:

The information signal is transmitted repeatedly in time at regularly intervals. The separation between the transmit times should be greater than the coherence time, T_c. The

*Taught by Prof. Narayan Mandayam, Rutgers University time interval depends on the fading rate, and increases with the decrease in the rate of fading.

3. **Polarization diversity**:

Here, the electric and magnetic fields of the signal carrying the information are modified and many such signals are used to send the same information. Thus orthogonal type of polarization is obtained.

4. **Angle Diversity**:

Here, directional antennas are used to create independent copies of the transmitted signal over multiple paths.

5. **Space Diversity**:

In Space diversity, there are multiple receiving antennas placed at different spatial locations, resulting in different (possibly independent) received signals.

The difference between the diversity schemes lies in the fact that in the first two schemes, there is wastage of bandwidth due to duplication of the information signal to be sent. Thus problem is avoided in the remaining three schemes, but with the cost of increased antenna complexity.

The correlation between signals as a function of distance between the antenna elements is given by the relation:

$$\rho = J_0^2\left(\frac{2\pi d}{\lambda}\right)$$

Where, $J_0(x)$ = Bessel function of zero order and first kind.

d = distance of separation in space of antenna elements

λ = carrier wavelength

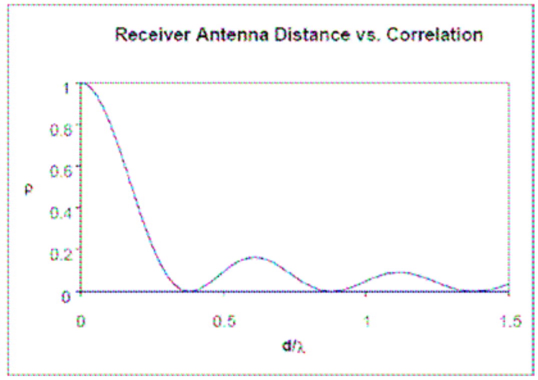

Figure 1: Plot of $J_0(x)$

From the graph, we can notice that as long as we have a separation of 'd' equaling half wavelength $\lambda/2$, we can attain zero correlation.

When f_c=5MHz, required distance for diversity = d = 30 m, which is very high.

When the carrier frequency is increased to 500 MHz, the distance required is d=30 cm, which tends to be a moderate value.

With a carrier frequency of 5 GHz, the distance required is d=3 cm.

High values of 'd' cannot be applied to the present mobile equipment, the reason being their form factor. Thus a high frequency of operation is desired in those cases.

The various schemes suggested depends on two factors:

a. Number of branches
b. Type of combining used

COMBINING METHODS:

Assume that there are M independent copies of the transmitted signal are available from M independent paths or branches in a diversity system.

We shall analyze the various combining methods of diversity systems and choose the optimum one for practical systems:

SELECTION DIVERSITY

Consider M branches assuming that the signal to noise ratio achieved on each branch is γ_i (i=1, 2….M). Further, assume that the received signal on each branch is independent and Rayleigh distributed with mean power of $2\sigma^2$. This type of diversity is microscopic in nature and is designed to combat small scale fading. Each of γ_i are thus distributed exponentially.

The pdf of γ_i is given by

$$P(\gamma_i) = (1/\gamma_0)\exp(-\gamma_i/\gamma_0)$$

Where i=1,2…..M

$\gamma_i \geq 0$

and $\gamma_0 = 2\sigma^2 (E_b/N_0)$ is the mean signal to noise ratio. E_b/N_0 is the SNR without fading.

The CDF of the SNR is thus,

$$P\{\gamma_i \leq \gamma\} = \int_0^{\gamma} p(x)dx = 1 - \exp(-\frac{\gamma}{\gamma_0})$$

The combining method of selection diversity is by picking the best branch of the set of received branches by comparing each one with every other branch. In short, it picks the $\max_i\{\gamma_i\}$.

Then the probability that the selected SNR of the branch is less than γ is $P(\gamma) = P_\gamma(\max_i \gamma_i \leq \gamma)$

$$= P_\gamma(\gamma_1 \leq \gamma, \gamma_2 \leq \gamma, \gamma_3 \leq \gamma, \ldots. \gamma_M \leq \gamma)$$

$$= \prod_{i=1}^{M} (1 - \exp(\gamma/\gamma_0))$$

or

$$P(\gamma) = (1-\exp(-\gamma/\gamma))^M$$

The above given expression is the resulting CDF when selection diversity is used.

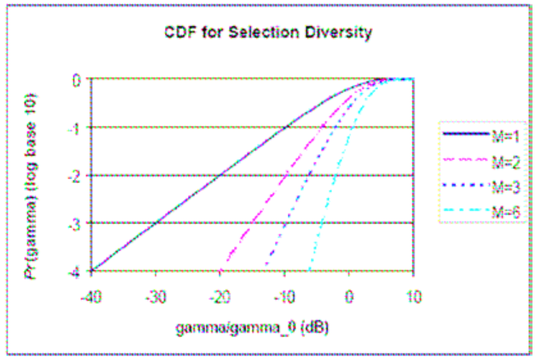

Figure 2: Plot of CDF of Selection diversity

The performance of diversity systems is usually measure by comparing the CDFs of various combining methods. From the given graph above, we can notice that the probability of γ falling a particular value below γ_0 reduces as M increases. Thus the receiver looks at M different branches and chooses the best of γ_i. Also note that the probability of a very low SNR decreases quite rapidly (for example, 10 dB below the mean γ_0) when diversity is used as opposed to the non-diversity case.

For M=1, Probability is 0.1
For M=2, Probability is 0.01.
For M=3, Probability is 0.001

When the SNRs are low, i.e far below the mean value of γ_0,

$\exp(-\gamma / \gamma_0) \approx 1 - (\gamma / \gamma_0)$, then

$P(\gamma) = (\gamma/ \gamma_0)^M$ for $\gamma << \gamma_0$. Therefore in log scale(dB scale), these correspond to straight lines with slopes M.

Average signal to noise ratio due to selection diversity combining is

$$E[\max\{\gamma_i\}] = \int_0^\infty \gamma\, p(\gamma)\, d\gamma$$

$$\therefore E[\max_i \gamma_i] = \gamma_0 \sum_{k=1}^{M} 1/k$$

M=1 to M=2, the improvement in the average SNR is 1.8 dB.

M=2 to M=3, the improvement in the average SNR is 0.9 dB.

Note:

- As M increases, the relative order of magnitude of improvement diminishes
- Diversity of order 2 is generally preferred for the best trade-offs.
- In the practical 2G CDMA system, a diversity system with M=6 is used.

The main handicap of selection diversity is that the signals must be monitored at a rate faster than that of the fading process if the largest of them all is to be selected.

MAXIMUM RATIO COMBINING (MRC):

In MRC, all the branches are used simultaneously. Each of the branch signals is weighted with a gain factor proportional to its own SNR.

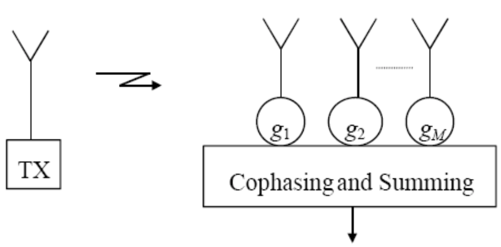

Figure 3: Maximum Ratio Combining

Co-phasing and summing is done for adding up the weighted branch signals in phase.

The gain associated with the i^{th} branch is decided by the SNR of the corresponding branch. i.e.,

$$g_i=(S/N)_i$$

The MRC scheme requires that the signals be added up after bringing them to the same phase, If a_i is the signal envelope, in each branch then the combined signal envelope is given as

$$a = \sum_{i=1}^{M} a_i\, g_i$$

Assuming that the noise components in the channel are independent and identically distributed in each branch, total noise power is

$$N_t= N_0 \sum_{i=1}^{M} g_i^2$$

The resulting SNR is thus given by

$$\gamma = a^2\, E_b/N_0= (E_b/N_0)\, (\Sigma g_i\, a_i)^2/(\Sigma\, g_i^2)^2$$

Utilizing the Cauchy-Schwarz Inequality,

$$\gamma \leq (E_b/N_0)(\Sigma\, a_i^2)$$

The equality in this case is obtained when g_i=k * a_i, k being some constant. The maximum value of the output SNR after MRC is given by

$$\gamma = (E_b/N_0)(\Sigma\, a_i^2)$$

i.e.,

$$\gamma = \Sigma\, (E_b/N_0)(a_i^2) = \Sigma\, \gamma_i$$

(All summations between i=1,2,....M).
Thus we notice that the sum of the SNRs of the individual branches yields the final SNR of the output.

To obtain the distribution of the combined signal, observe that

$\gamma_i = (E_b/N_0)\, a_i^2 = (E_b/N_0) * (x_i^2 + y_i^2)$

γ_i is χ^2 distributed with degree 2 which is the same as an exponential distribution.

Let $\gamma = \Sigma\gamma_i$. Then we can see that γ is χ^2 distributed with degree 2M. Then the PDF of γ is given by

$p(\gamma) = 1/(M-1)! * \gamma^{M-1}/\gamma_0^{M} * \exp(-\gamma/\gamma_0)$

with $\gamma \geq 0$;

γ_0 is the mean SNR in each branch and is given by $2\sigma^2 E_b/N_0$.

The CDF of γ is

$$P(\gamma) = \int_0^{\gamma} \frac{1}{(M-1)!} \frac{x^{M-1}}{\gamma_0^M} \exp(-\frac{x}{\gamma_0}) dx$$

$$= 1 - \exp(-\frac{\gamma}{\gamma_0}) \sum_{i=1}^{M} \frac{1}{(i-1)!} \left(\frac{\gamma}{\gamma_0}\right)^{i-1}$$

It can be noticed that as compared to selection combining, the fall of the probability is more rapid.

Eg: at a level of 10 dB below γ_0,
M=1 Probability=0.1
M=2 Probability=0.005

The main challenge in MRC combining is the co-phasing of the incoming branches after weighting them.

The expected value of the signal strength,

$E[\text{Signal Strength}] = E[\Sigma\gamma_i] = M*\gamma_0$;

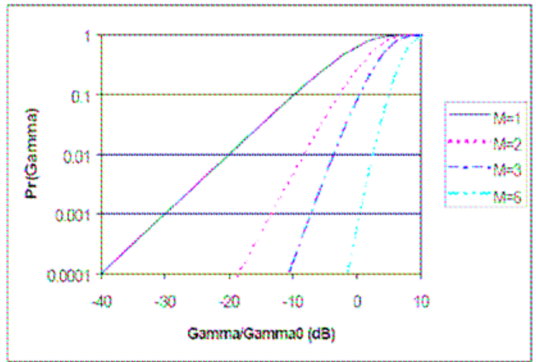

Figure 4: Plot of CDF of MRC

Thus the expected value increases linearly with the increase in M. The mean value increases considerably as compared to the Selection combining. But as in Selection combining, the 3-dB improvement from M=k to M=k+1 reduces with the increase in M, as shown in the figure below:

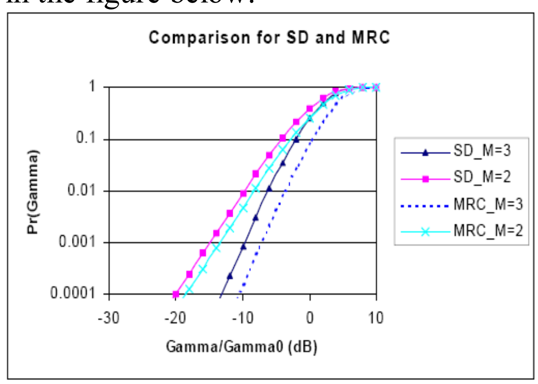

Figure 5: Comparison between Selection Diversity and MRC

EQUAL GAIN COMBINING:

The third type of combining method is the Equal Gain combining wherein the g_i's of the MRC scheme are all made equal to 1, for all i=1,2,….M. Thus co-phasing and summing is done on the branches which are received directly.

$$a = \sum_{i=1}^{M} a_i$$

and the resulting SNR is

$\gamma = a^2 E_b/(M{*}N_0)$.

$= (E_b/M{*}N_0){*}(\Sigma\ a_i)^2$.

Since γ can be represented the square of the sum of M Rayleigh distributed random variables, there is no closed form expression to it.
The performance of the Equal Gain Combining is worse than the MRC.

The expected value of the SNR is given by,

$E[\gamma] = (E_b/MN_0)\ E[(\Sigma\ A_i)^2]$ where A_i is a Rayleigh random variable.

$$= (E_b/MN_0)\ E[\sum_i \sum_j A_i A_j]$$
$$= (E_b/MN_0)\ \Sigma\Sigma\ E[A_i A_j]$$

For uncorrelated branches, we have

$$E[A_i A_j] = E[A_i]E[A_j]$$

Further, we assumed each of them to be Rayleigh,

Thus, $E[A_i^2] = 2\sigma^2$ which implies that

$$E[A_i] = \sqrt{(\pi\sigma^2/2)}$$
$$E[\gamma] = (E_b/MN_0){*}[2M\sigma^2 + M(M-1)\ \pi\sigma^2/2]$$
$$= \gamma_0[1 + (M-1)\ \pi/4]$$

where $\gamma_0 = (E_b/N_0)\ 2\sigma^2$

For large M,

$\therefore E[\gamma] = \pi\ M\ \gamma_0/4$

Thus, Equal gain combining is worse than MRC approximately by a factor of $\pi/4 \approx 0.8$. Comparison between MRC, Equal Gain Combining and Selection Diversity is as shown in the graph below. The analysis is done using the expected SNR value $E[\gamma]$.

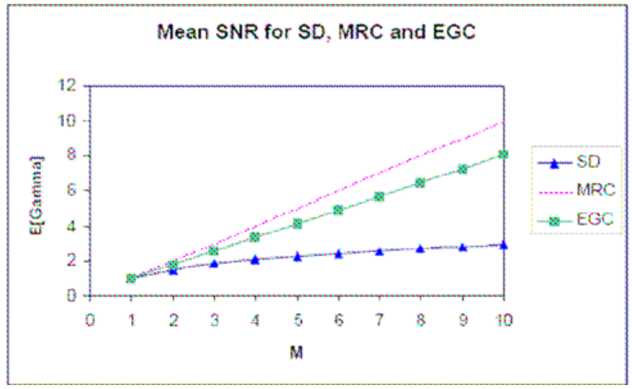

Figure 6: Comparison between Selection Diversity, MRC and EGC.

<u>SWITCHED DIVERSITY</u>:

One branch at a time is used in switched diversity and the combining strategy is to remain using the current branch until the signal envelope drops below a predetermined threshold value x. When the signal value falls below x, two strategies could be followed:

(a) Switched and Stay: In this method, the other signal or branch is always selected when the envelope in the current branch falls below x.
(b) Switched and examine: Here, the other branch is selected only if that SNR is above the threshold.

The order of diversity in this scheme is 2.The advantage of this scheme is that all the branches need not be monitored at all times. This leads to two advantages:

Simplicity in implementation as only one receiver front end is required in contrast with selection diversity where M such front ends are required for each of the M branches (All branches have to be monitored continuously at the same time). This results in discontinuities at switching times leading to amplitude and phase transients in selection diversity.

➢

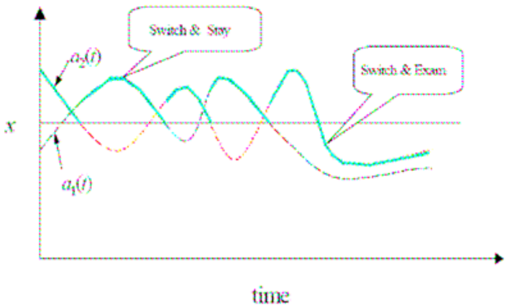

Figure 7: Types of Switched Diversity

➢ Unnecessary switching as in the case of selection switching is avoided.

Note that $a_1(t)$ and $a_2(t)$ are both independent Rayleigh processes. Assume identical mean and autocorrelation.

Let r(t) be the combined process composed with $a_1(t)$ and $a_2(t)$ in parts according to the switching level of x.

Let γ_x be the SNR level corresponding to x.

The probability that the SNR in one branch is below the threshold of γ_x is given as:

$q = P_\gamma[\gamma_{a1 \text{ or } a2} < \gamma_x]$

$= 1- \exp\{\gamma_x/\gamma_0\}$ since the SNR in each branch is exponential.

The SNR of the combined signal has the PDF given by

$$Pc(\gamma) = \begin{cases} 1+q*p_\gamma(\gamma) \; ; & \gamma \geq \gamma_x \\ q*p_\gamma(\gamma) \; ; & \gamma < \gamma_x \end{cases}$$

Where $p_\gamma(\gamma)$ is the exponential distribution in each branch.

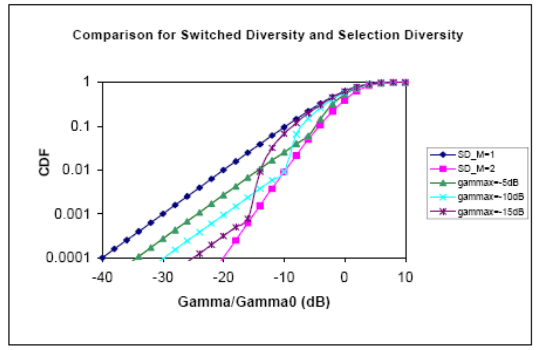

Figure 8: Comparison of Switch Diversity and Selection Diversity

NOTES ON SWITCHED DIVERSITY:

- Switched diversity is not continuously monotonic
- It is much worse than Selection diversity in terms of performance
- It equals Selection diversity at a few points.
- The best diversity scheme is chosen with a trade-off between complexity and performance.

CORRELATED SIGNALS:

In a particular system it can be difficult to achieve signals in branches that are completely uncorrelated. For eg., limited frequency separation between the frequency diverse branches or closely mounted antenna structures in space diversity systems.

Consider the case of M=2 branch diversity. Taking the example of only MRC, The CDF of the SNR of the resulting combined signal is

$$p_\gamma(\gamma)=1-(1/2\rho)*[(1+\rho)\exp\{-\gamma/(\gamma_0(1+\rho))\} - (1-\rho)\exp\{-\gamma/(\gamma_0(1-\rho))\}]$$

ρ^2 is a parameter that captures the normalized covariance of the two signal envelopes.

If $\rho^2 = 1$, fully correlated branches can be obtained, which indicates that there is no diversity.

If $\rho^2 = 0$, completely uncorrelated branches can be obtained.

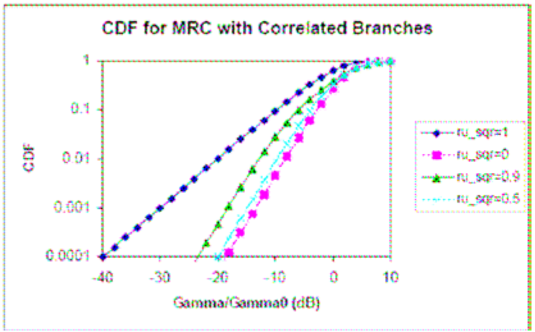

Figure 9: CDF of MRC with correlated branches

For Selection diversity,

$p(\gamma) = 1-\exp(-\gamma/\gamma 0)*[1-Q(a, b)+Q(b, a)]$

where Q(a,b) is the Marcum's Q function defined as

$$Q(a,b) = \int_b^{\infty} x \exp\left(-\frac{1}{2}(a^2 + x^2)\right) I_0(ax)dx$$

Note that, Even when $\rho^2 = 0.9$, i.e., the correlation is high, there are substantial diversity gains still obtained.

References:

[1] Wireless Communications Principles and Practice, Dr. Rappaport.

[2] Digital Communications, Dr. Proakis.

[3] Wireless Communication Technologies, lecture notes, Spring 2005, Dr. Narayan Mandayam, Rutgers University.