16582/16418 Wireless Communication

Lecture Notes 7: Mobile Radio Channel Modeling II

Statistical Models for Fading Processes

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- Quick Review of Fading Models
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  - Rayleigh
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- Combating Fading
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Quick Review of Fading Models

- Dispersion in Time and Frequency Effect Channel model
- In Time, look at relation between multipath spread and bit duration
  - Selective or Flat Fading
  - BW of channel vs. BW of signal
- In frequency look at Doppler Spread relative to inverse of Bit Duration
  - Fast or Slow Fading
  - Signaling rate vs. channel change rate

Types of Small-scale Fading

Small-scale Fading
(Based on Multipath Time Delay Spread)

Flat Fading

1. BW Signal < BW of Channel
2. Delay Spread < Symbol Period

Frequency Selective Fading

1. BW Signal > Bw of Channel
2. Delay Spread > Symbol Period

Small-scale Fading
(Based on Doppler Spread)

Fast Fading

1. High Doppler Spread
2. Coherence Time < Symbol Period
3. Channel variations faster than baseband signal variations

Slow Fading

1. Low Doppler Spread
2. Coherence Time > Symbol Pe
3. Channel variations smaller tha signal variations

Impulse Response of the Fading Multipath Model

2) Discrete Multipath Model (resolvable)

$$\tilde{y}(t)=\sum_{k=1}^{N(t)}\tilde{a}_k(t)\tilde{x}(t-\tau_k(t)),\qquad \tilde{h}(t,\tau)=\sum_{k=1}^{N(t)}\tilde{a}_k(t)\delta(t-\tau_k(t))$$

- For discrete multipath channels, the above PDF's are used to model $|a_k(t)|$

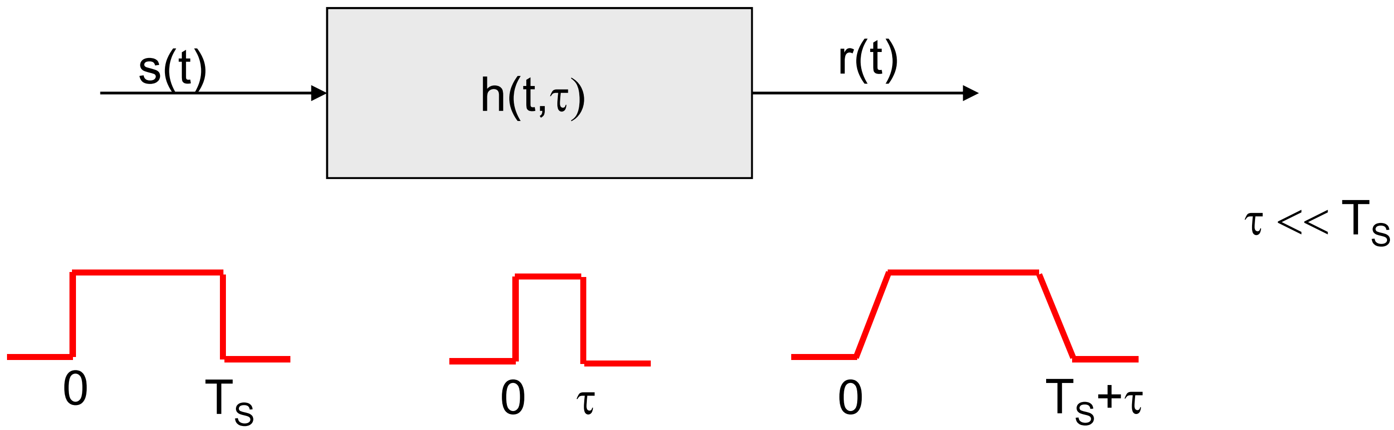
3) Continuous Multipath (unresolvable)

$$\tilde{y}(t)=\int_{-\infty}^{\infty}\tilde{h}_k(t)\tilde{x}(t-\tau_k(t))d\tau$$

Flat Fading

- Occurs when symbol period of the transmitted signal is much larger than the Delay Spread of the channel
  - – Bandwidth of the applied signal is narrow.
- Occurs when the amplitude of the received signal changes with time
  - For example according to Rayleigh Distribution
- May cause deep fades.
  - – Increase the transmit power to combat this situation.

Flat Fading



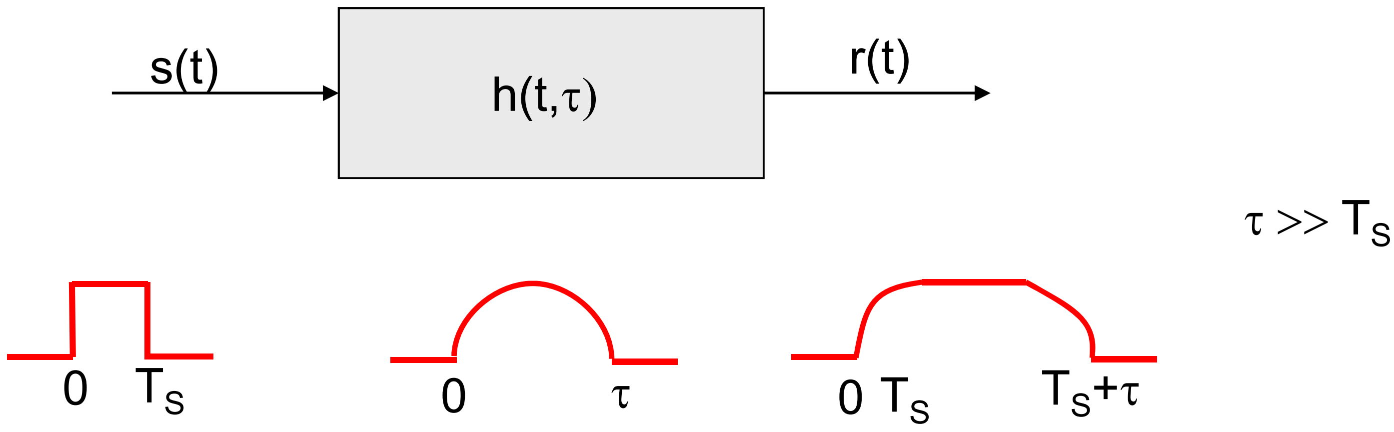
Occurs when:
$B_S << B_C$
and
$T_S >> \sigma_\tau$

B_C: Coherence bandwidth
B_S: Signal bandwidth
T_S: Symbol period
σ_τ: Delay Spread

Frequency Selective Fading

- Occurs when channel multipath delay spread is greater than the symbol period.
  - Symbols face time dispersion
  - Channel induces Intersymbol Interference (ISI)
- Bandwidth of the signal s(t) is wider than the channel impulse response.

Frequency Selective Fading



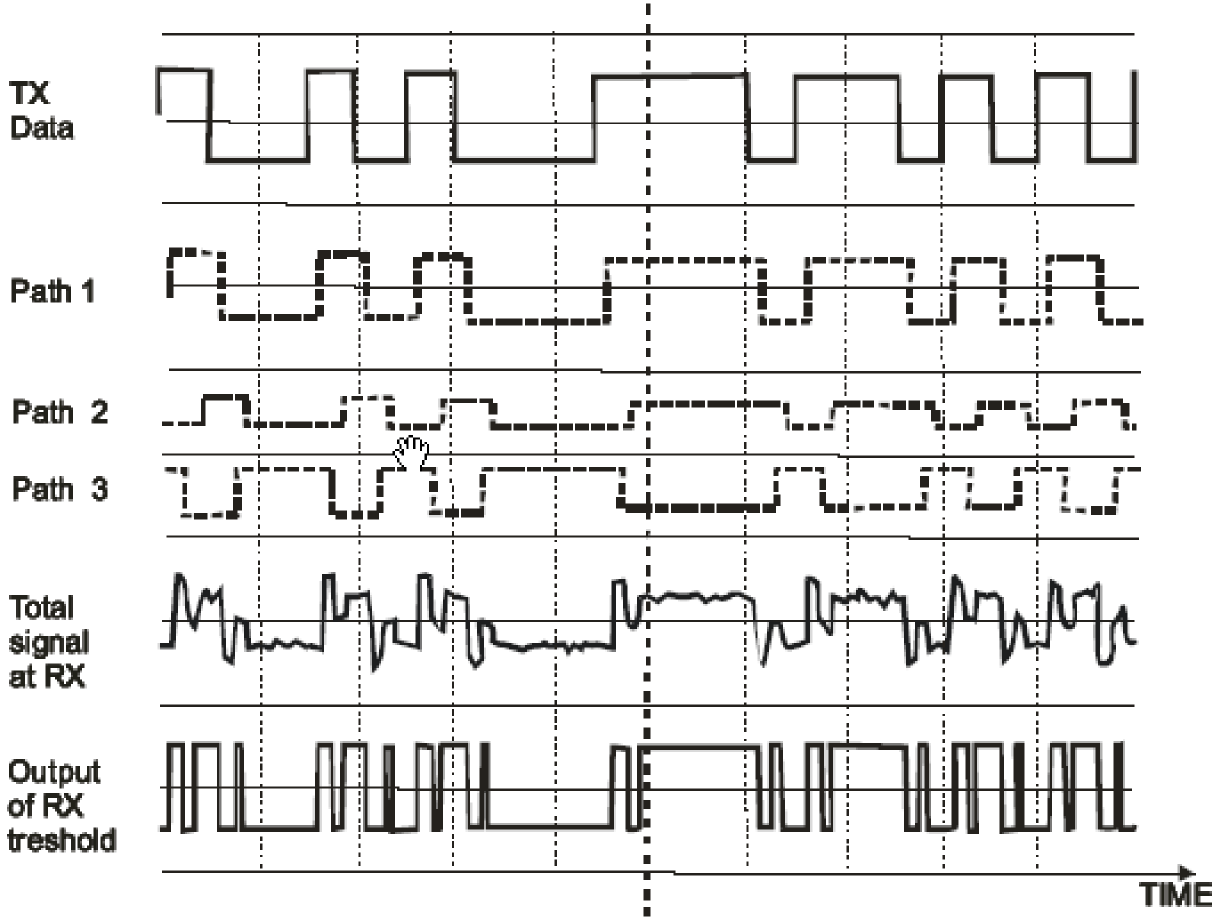
Causes distortion of the received baseband signal

Causes Inter-Symbol Interference (ISI)

Occurs when:
$B_S > B_C$
and
$T_S < \sigma_\tau$

As a rule of thumb: $T_S < \sigma_\tau$

ISI is result of Selective Fading



Fast Fading

- Due to Doppler Spread
  - Rate of change of the channel characteristics
    is **larger** than the
    Rate of change of the transmitted signal
  - The channel changes during a symbol period.
  - The channel changes because of receiver motion.
  - Coherence time of the channel is smaller than the symbol period of the transmitter signal

Occurs when:
$B_S < B_D$
and
$T_S > T_C$

B_S: Bandwidth of the signal
B_D: Doppler Spread
T_S: Symbol Period
T_C: Coherence Bandwidth

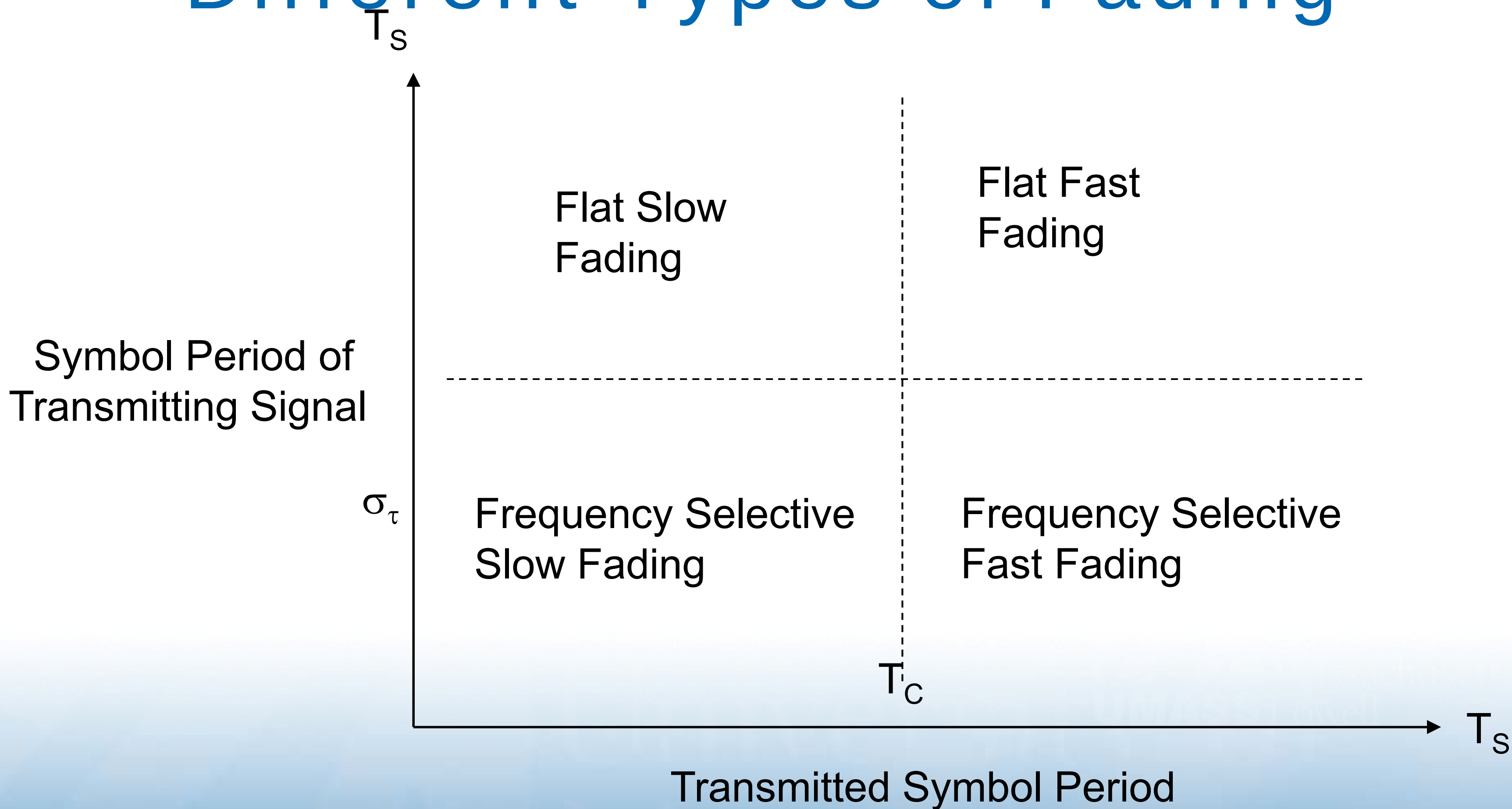
Slow Fading

- Due to Doppler Spread
  - Rate of change of the channel characteristics is **much smaller** than the Rate of change of the transmitted signal

Occurs when:
$B_S >> B_D$
and
$T_S << T_C$

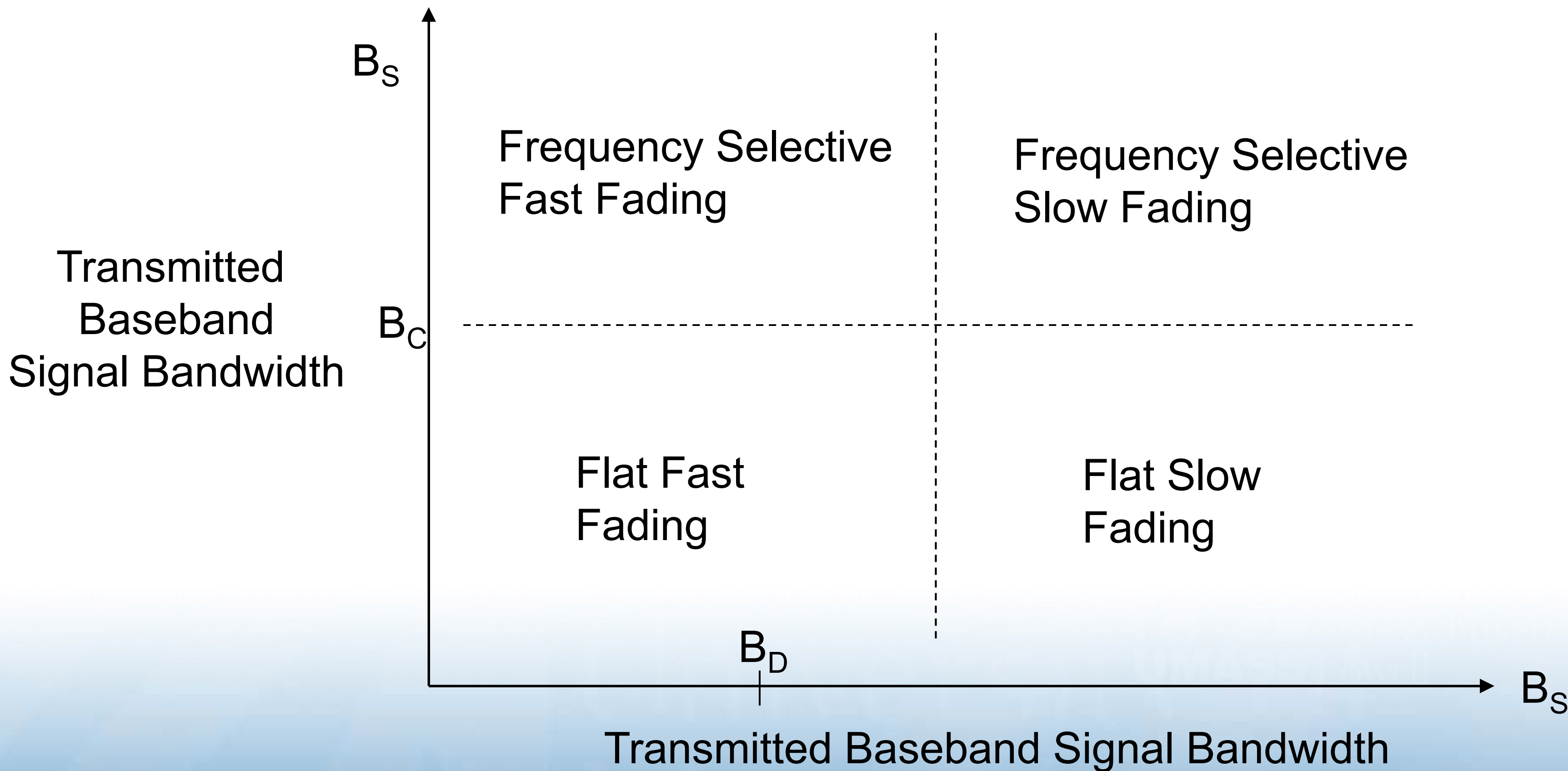
B_S: Bandwidth of the signal
B_D: Doppler Spread
T_S: Symbol Period
T_C: Coherence Bandwidth

Different Types of Fading



With Respect To SYMBOL PERIOD

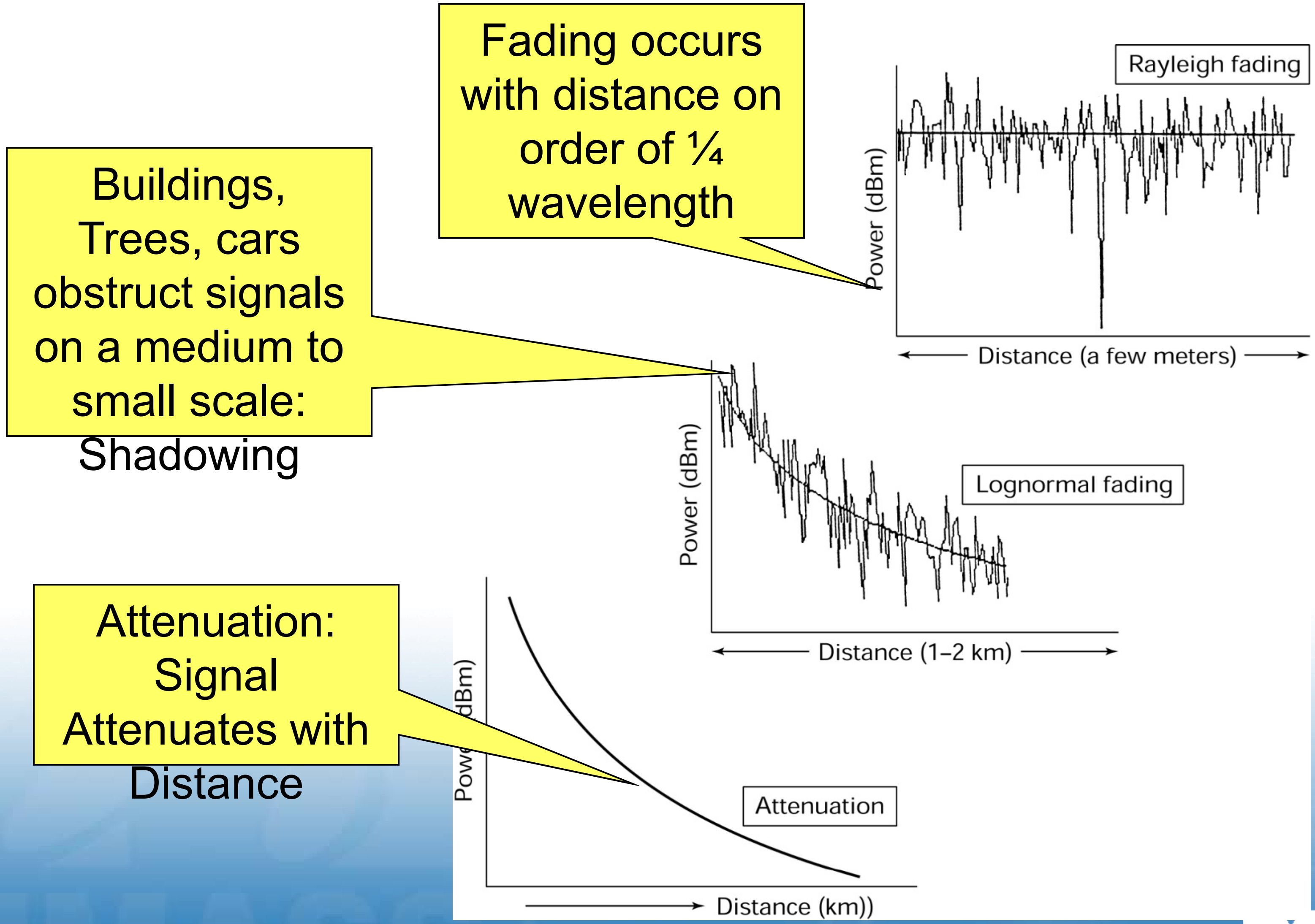
Different Types of Fading



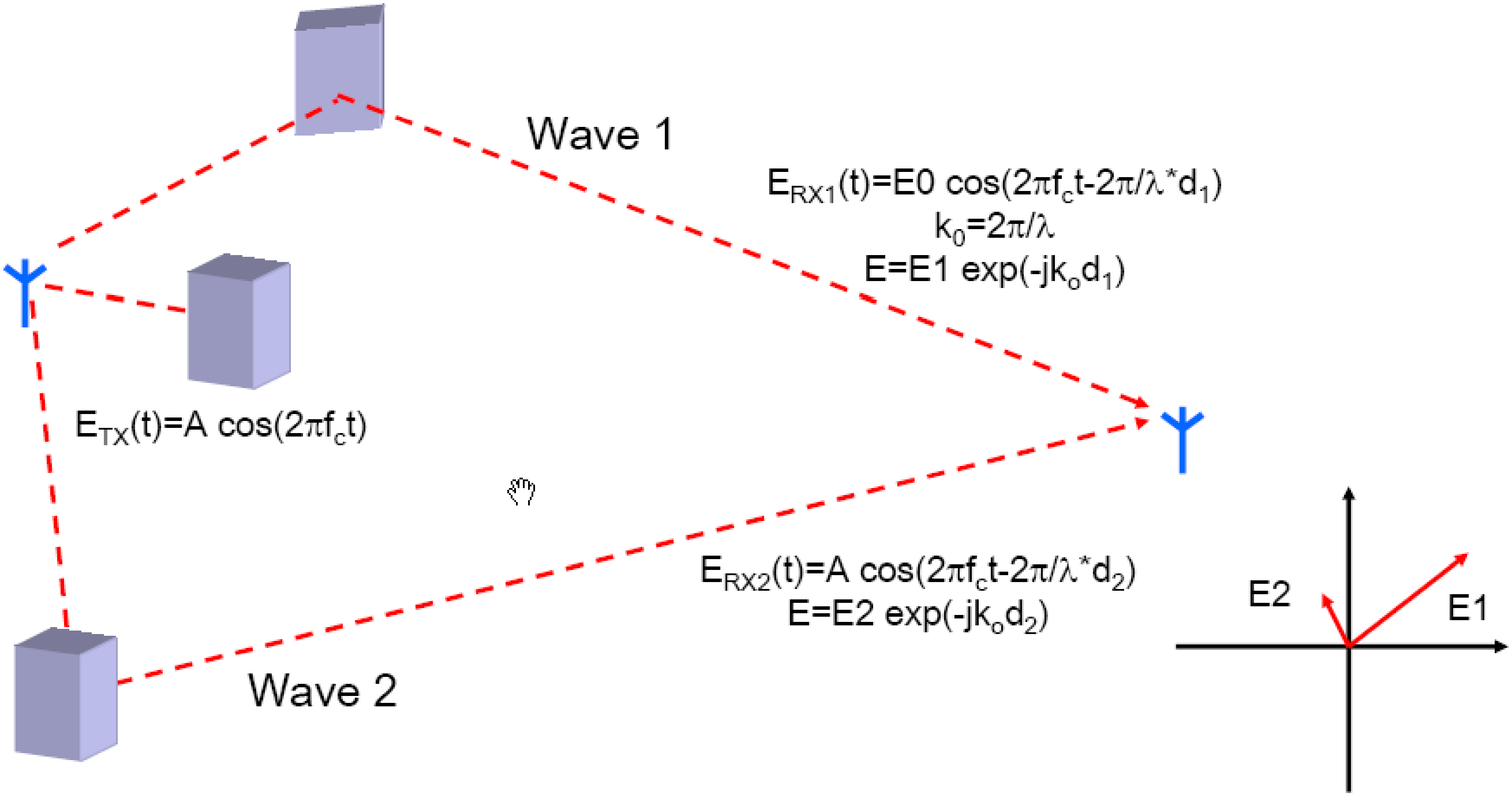
With Respect To BASEBAND SIGNAL BANDWIDTH

Statistical Models For Small Scale Fading

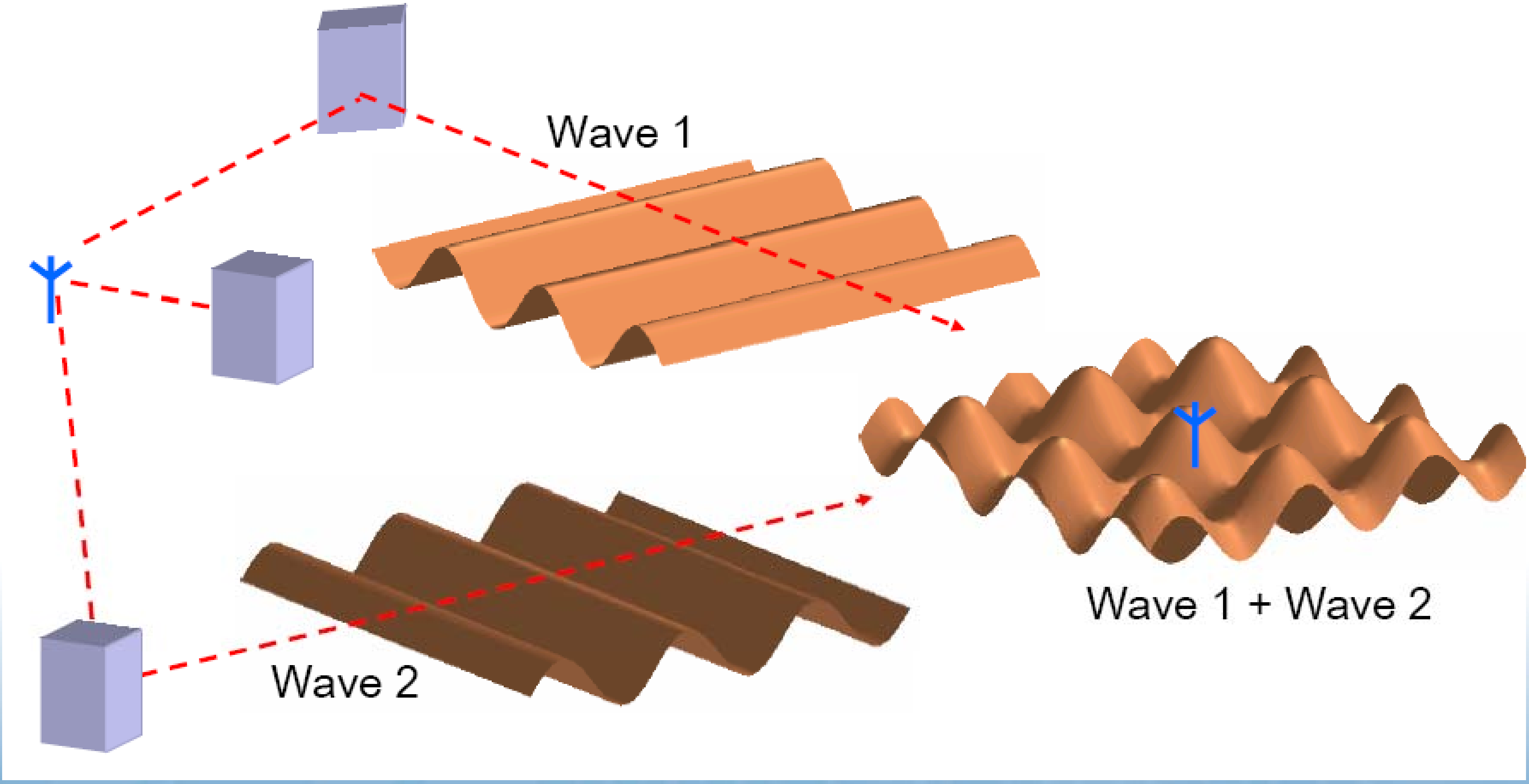
Three Major Effects: Attenuation, Long-term Fading (Shadowing), and Short-term Fading.



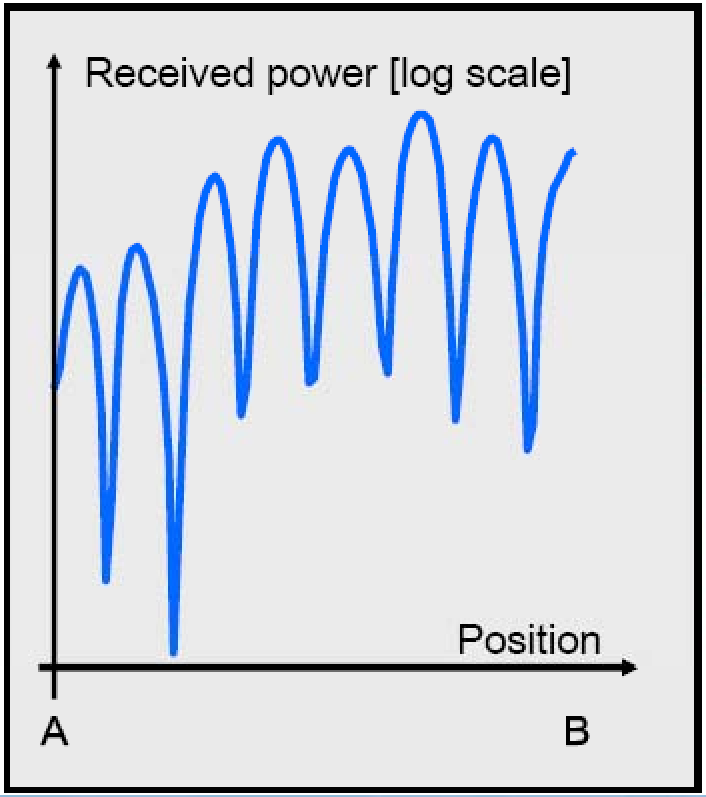
Fading Is the Result of Constructive and Destructive Wave Combining

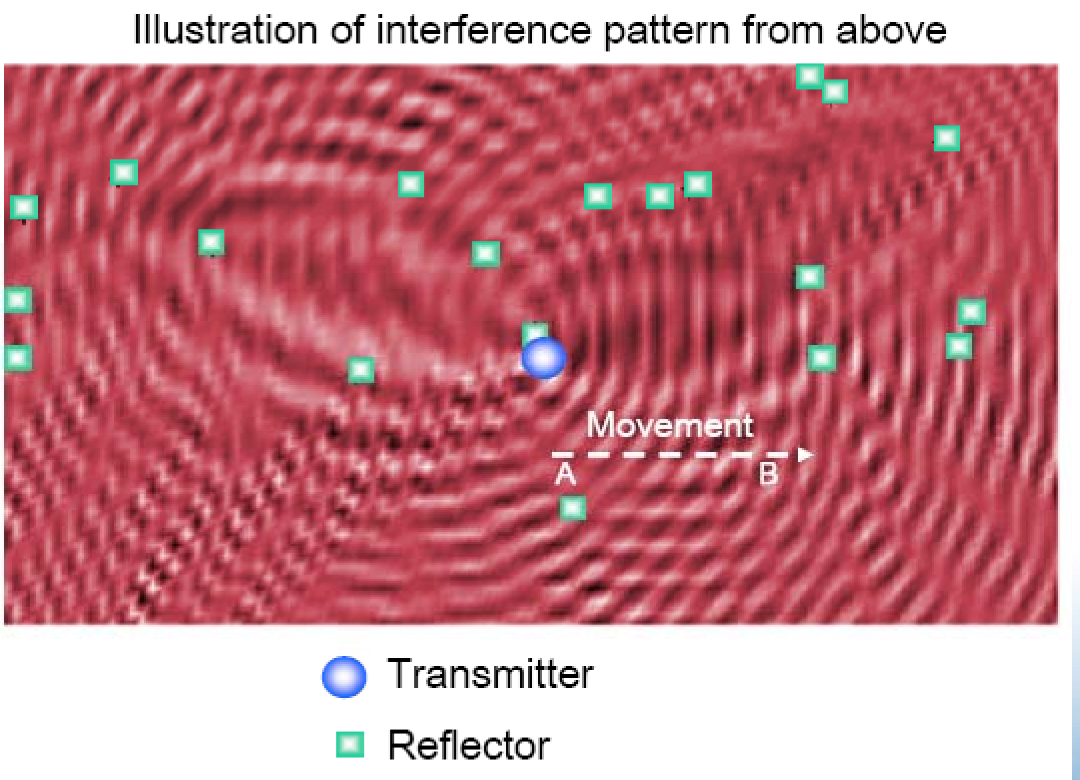


Small Scale Fading in Space and Time

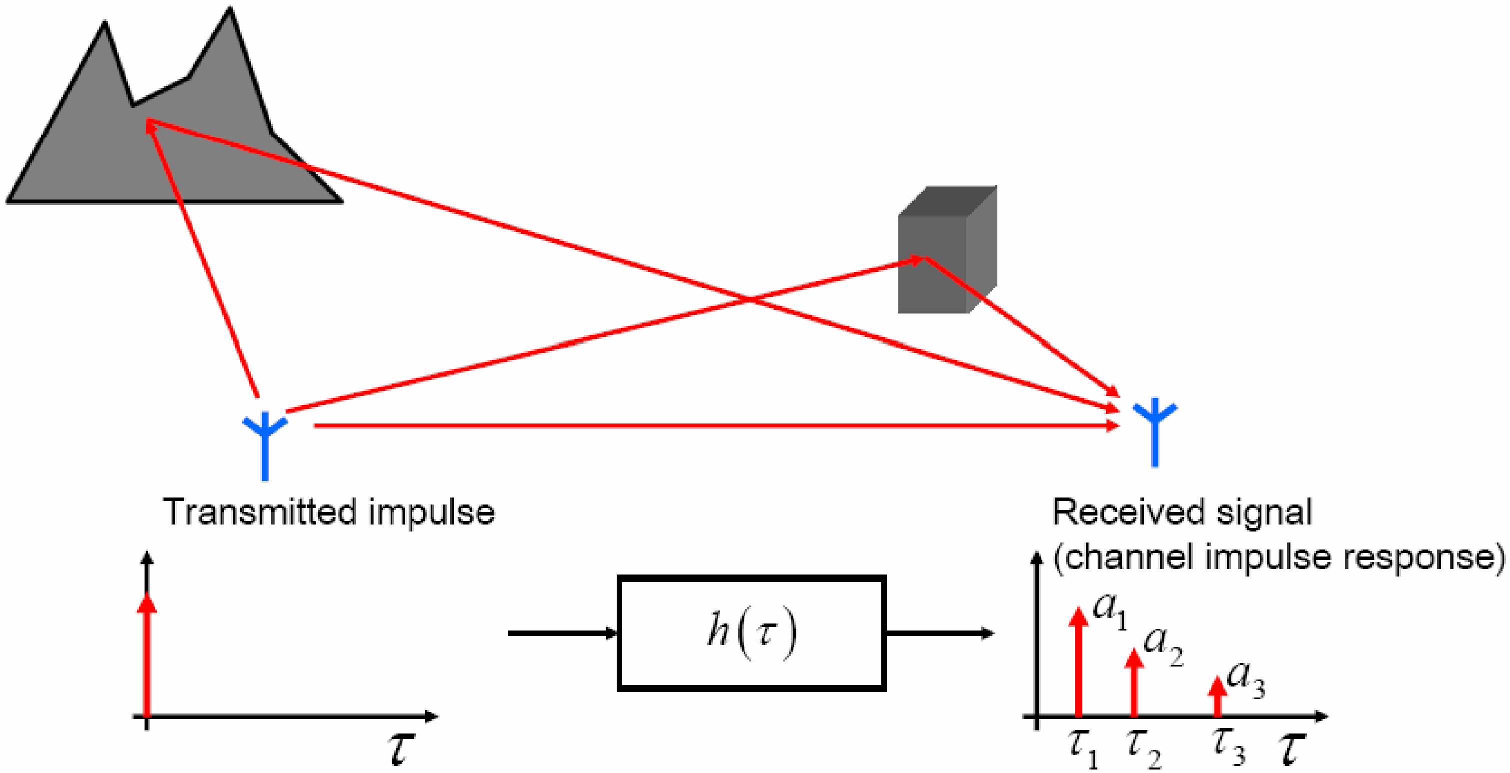


Space/Time Interference patterns





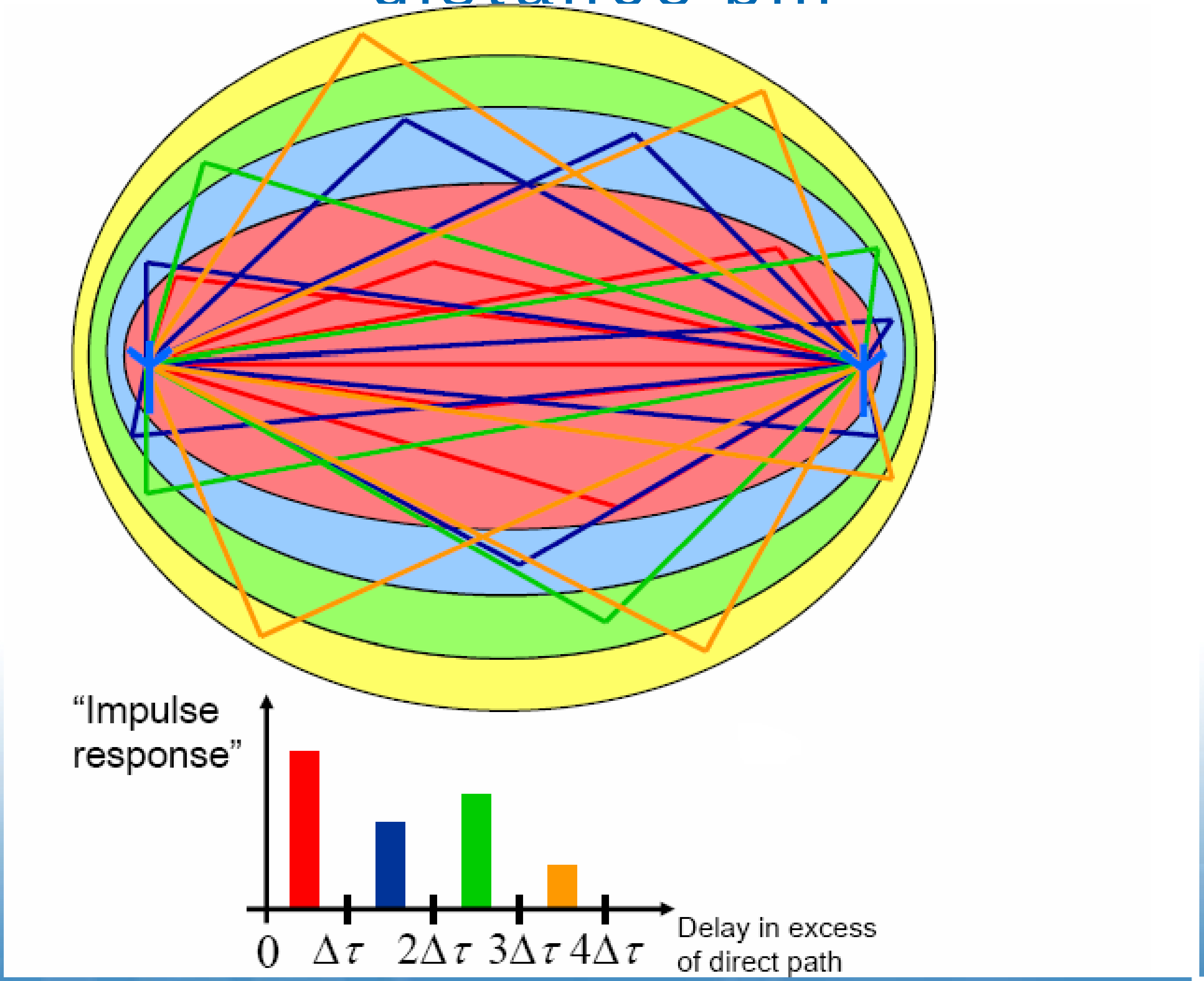
Impulse Response of a Multipath Channel



$$h(\tau) = a_1\delta(\tau - \tau_1) + a_2\delta(\tau - \tau_2) + a_3\delta(\tau - \tau_3)$$

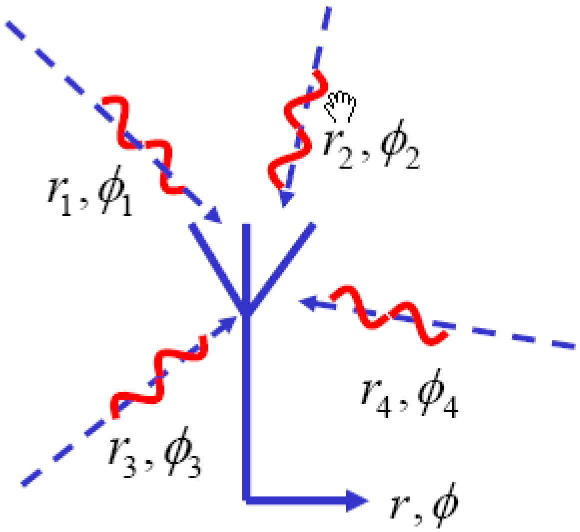
A_i can be deterministic or random complex Gaussian Variables

Many Scatterers from same distance results in random fading at each distance bin

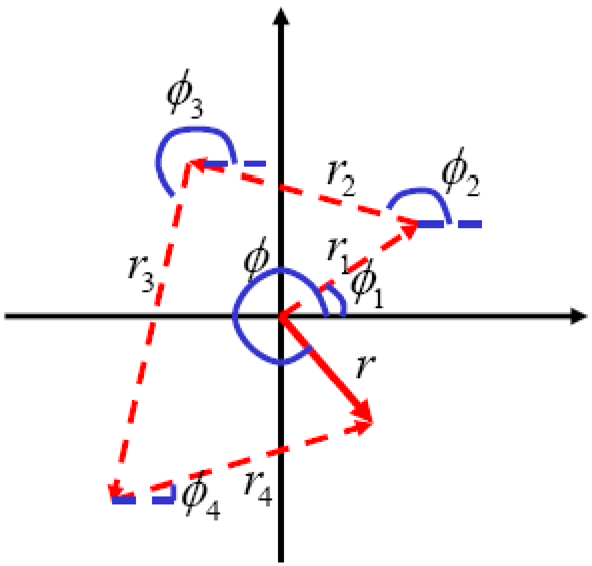


Many Waves Combine Due to Scattering

Many incoming waves with independent amplitudes and phases



Add them up as phasors



$$r\exp(j\phi) = r_1\exp(j\phi_1) + r_2\exp(j\phi_2) + r_3\exp(j\phi_3) + r_4\exp(j\phi_4)$$

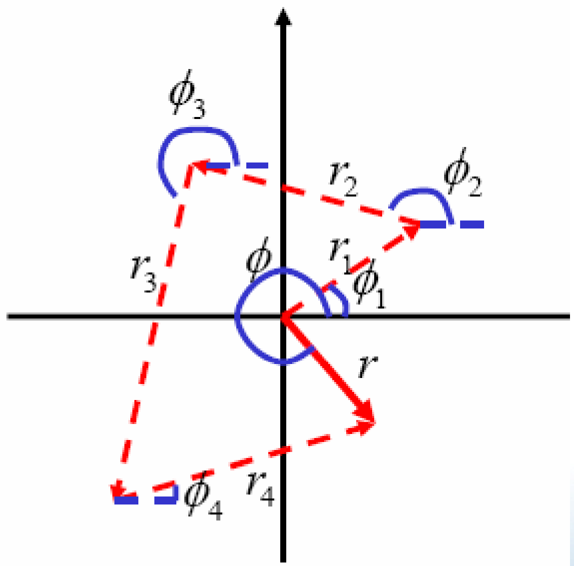
Real and Imaginary Parts are Gaussian Due to Central Limit Theorem

Re and Im components are sums of many independent equally distributed components

$$\mathrm{Re}(r) \in N(0, \sigma^2)$$

Re(r) and Im(r) are independent

The phase of r has a uniform distribution



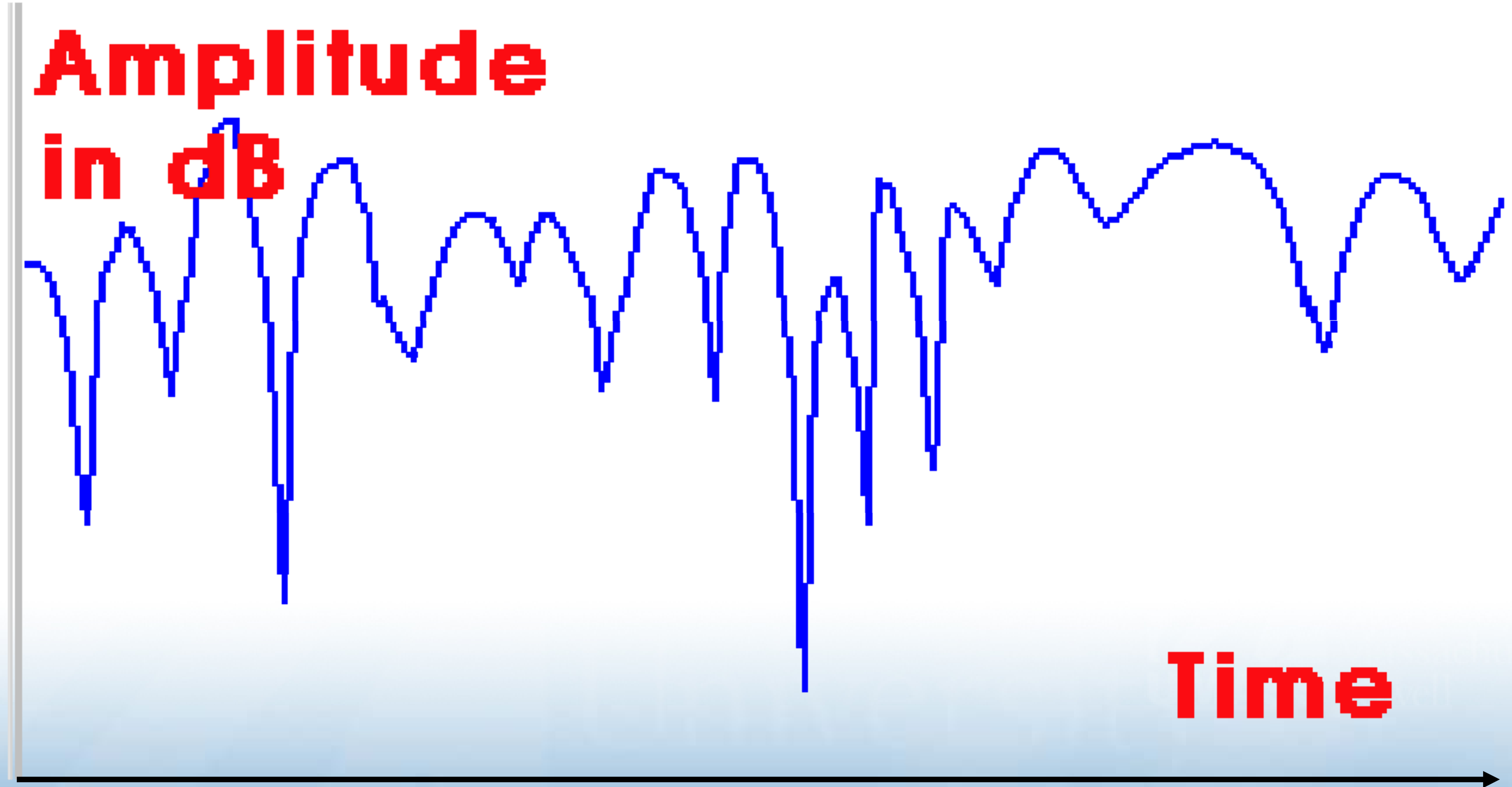
Fading Distributions

- Describes how the received signal amplitude changes with time.
  - Remember that the received signal is combination of multiple signals arriving from different directions, phases and amplitudes.
  - With the received signal we mean the baseband signal, namely the envelope of the received signal (i.e. r(t)).
- Its is a statistical characterization of the multipath fading.
- Often used distributions
  - Rayleigh Fading
  - Ricean Fading
  - Nakagami Fading

Rayleigh and Rician Distributions

- Rayleigh Describes the received signal envelope distribution for channels, where all the components are non-LOS:
  - i.e. there is no line-of–sight (LOS) component.
- Rician Describes the received signal envelope distribution for channels where one of the multipath components is LOS component.
  - i.e. there is one LOS component.

Rayleigh Fading



Rayleigh Fading

Rayleigh distribution has the probability density function (PDF) given by:

$$p(r)=\begin{cases}\dfrac{r}{\sigma^2}e^{\left(-\frac{r^2}{2\sigma^2}\right)} & (0\le r\le\infty)\\ 0 & (r<0)\end{cases}$$

σ^2 is the time average power of the received signal before envelope detection.
σ is the rms value of the received voltage signal before envelope detection

Remember: $\overline{P}\,(\text{average power})\propto V_{rms}^2$ (see end of slides 5)

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Rayleigh Fading (cont'd)

The probability that the envelope of the received signal does not exceed a specified value of R is given by the CDF:

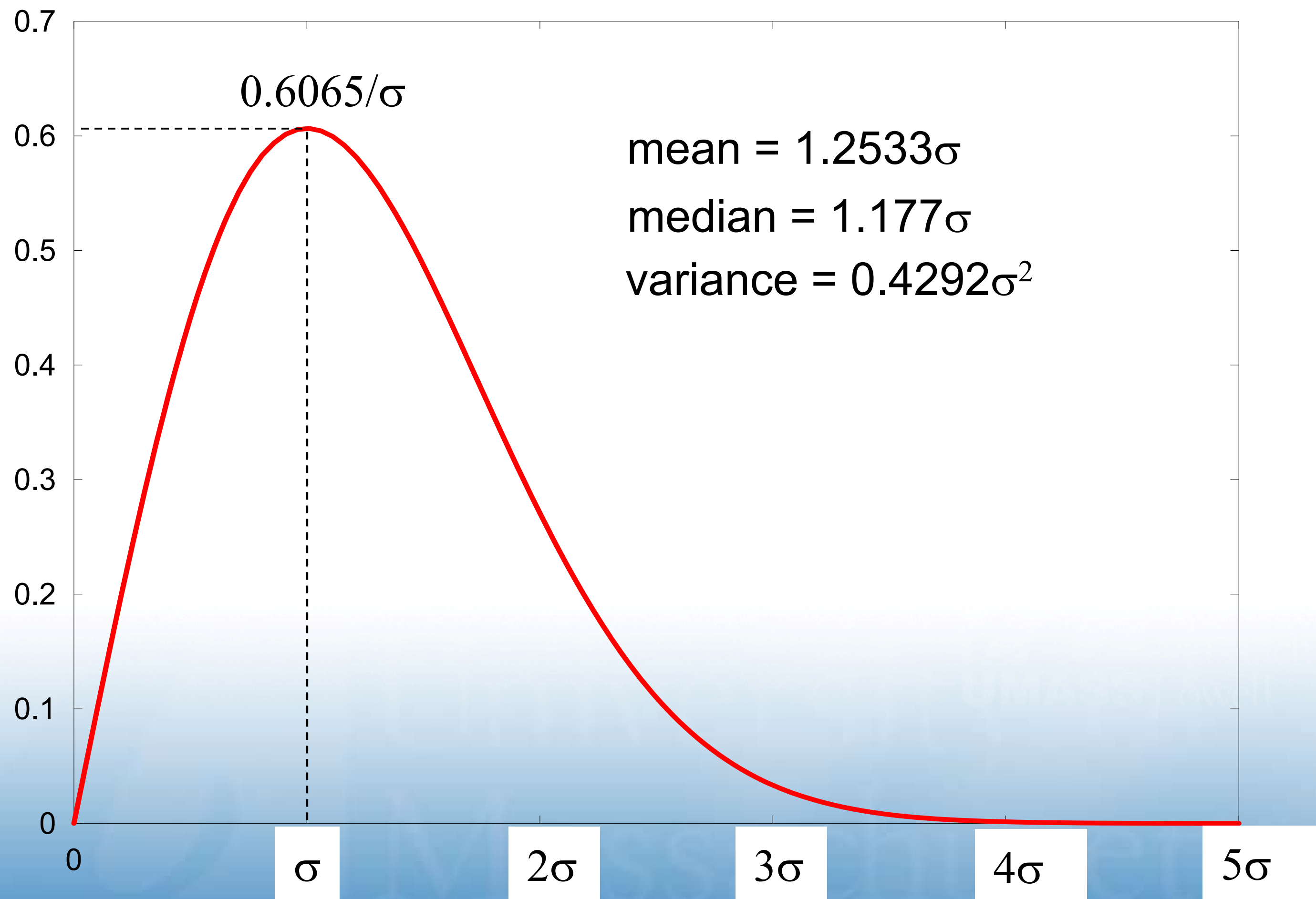
$$P(R) = P_r(r \le R) = \int_0^R p(r)dr = 1 - e^{-\frac{R^2}{2\sigma^2}}$$

$$r_{mean} = E[r] = \int_0^\infty rp(r)dr = \sigma\sqrt{\frac{\pi}{2}} = 1.2533\sigma$$

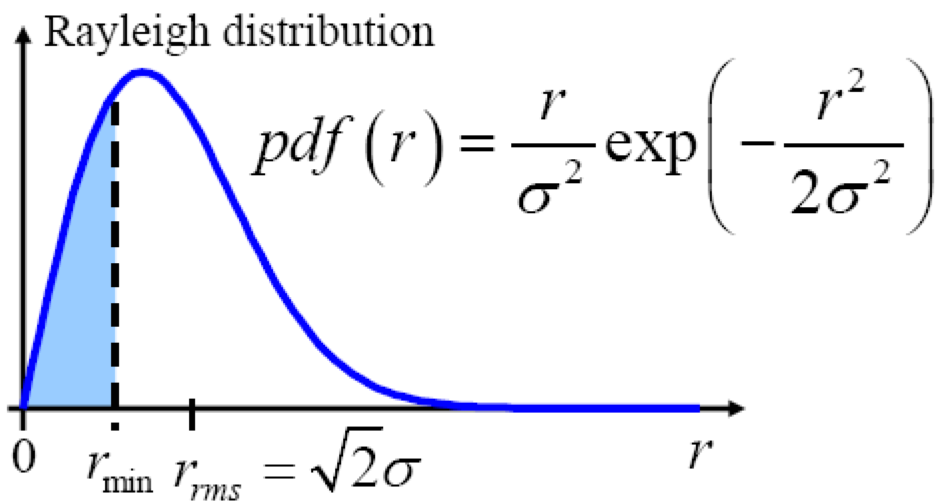
$$r_{median} = 1.177\sigma \quad \text{found by solving } \frac{1}{2} = \int_0^{r_{median}} p(r)dr$$

$$r_{rms} = \sqrt{2}\sigma$$

Rayleigh PDF

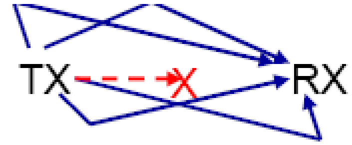


Pdf and Cdf of Rayleigh Fading



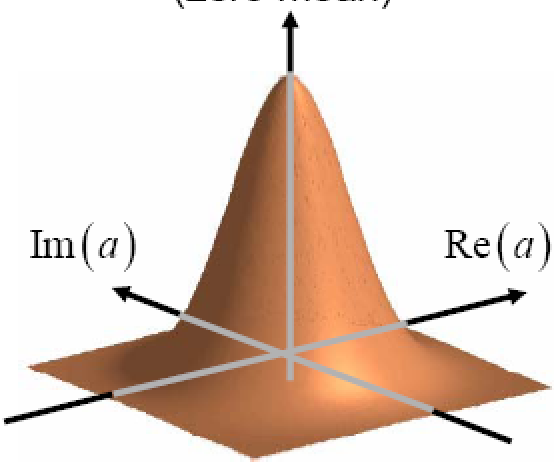
$$\Pr(r < r_{\min}) = \int_0^{r_{\min}} pdf(r)\,dr = 1 - \exp\left(-\frac{r_{\min}^{\ 2}}{r_{rms}^{\ 2}}\right)$$

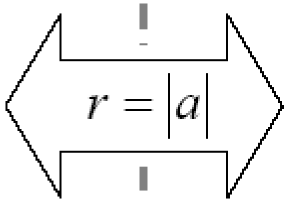
The Envelope is Rayleigh Distributed



No dominant component
(no line-of-sight)

Tap distribution
2D Gaussian
(zero mean)

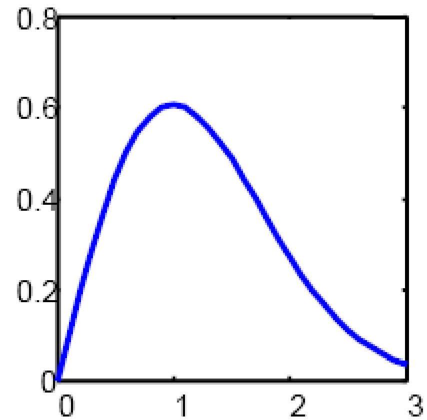




No line-of-sight component

Amplitude distribution

Rayleigh



$$pdf(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right)$$

Rayleigh Fading Margin

How many dB fading margin, against Rayleigh fading, do we need to obtain an outage probability of 1%?

$$\Pr\left(r < r_{\min}\right) = 1 - \exp\left(-\frac{r_{\min}^{2}}{r_{rms}^{2}}\right) = 1\% = 0.01$$

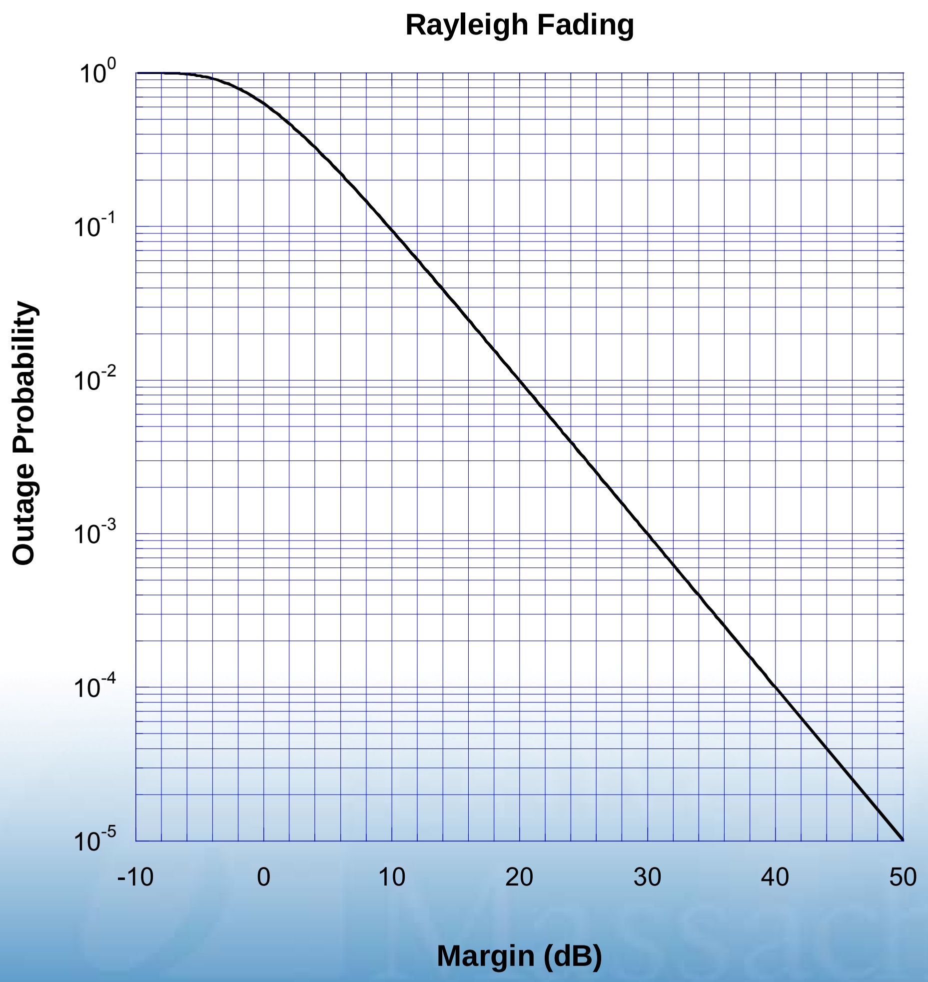
Some manipulation gives

$$1 - 0.01 = \exp\left(-\frac{r_{\min}^{2}}{r_{rms}^{2}}\right) \Rightarrow \ln\left(0.99\right) = -\frac{r_{\min}^{2}}{r_{rms}^{2}}$$

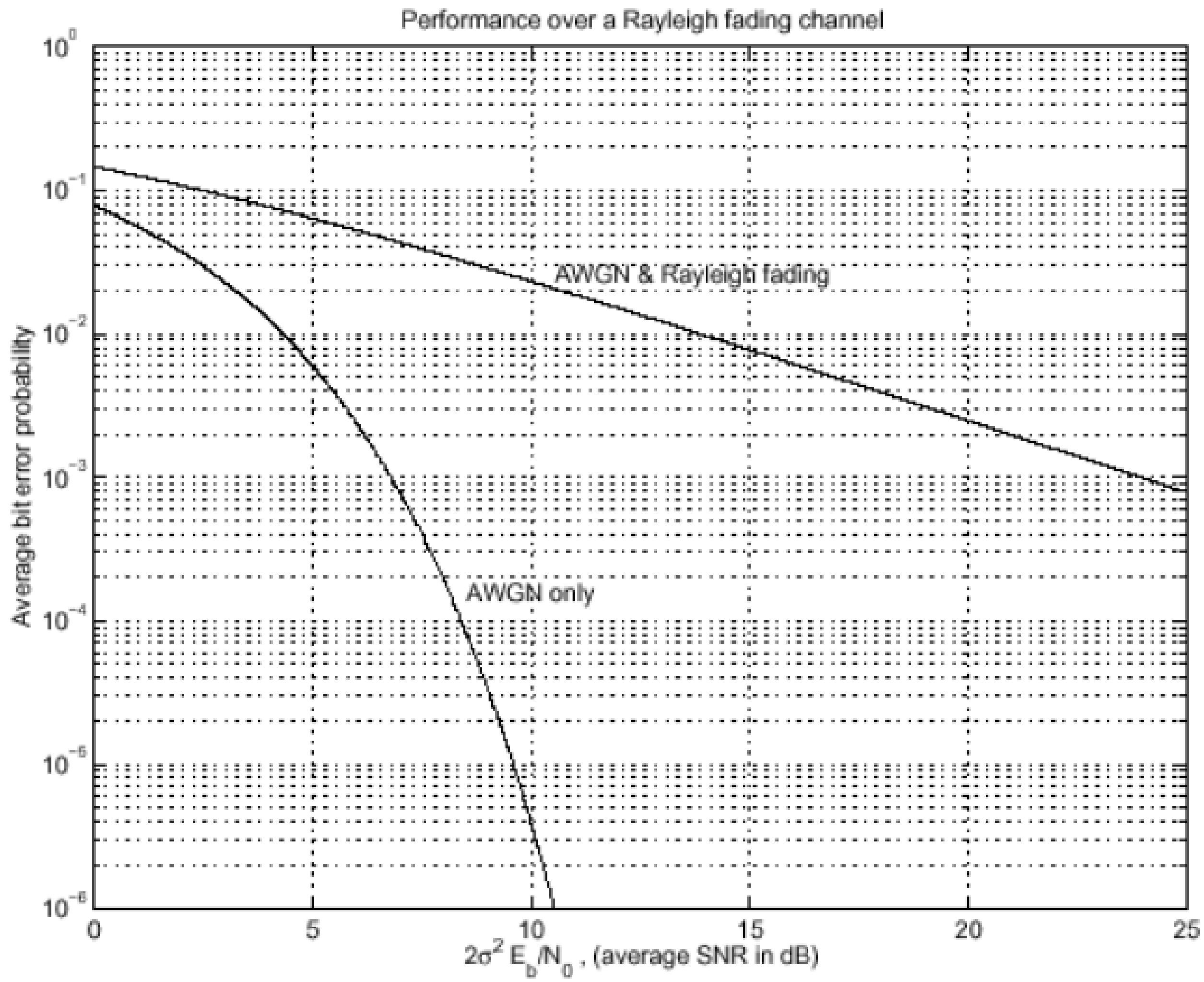
$$\Rightarrow \frac{r_{\min}^{2}}{r_{rms}^{2}} = -\ln\left(0.99\right) = 0.01 \quad \Rightarrow M = \frac{r_{rms}^{2}}{r_{\min}^{2}} = 1/0.01 = 100$$

$$\Rightarrow \boxed{M_{|dB} = 20}$$

Rayleigh Outage Probability



Digital Communication in Rayleigh Fading is Difficult



Ricean Distribution

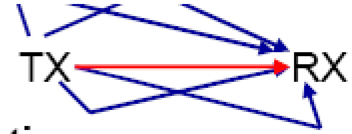
- When there is a stationary (non-fading) LOS signal present, then the envelope distribution is Ricean.
- The Ricean distribution degenerates to Rayleigh when the dominant component

- The ratio between the power of the LOS component and the diffuse components is called Ricean K-factor

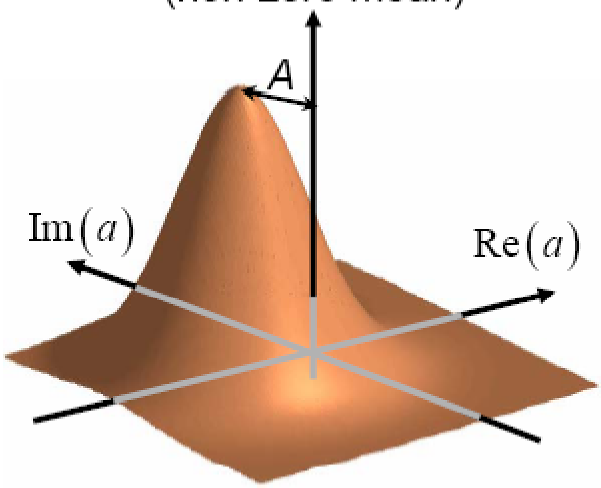
$$k = \frac{\text{Power in LOS component}}{\text{Power in random components}} = \frac{A^2}{2\sigma^2}$$

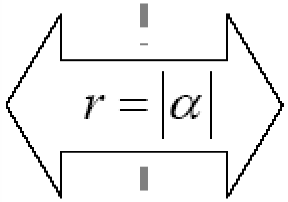
Rician PDF

A dominant component (line of sight)



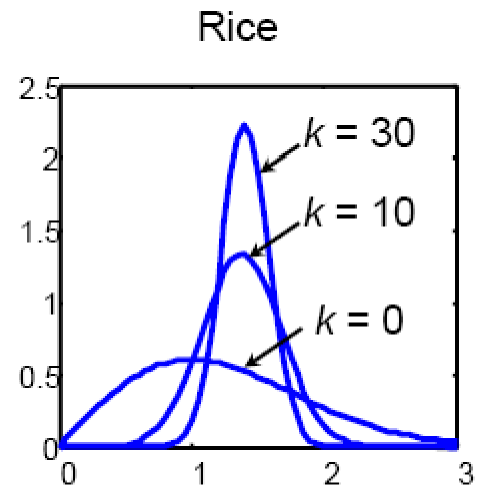
Tap distribution
2D Gaussian
(non-zero mean)





Line-of-sight (LOS) component with amplitude *A*.

Amplitude distribution



$$pdf(r) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2+A^2}{2\sigma^2}\right)I_0\left(\frac{rA}{\sigma^2}\right)$$

Rician Fading

In case of Line-of-Sight (LOS) one component dominates.

- Assume it is aligned with the real axis

$$\mathrm{Re}(r) \in N(A, \sigma^2) \quad \mathrm{Im}(r) \in N(0, \sigma^2)$$

- The received amplitude has now a Ricean distribution instead of a Rayleigh

- The ratio between the power of the LOS component and the diffuse components is called Ricean K-factor

$$k = \frac{\text{Power in LOS component}}{\text{Power in random components}} = \frac{A^2}{2\sigma^2}$$

Nakagami Probability Distribution

- In many cases the received signal can not be described as a pure LOS + diffuse components
- The Nakagami distribution is often used in such cases

$$pdf(r) = \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m r^{2m-1} \exp\left(-\frac{m}{\Omega} r^2\right)$$

$\Gamma(m)$ is the gamma function

$$\Omega = \overline{r^2}$$

$$m = \frac{\Omega^2}{(r^2 - \Omega)^2}$$

with m it is possible to adjust the dominating power

Nakagami Shape Factor

where

$$\Omega = E[R^2]$$

$$m = \frac{(E[R^2])^2}{Var(R)} \geq \frac{1}{2}$$

$$m = \begin{cases} \frac{1}{2}, & one\ sided\ Gaussian \\ 1, & Rayleigh\ distribution \\ > \frac{1}{2}, & approximates\ Ricean \\ \to \infty, & no\ fading \end{cases}$$

- The parameter m is called the 'shape factor' of the Nakagami or Nakagami m-parameter
- When $m = 1$, Nakagami becomes Rayleigh fading is recovered, with an exponentially distributed instantaneous power
- Nakagami fading occurs for multipath scattering with relatively large delay-time spreads and different clusters of reflected waves
- m-parameter can also be written in terms of the Ricean K factor

$$m = \frac{(K+1)^2}{2K+1} \qquad K = \frac{A^2}{2\sigma^2}$$

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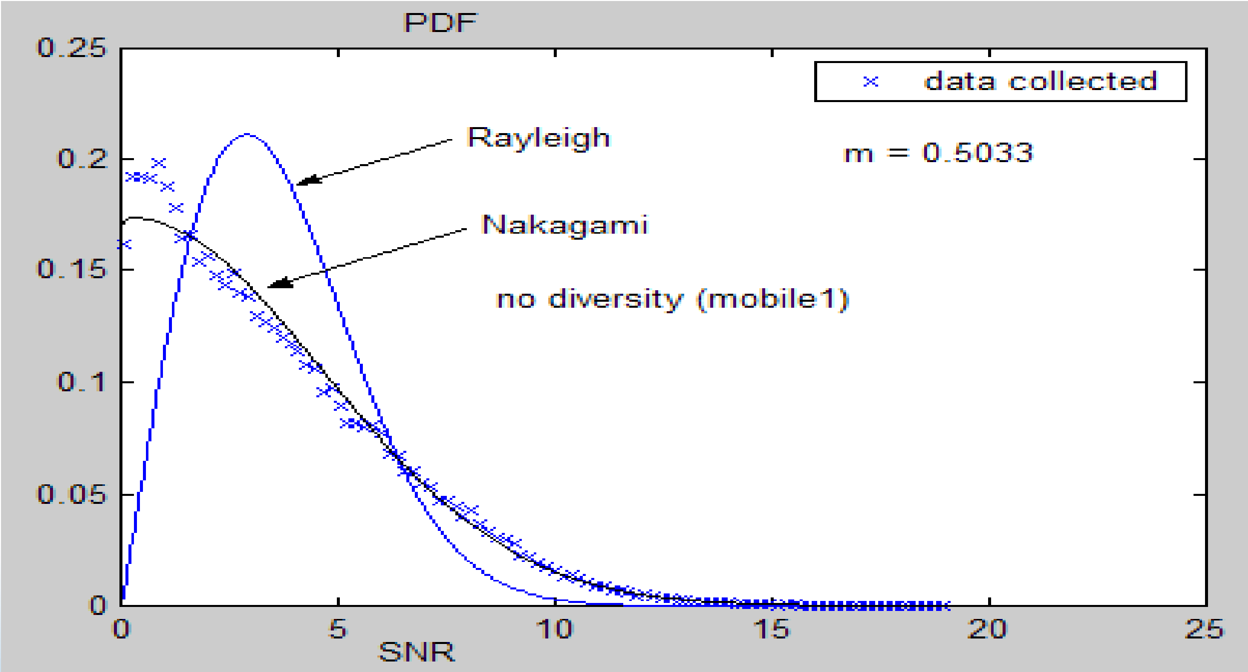
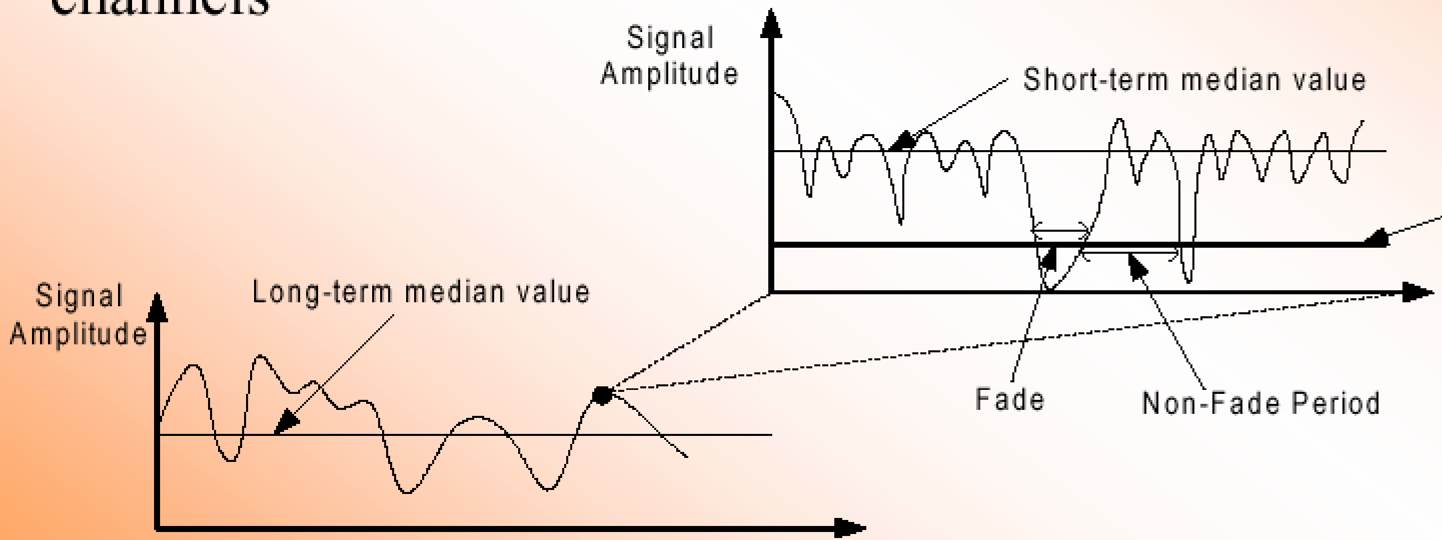
Nakagami Fading for stationary user

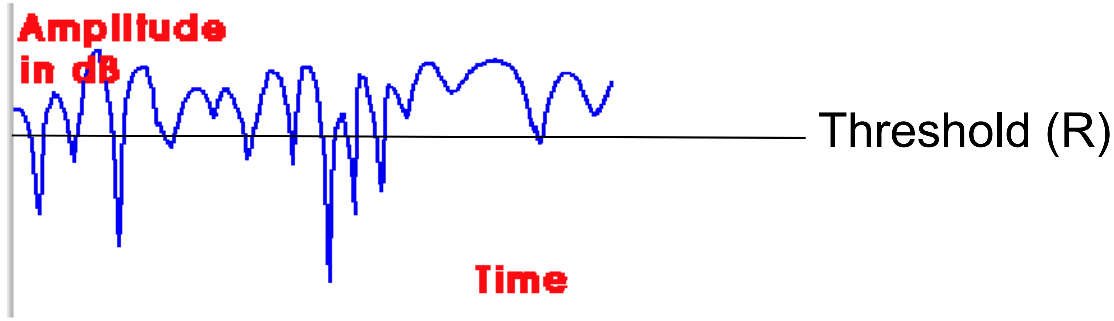


Level Crossing and Fade Rates

- LCR is the average number of times per second that a fading signal crosses a certain threshold
- It relates the time rate of change to the received signal envelope
- It can be used to characterize the nature of burst error in fading channels



Level Crossing Rate (LCR)



LCR is defined as the expected rate at which the Rayleigh fading envelope, normalized to the local rms signal level, crosses a specified threshold level R in a positive going direction. It is given by:

$$N_R = \sqrt{2\pi}\, f_m \rho e^{-\rho^2}$$

where

$$\rho = R / r_{rms} \quad \text{(specfied envelope value normalized to rms)}$$

N_R : crossings per second

Average Fade Duration

Defined as the average period of time for which the received signal is below a specified level R.

For Rayleigh distributed fading signal, it is given by:

$$\bar{\tau} = \frac{1}{N_R} \Pr[r \le R] = \frac{1}{N_R}\left(1 - e^{-\rho^2}\right)$$

$$\bar{\tau} = \frac{e^{\rho^2} - 1}{\rho f_m \sqrt{2\pi}}, \quad \rho = \frac{R}{r_{rms}}$$

ADF for Different Distributions

- It is mathematically defined as

$$ADF = N(\rho) = \hat{\tau} = \frac{P[r \leq R]}{N_R}$$

- **For Rayleigh:**

$$ADF = N(\rho) = \hat{\tau} = \frac{e^{\rho 2} - 1}{\sqrt{2\pi}\,\rho f_m}$$

where

$$\rho = \frac{R}{R_{rms}} = \text{ratio of threshold to rms amplitude}$$

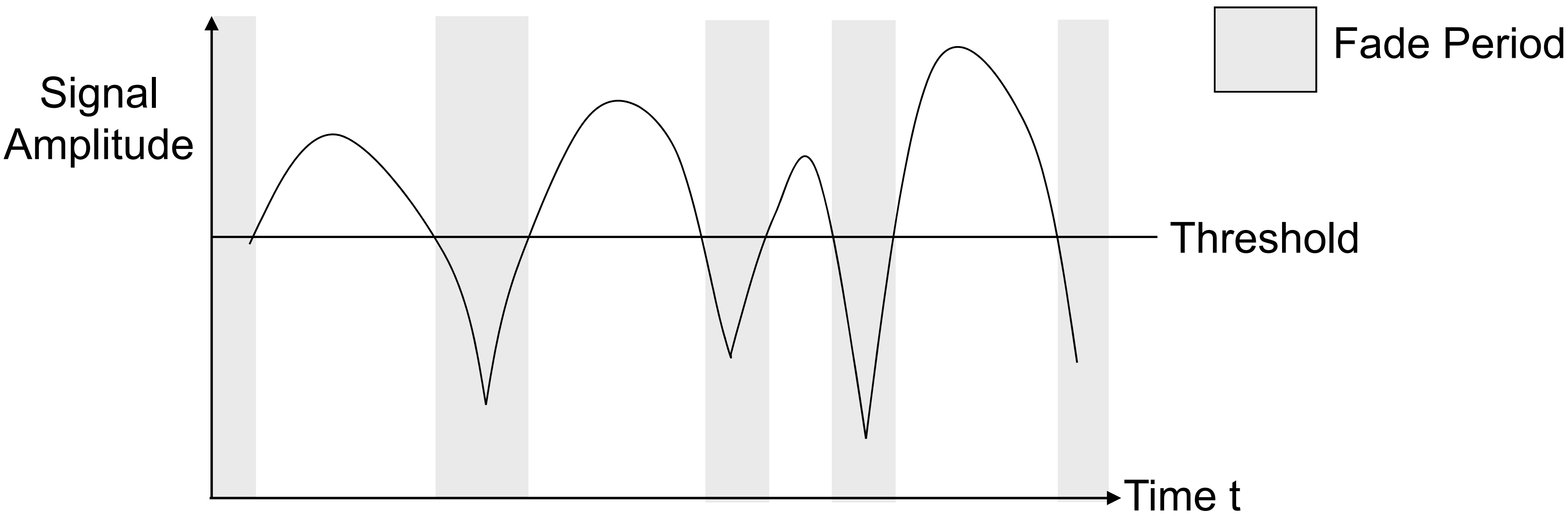
- **For Ricean:**

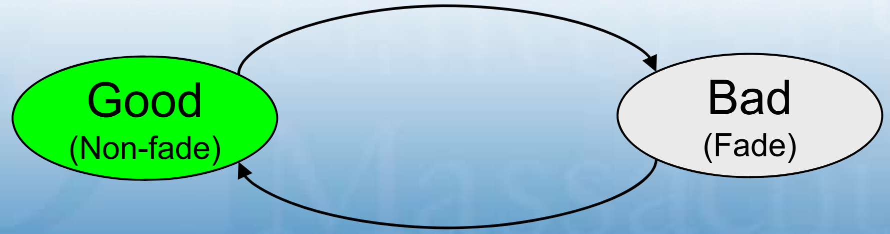
$$ADF = \hat{\tau} = \frac{1 - Q\left(\sqrt{2\pi K},\ \sqrt{2(K+1)\rho^2}\right)}{\sqrt{2\pi(K+1)}\, f_m \rho\, e^{-K-(K+1)\rho^2}\, I_o(2\rho\sqrt{K(K+1)})}$$

where Q(a,b) = Marcum Q-function

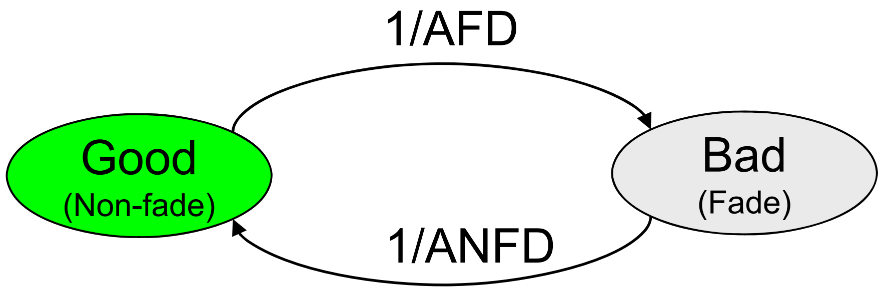
$$Q(a, b) = \int_b^\infty x \exp\left(-\frac{a^2 + x^2}{2}\right) I_o(ax)\,dx \qquad Q(a, 0) = 1, \quad Q(0, b) = \exp\left(-\frac{b^2}{2}\right)$$

Fading Model – Gilbert-Elliot Model





Gilbert-Elliot Model



The channel is modeled as a Two-State Markov Chain.
Each state duration is memory-less and exponentially distributed.

The rate going from Good to Bad state is: 1/AFD (AFD: Avg Fade Duration)
The rate going from Bad to Good state is: 1/ANFD (ANFD: Avg Non-Fade Duration)

16.582 Case Study: Channel Measurements for 2G MMDS and applicability to 4G LTE and WiMax

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Credits

- Based on slides from, Dhananjay Gore, Stanford University
- Conducted for Sprint Broadband, 1999-2000

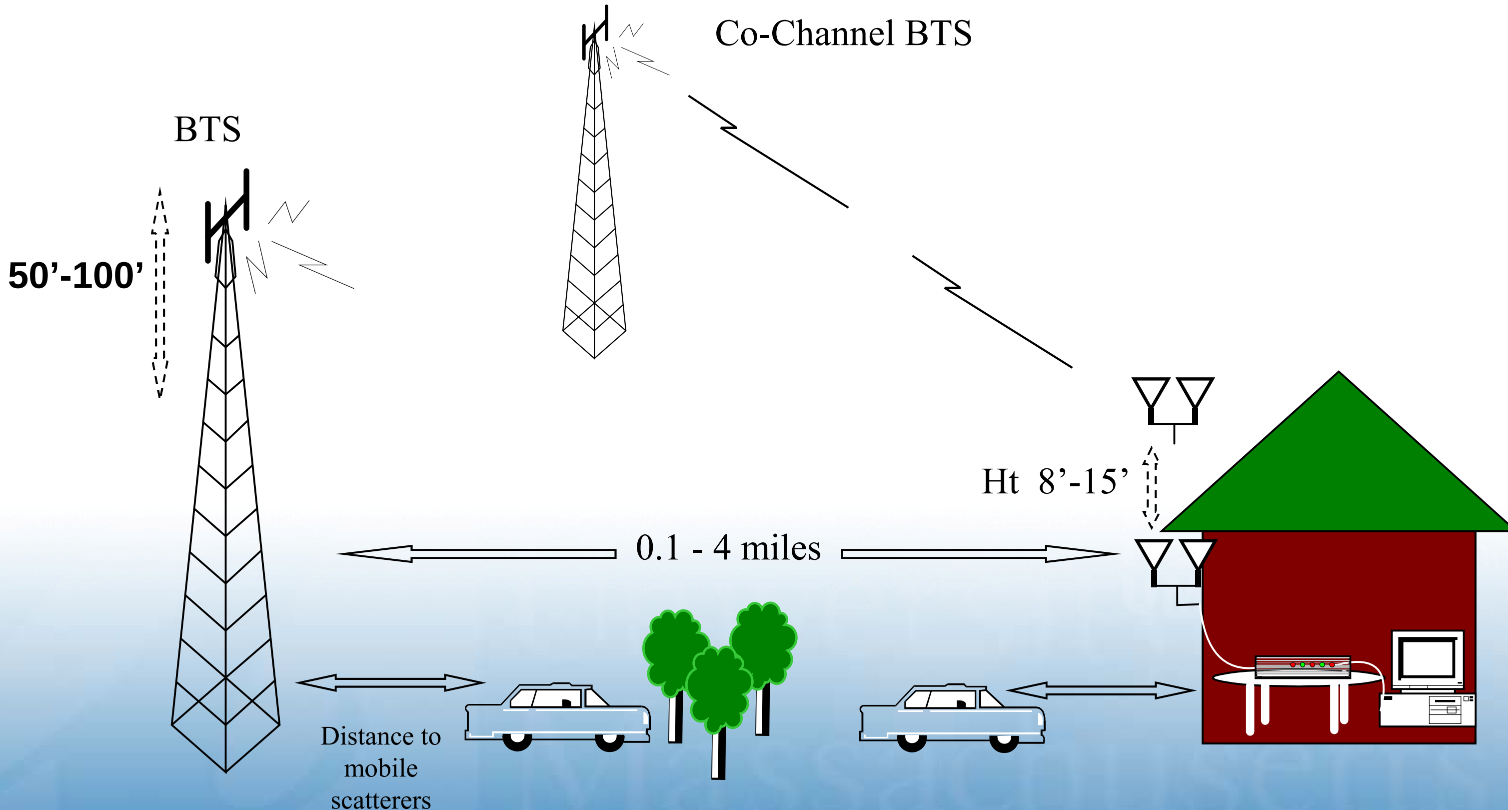
Goal of Program

To characterize wireless channels for 2G MMDS but 4G has been deployed in this band

What Is MMDS?

- MMDS (Microwave Multipoint distribution System), is a band of frequencies at 2.5 GHz, allocated for fixed and mobile digital communication
  - Originally viewed as a “wireless cable” system for broadcast digital services
  - Viewed as mostly TDD
- Business case required self installable CPE antennas and need to know reliability and channel characteristics

Typical Scenario



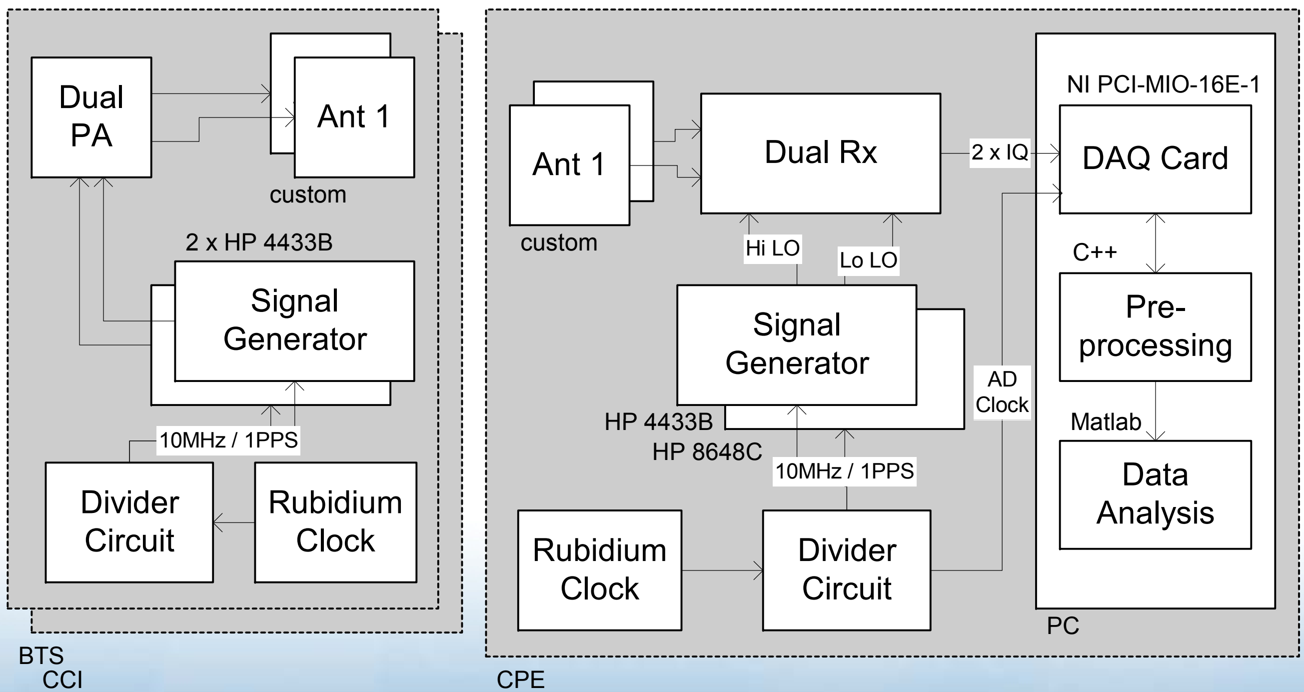
Scenario Dimensions

- Terrain
  - Rural, Suburban, Urban, Hilly
- Antenna Configuration
  - BTS, CPE antenna heights & spacing
  - Polarization, Beam-width
- Reuse Factor
  - 1 and 3
- Sectorization
  - 3

Antenna Configurations

- BTS antenna heights
  - 35’, 50’, 80’,120’ (35-120 ft)
- CPE antenna heights
  - Under the eaves: 85” to 95”, (~7 ft)
  - Patio of a Condominium: 130” (~10 ft)
  - Rooftop: 175” to 220” (15-20 ft)
- CPE antenna spacing
  - 0.5 - 5 wavelengths
- Beam-width 90^0 at BTS and 50^0 at CPE

Measurement Set-up



2480 MHz
4 MHz BW

Measured Channel Parameters

- Path Loss
- K-factor
- Delay Spread
- Doppler Power Spectrum
- Level Crossing Rates (LCR)
- Average Duration of Fade (ADF)
- Antenna Correlation
- C/I ratios

Path-Loss Measurements

- Published literature (AT&T measurements)
- SU measurements only for 0.1-4 miles
- SU measurements made in multiple Bay area locations
- SU measurements agree with AT&T measurements

SU: Stanford University

G2 MMDS Path Loss Model

Median Path Loss:

$$PL(dB) = A + 10\gamma \log_{10}(d/d_0) + s + \Delta PL_f + \Delta PL_h$$

for $d > d_0$

where

$$A = 20\log_{10}(4\pi d_0/\lambda)$$ (free space path loss)

$$\gamma = \left(a - bh_b + {c}/{h_b}\right)$$, 10 meters < h_b < 80 meters

(mean path loss exponent)

λ is the wavelength

Path Loss Model (contd.)

- s is a lognormal shadow fading
  - zero mean
  - terrain dependent standard deviation
- h_b is the BTS height in meters
- a, b, c are constants dependent on the terrain category
- d_o is chosen as 100m (reference distance)
- d is the distance from BTS

Correction Terms

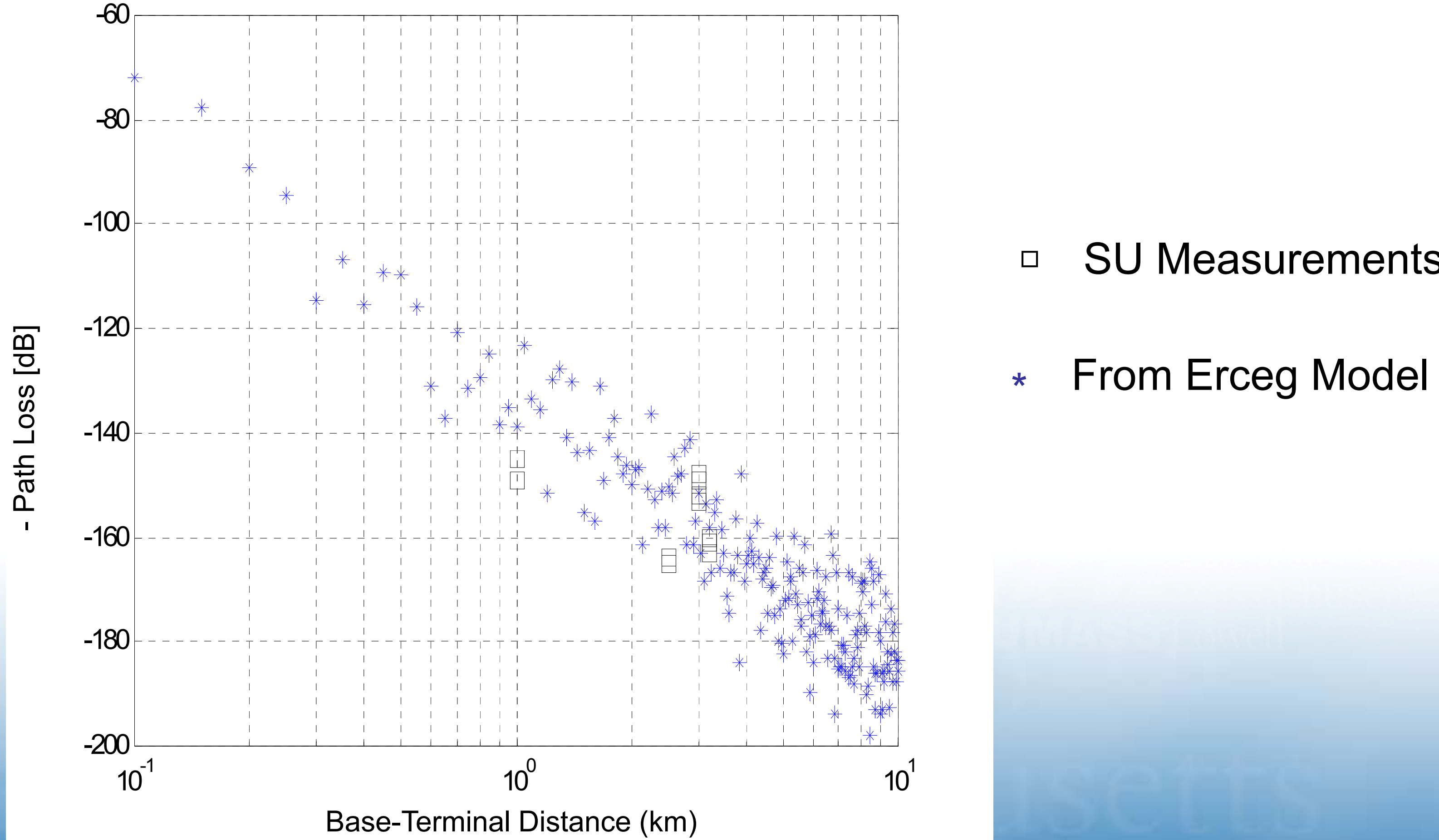
- Frequency correction terms

$$\Delta PL_f = 5.7\log\left(f/2000 \right)$$ *f* in MHz

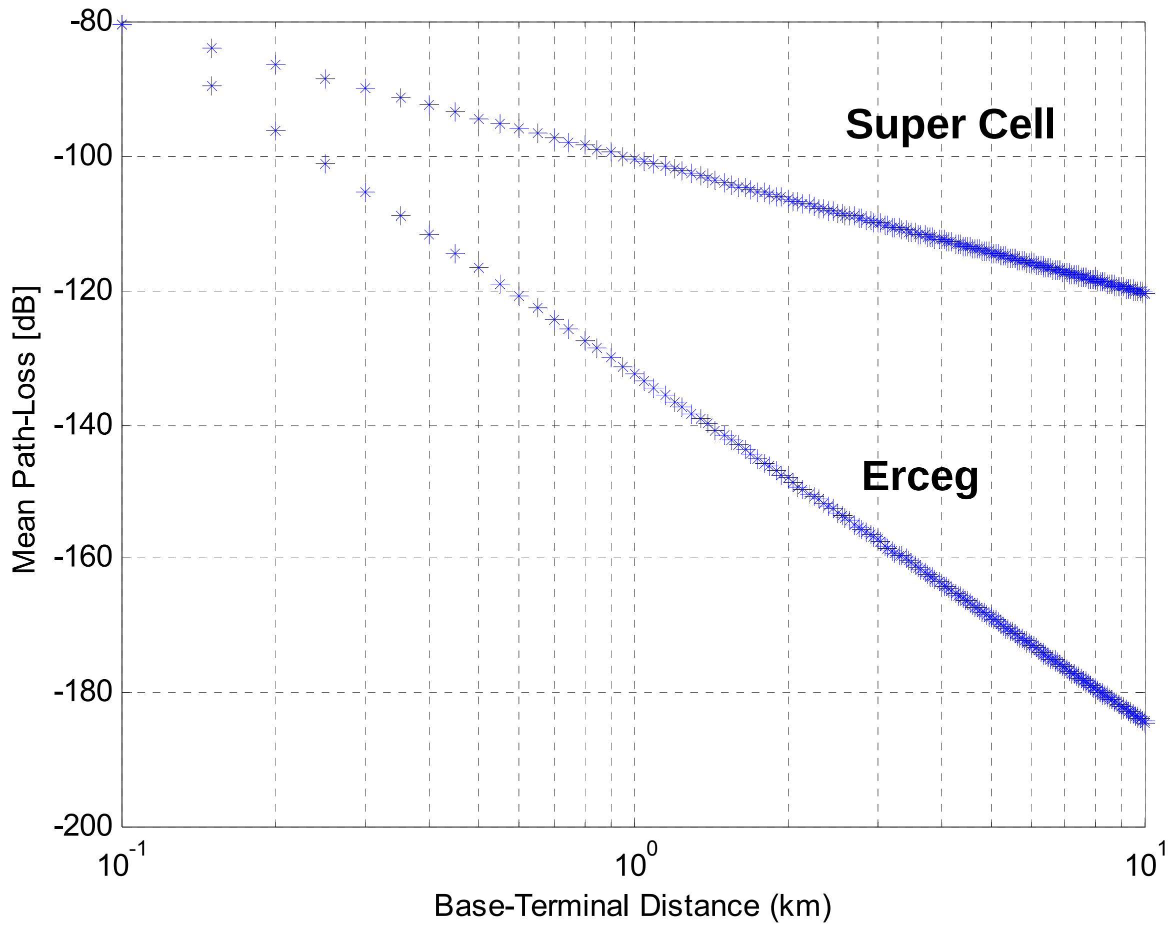
- CPE height correction term (> 2 meters)

$$\Delta PL_h = -10.8\log(h_{CPE}/2)$$ 1 meter < h_{CPE} < 8 meters

Path Loss Scatter Plot



Mean Path Loss vs Distance

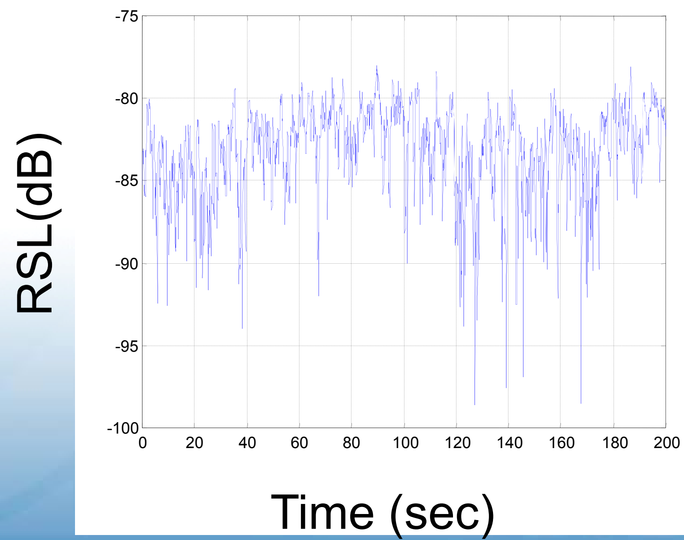


K-factor Measurements

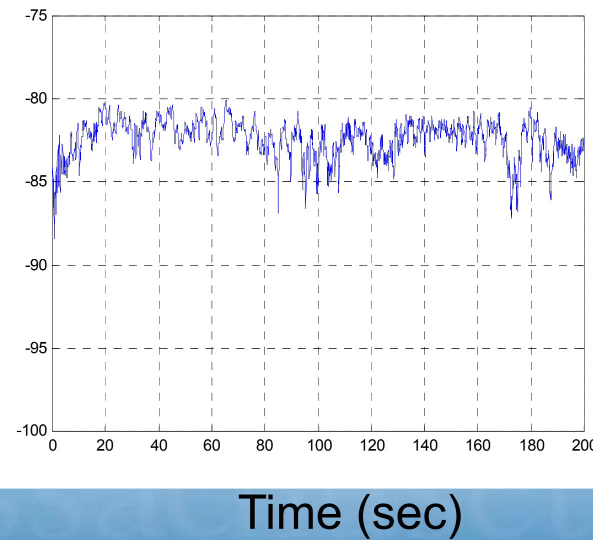
$$K = \frac{\text{power in fixed (mean) component}}{\text{power in varying (scattered) component}}$$

Typical Signal Envelope:

K = -10 dB



K = 6 dB



K-factor Model

- Erceg model for K-factor

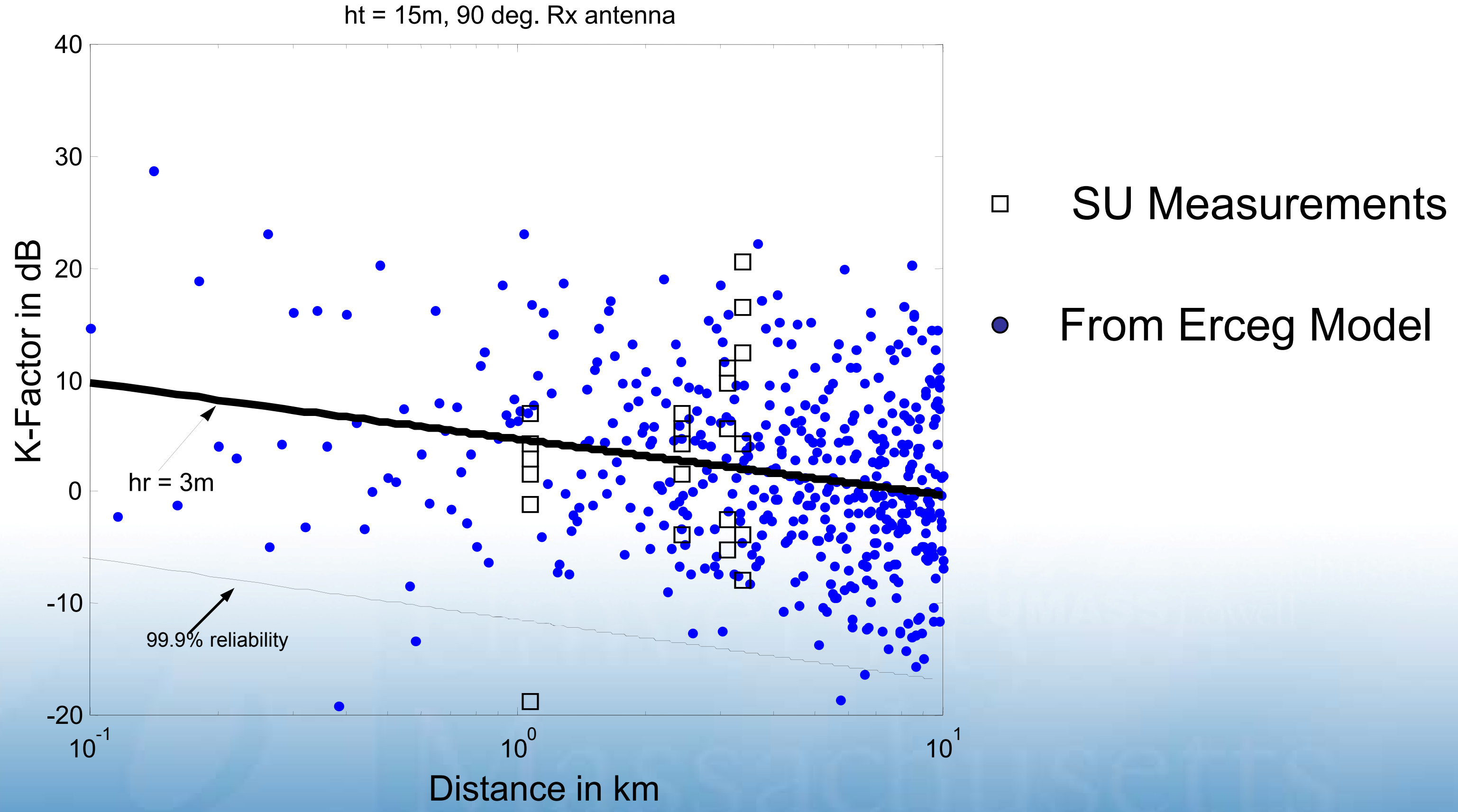
$$K = F_s F_h F_b K_o d^{\gamma} u$$

- F_s is a seasonal factor
  - 1.0; summer (leaves)
  - 2.5; winter (no leaves)
- F_h is the height factor
  - $(h/3)^{0.46}$ (h is the CPE height in meters)

K-factor Model (contd.)

- F_b is the beamwidth factor
  - $F_b = (b/10)^{-0.62}$; (b in degrees)
- K_o and γ are regression coefficients
  - $K_o = 10; \gamma = -0.5$
- u is a lognormal variable
  - zero mean
  - std. deviation of 8.0 dB

K-factor Scatter Plot



K-factor and Reliability

- K-factors are highly variable
- To ensure 99.9% reliability, systems must be designed for zero K-factor (Rayleigh fading)

Delay Spread Model

- Spike-Plus-Exponential Model (Erceg)

$$P(\tau) = A\delta(\tau) + B\sum_{i=0}^{\infty} e^{-i\Delta\tau/\tau_o}\delta(\tau - i\Delta\tau)$$

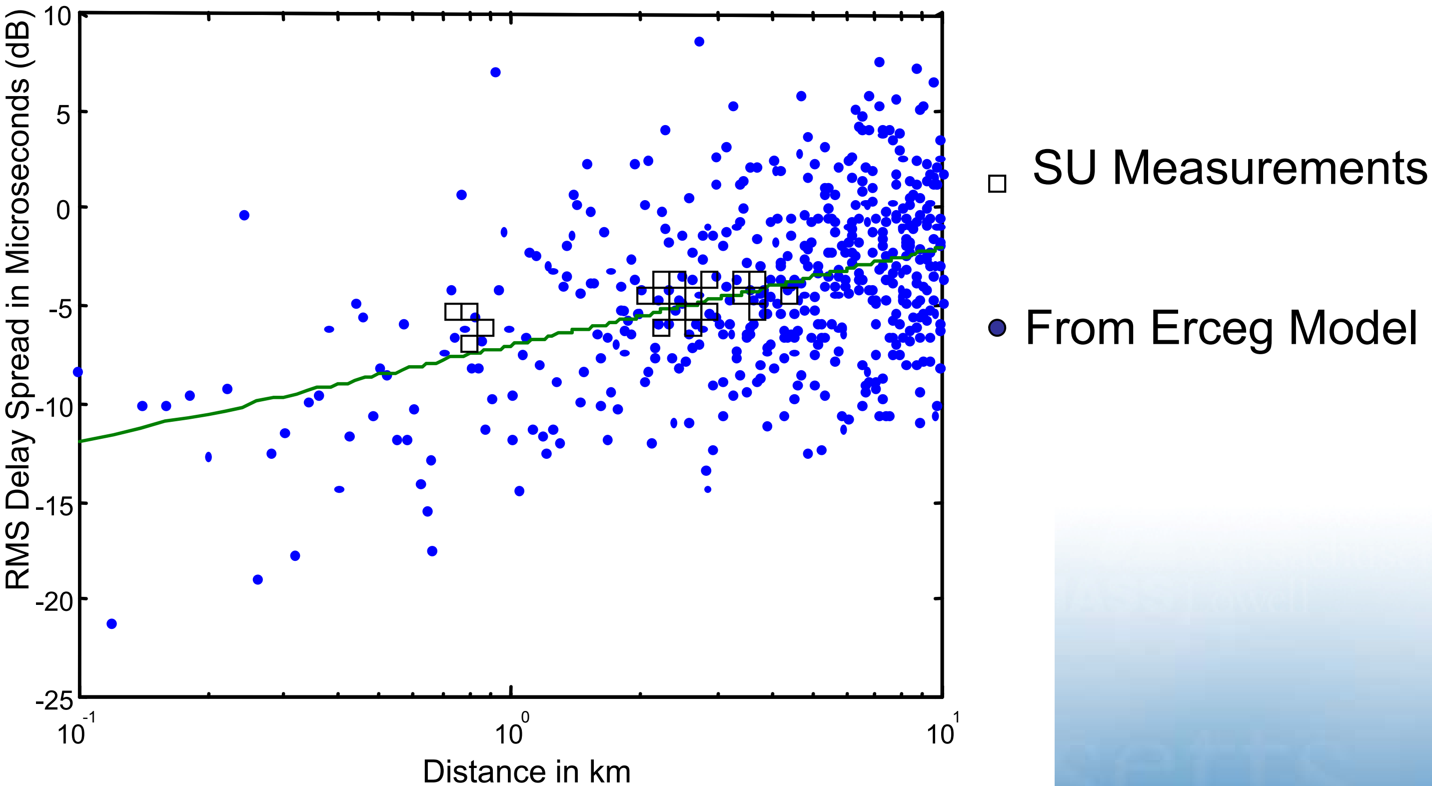
A, *B*, τ_o and $\Delta\tau$ are experimentally determined

$$T_{rms} = \frac{\Delta\tau}{e^{\Delta\tau/2\tau_o} - e^{-\Delta\tau/2\tau_o}}$$

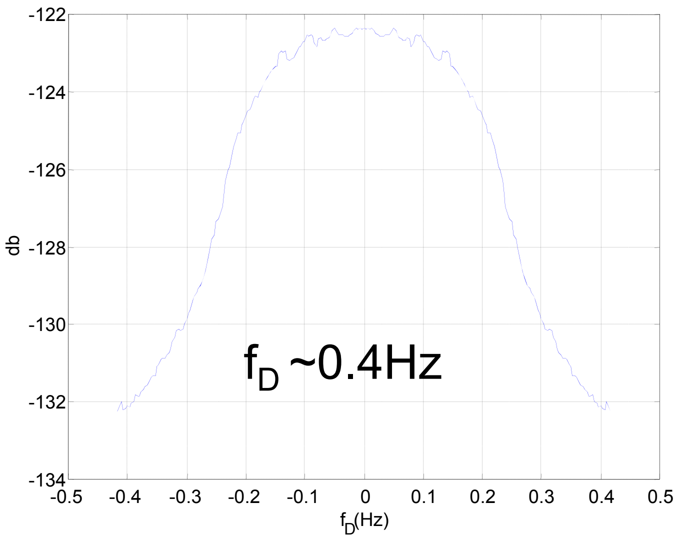
- Good Model for directive antennas

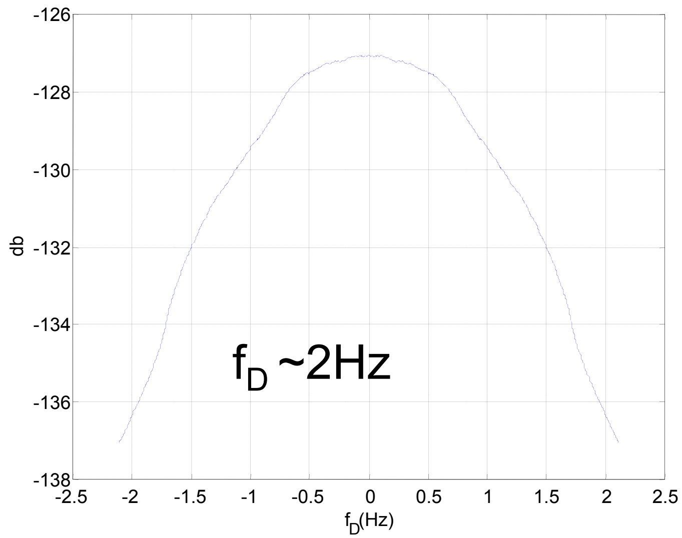
Delay Spread Scatter Plot

(Suburban)



Doppler Power Spectrum





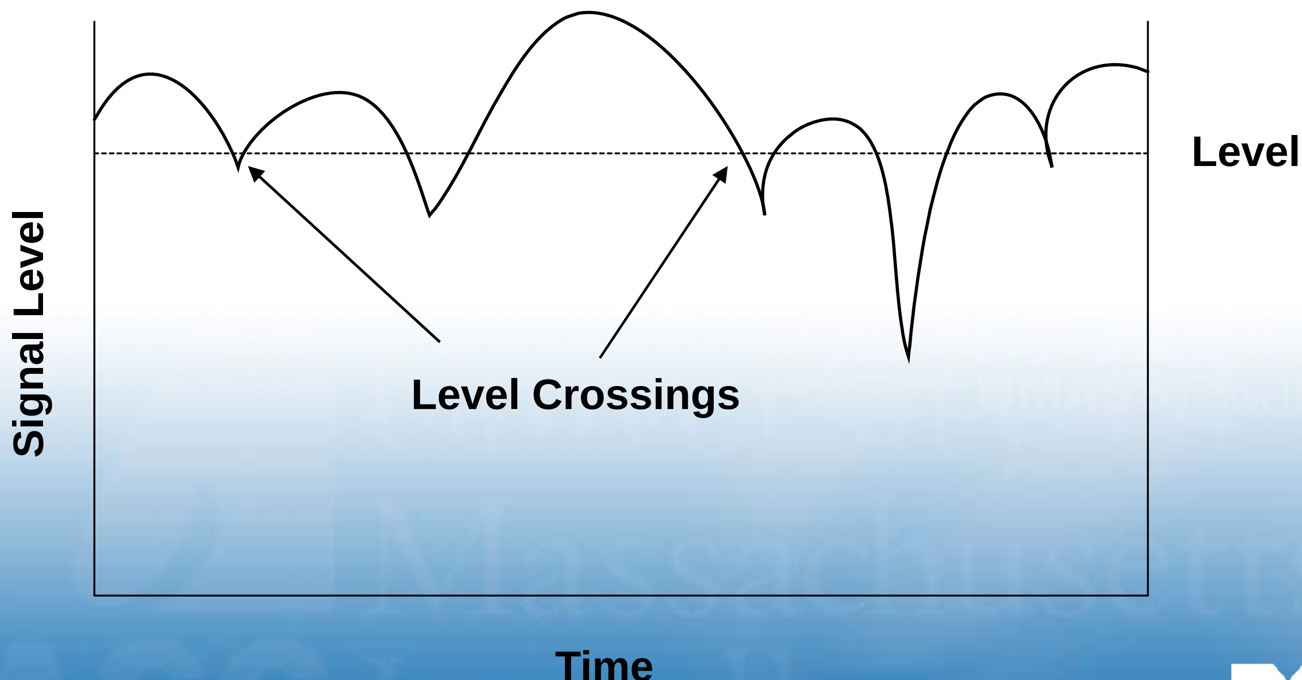
Low Wind

High Wind

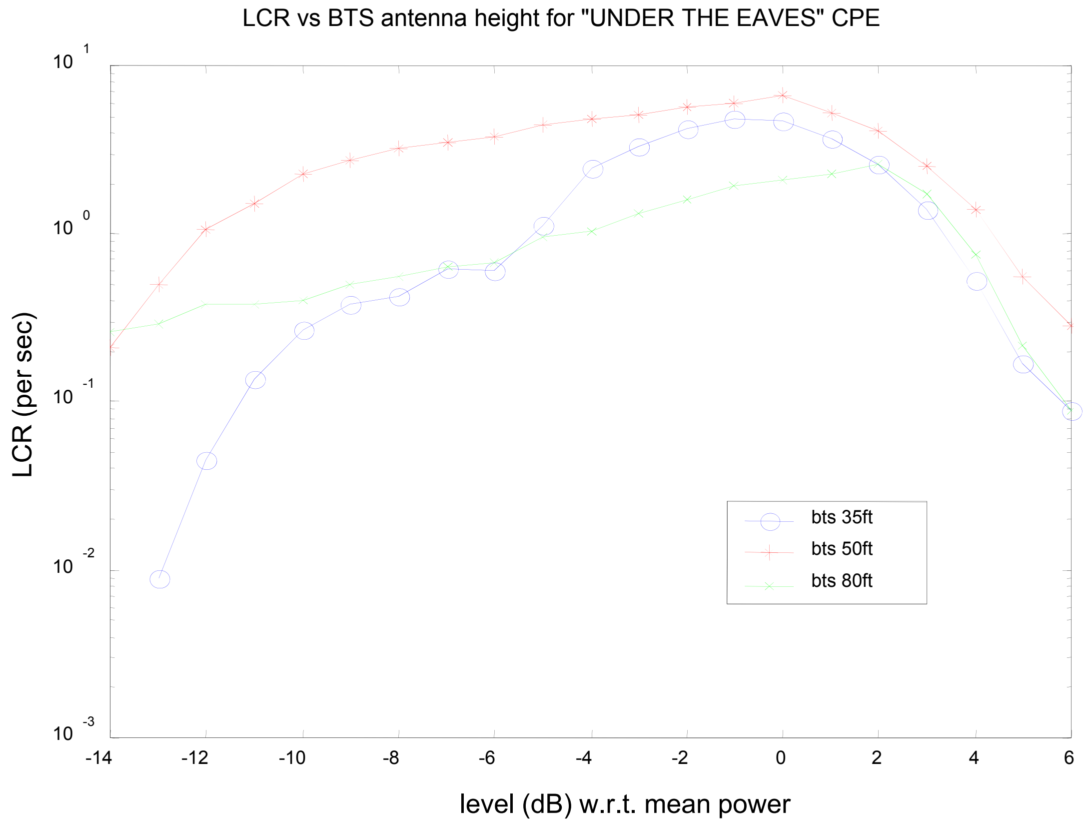
Rounded Spectrum with f_D~ 0.1Hz- 2Hz

Level Crossing Rate (LCR)

LCR is the rate (in sec) at which the signal crosses a certain level

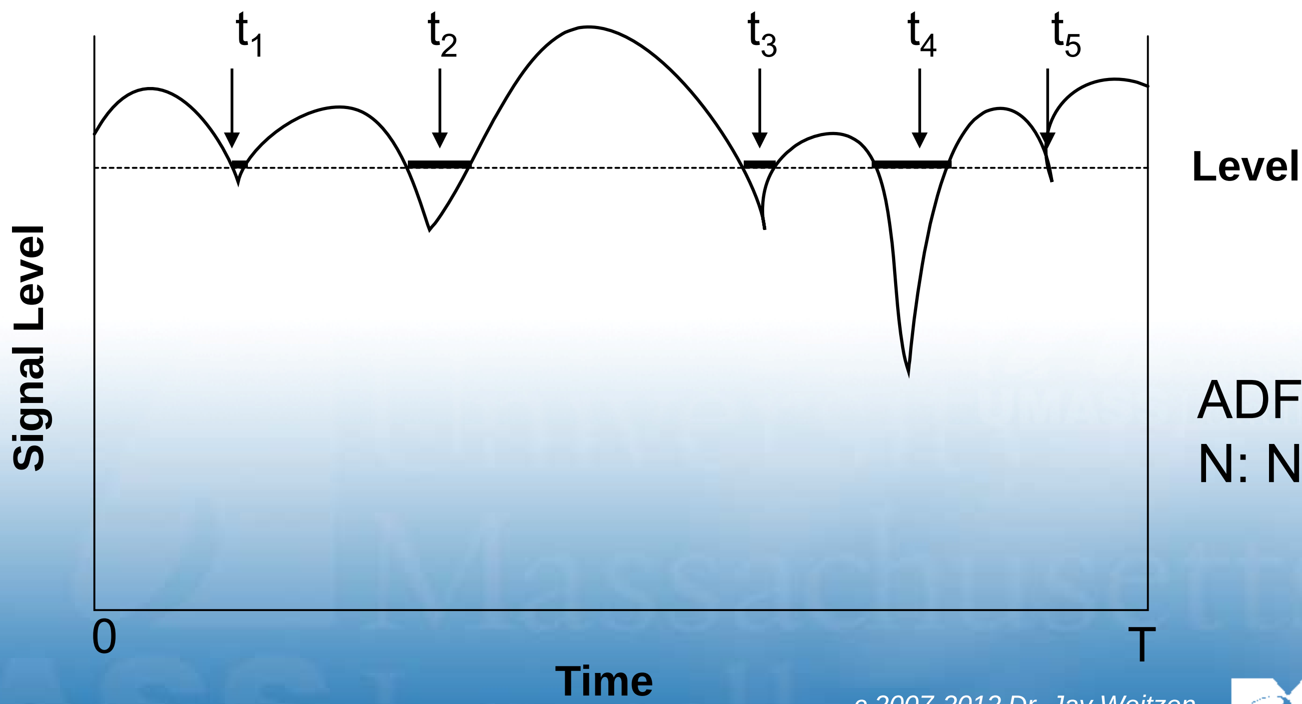


LCR (measured)



Average Duration of Fade (ADF)

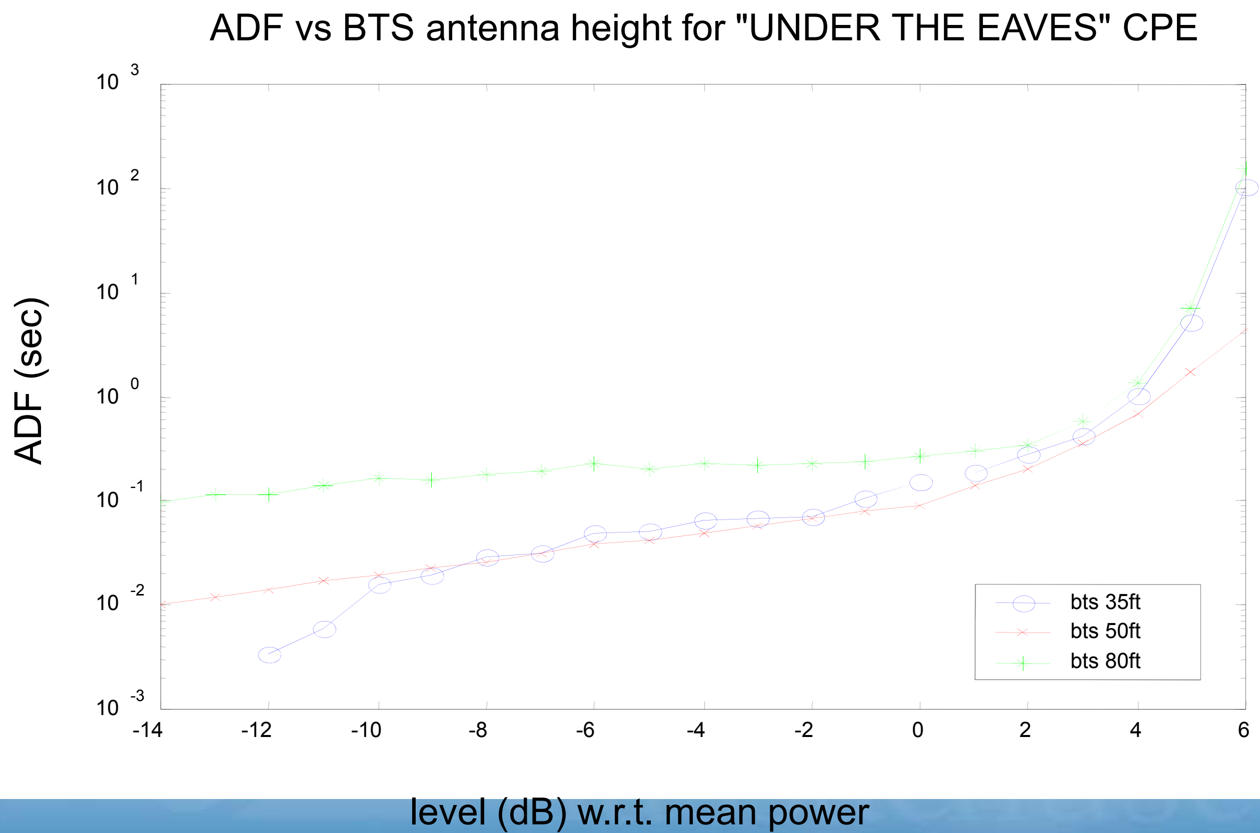
ADF is the average duration (in secs) for which the signal level stays below a certain threshold



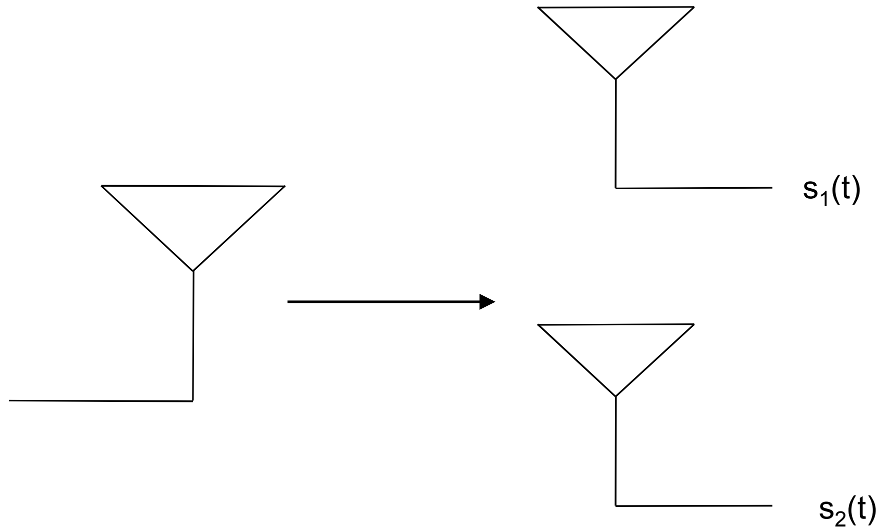
ADF = $(\Sigma\ t_i)/N$

N: No. of fades

ADF (measured)

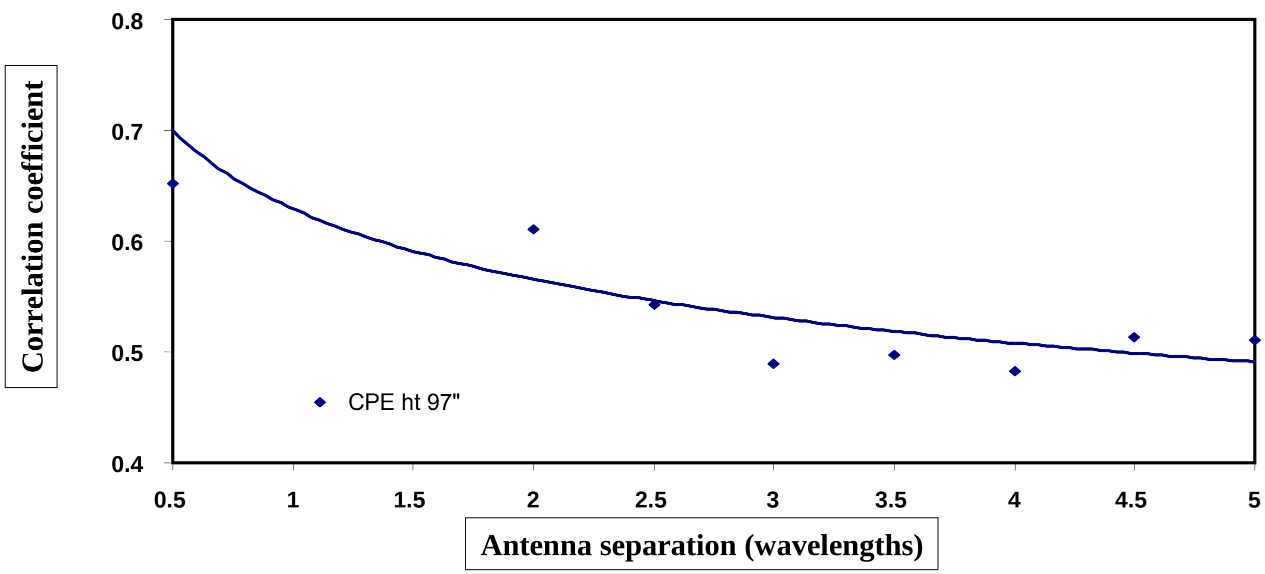


Antenna Correlation (Spatial)



$$\rho_{s_1,s_2} = \frac{E[|s_1 s_2|] - E[|s_1|]E[|s_2|]}{\sqrt{E[(|s_1| - E[|s_1|])^2]E[(|s_2| - E[|s_2|])^2]}}$$

CPE Antenna Correlation Coefficient vs Antenna Spacing

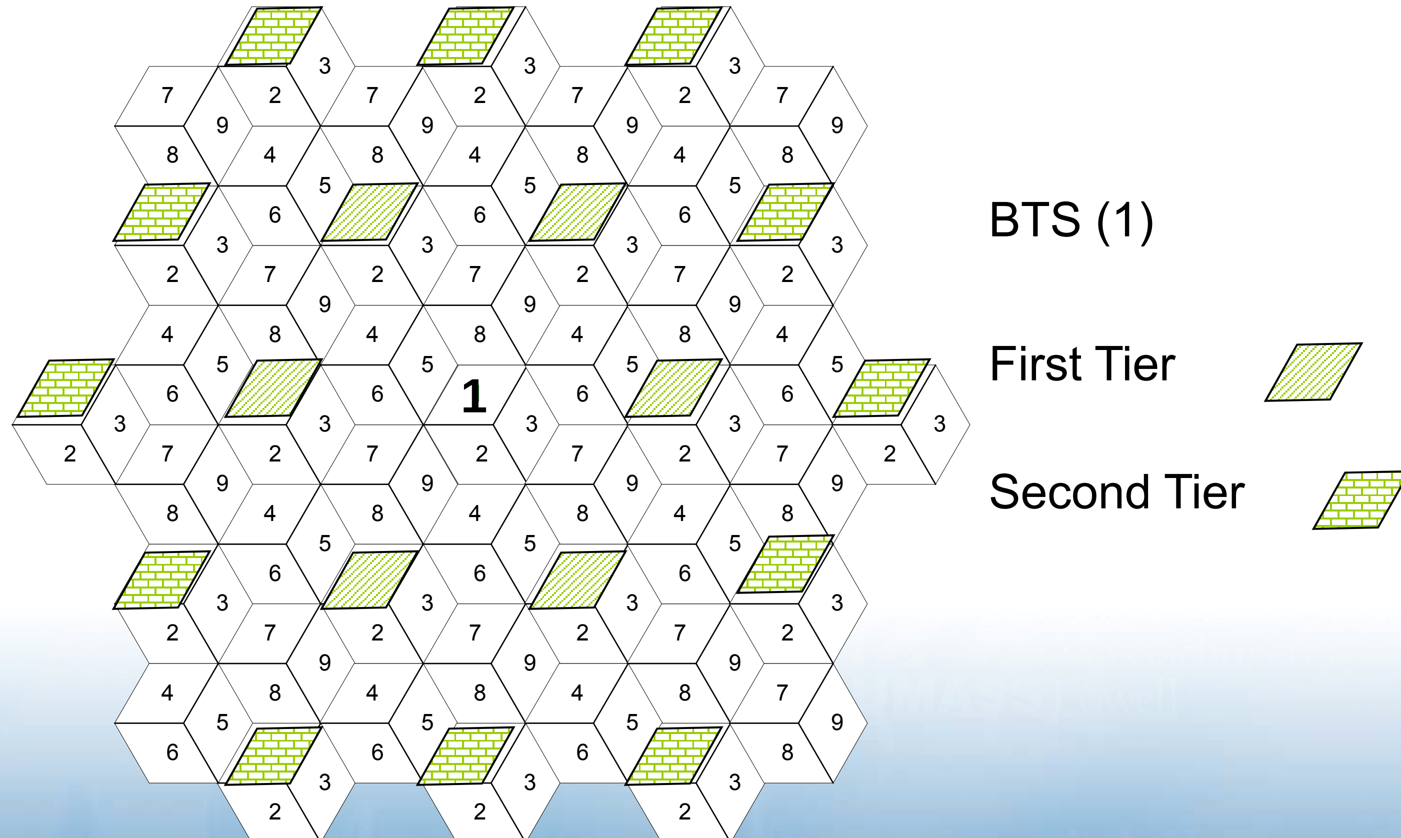


BTS ht 35'

CPE (V. Pol)

- 0.75 - 1 wavelength spacing adequate for under the eaves CPE
- 10 wavelengths sufficient for BTS antenna spacing

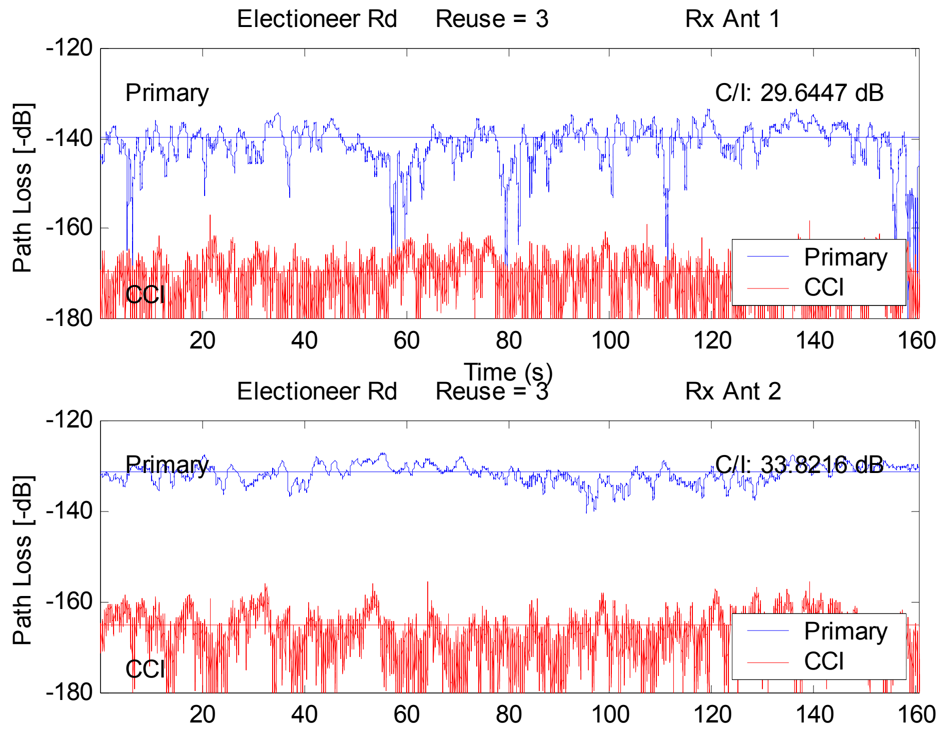
Frequency Reuse



Reuse Factor 3 x 9

Measured C/I (Cell Edge)

Excellent Conditions



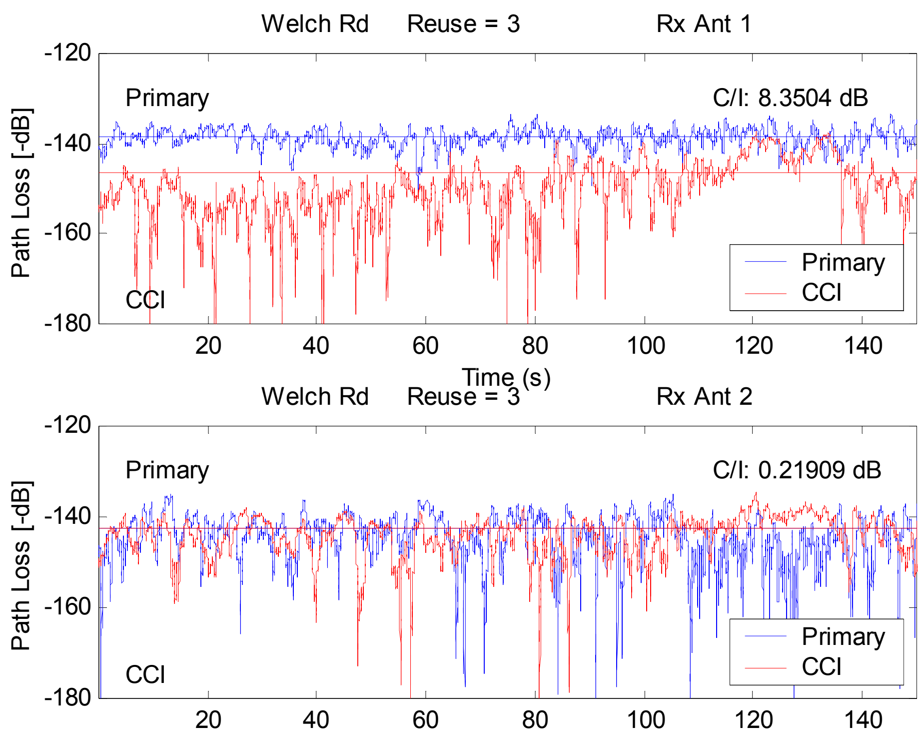
C/I = 29.6dB

C/I = 33.8dB

Reuse 3

Measured C/I (Cell Edge)

Poor Conditions

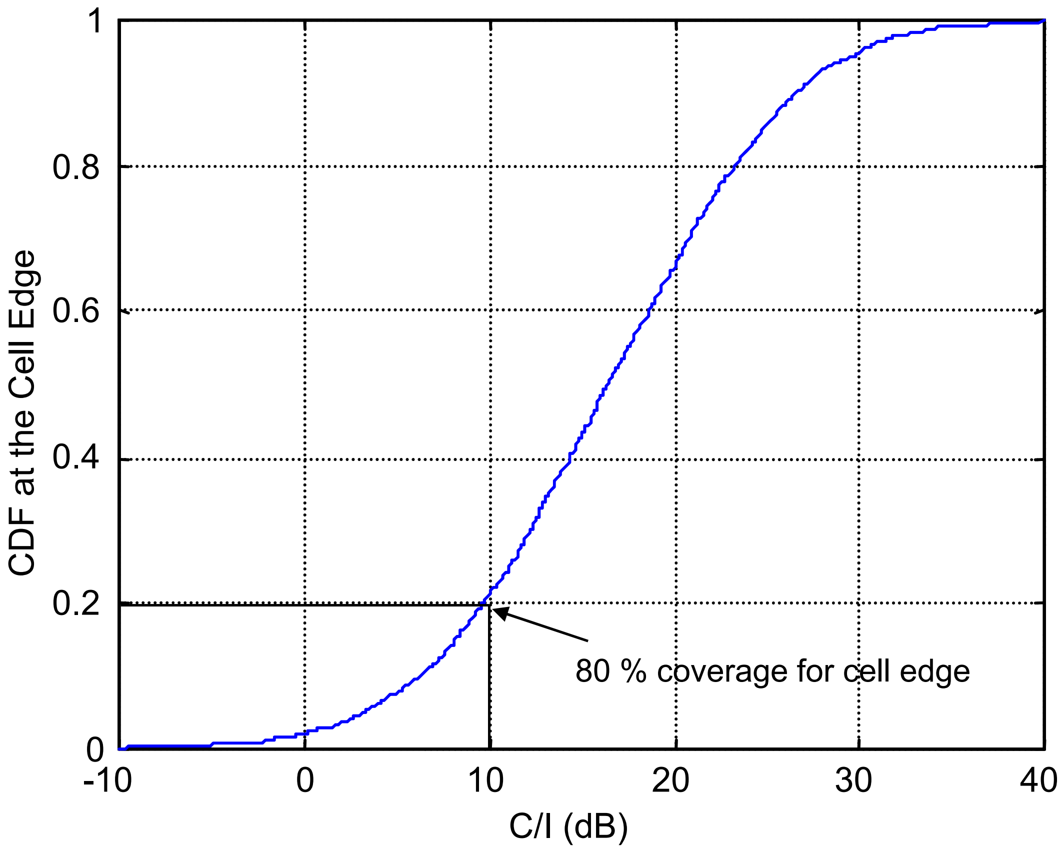


C/I = 8.3 dB

C/I = 0.21 dB

Reuse 3

CDF of C/ I at the Cell Edge
(Reuse= 3 x 9)



- C/I statistics
  - Randomly populate subs
  - Compute path loss and shadow loss
  - Compute C/I
  - Average over many trials

Summary

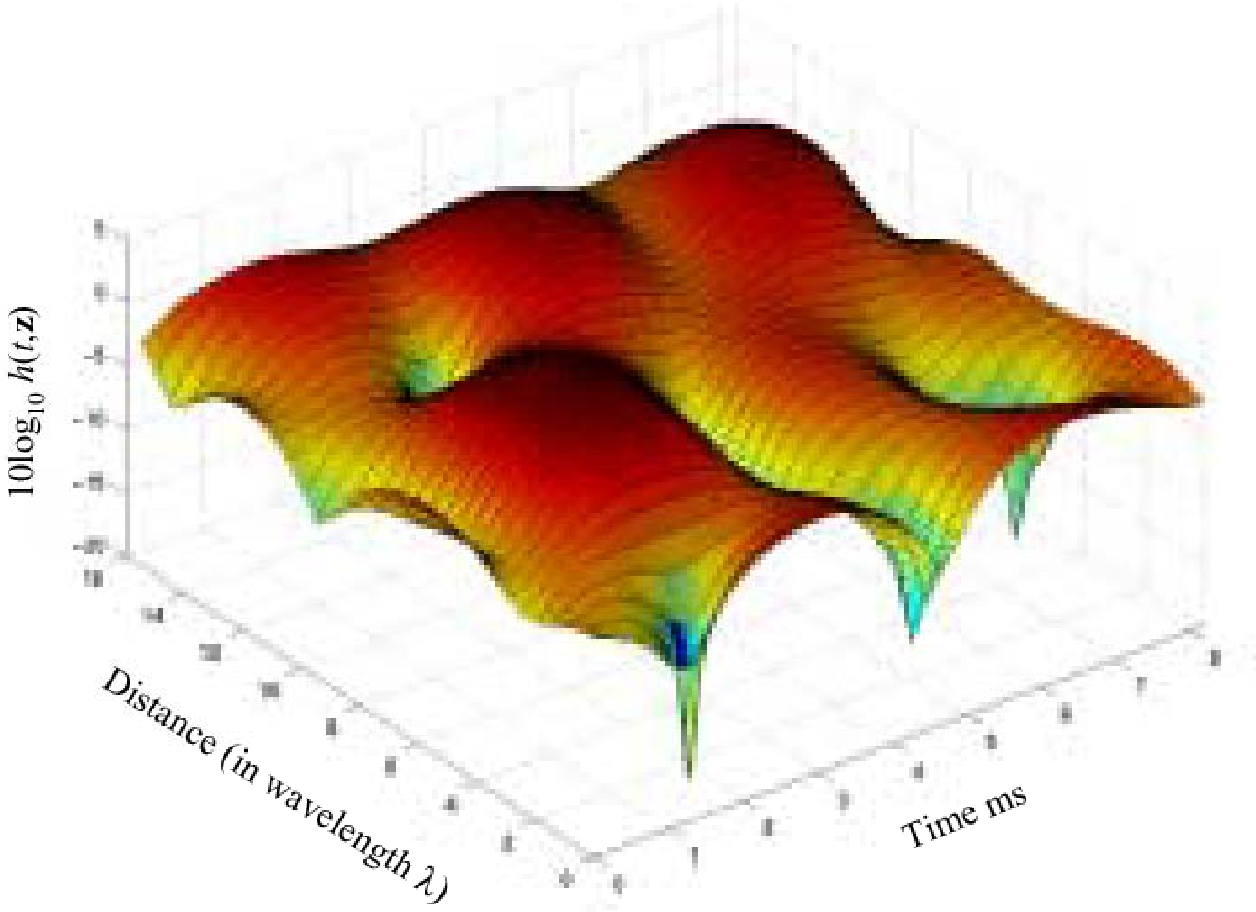
- Over 200 hrs of measurement effort
- Measured parameters (Path Loss, K-factor and Delay Spread) appear to conform to AT&T results
- Consistency in new measurements of Doppler, antenna correlation, LCR and ADF
- We feel reasonably comfortable that measurements capture the true nature of MMDS propagation
- More measurements planned

References

- V. Erceg et. al, “An empirically based path loss model for wireless channels in suburban environments,” *IEEE JSAC*, vol. 17, no. 7, July 1999, pp. 1205-1211.
- V. Erceg et.al, “A model for the multipath delay profile of fixed wireless channels,” *IEEE JSAC*, vol. 17, no.3, March 1999, pp. 399-410.
- Larry J. Greenstein et.al, “A new path-gain/Delay-spread propagation Model for digital Cellular Channels,” *IEEE Trans. On Vehicular Technology*, vol. 46, no. 2, May 1997.
- L.J. Greenstein, S. Ghassemzadeh, V.Erceg, and D.G. Michelson, “Ricean K-factors in narrowband fixed wireless channels: Theory, experiments, and statistical models,” *Proceedings of WPMC’99*, Amsterdam, September 1999.
- David Parsons, “*The Mobile Radio Propagation Channel*,” John Wiley and Sons, 1992.
- L. J. Greenstein and Vinko Erceg, “Gain Reductions Due to Scatter on Wireless Paths with Directional Antennas,” *IEEE Communications Letters*, vol. 3, No. 6, June 1999.
- L.J. Greenstein et.al, “Moment-method estimation of the Ricean K-factor,” *IEEE Communications Letters*, vol.3, no.6, June 1999, pp. 175-176.

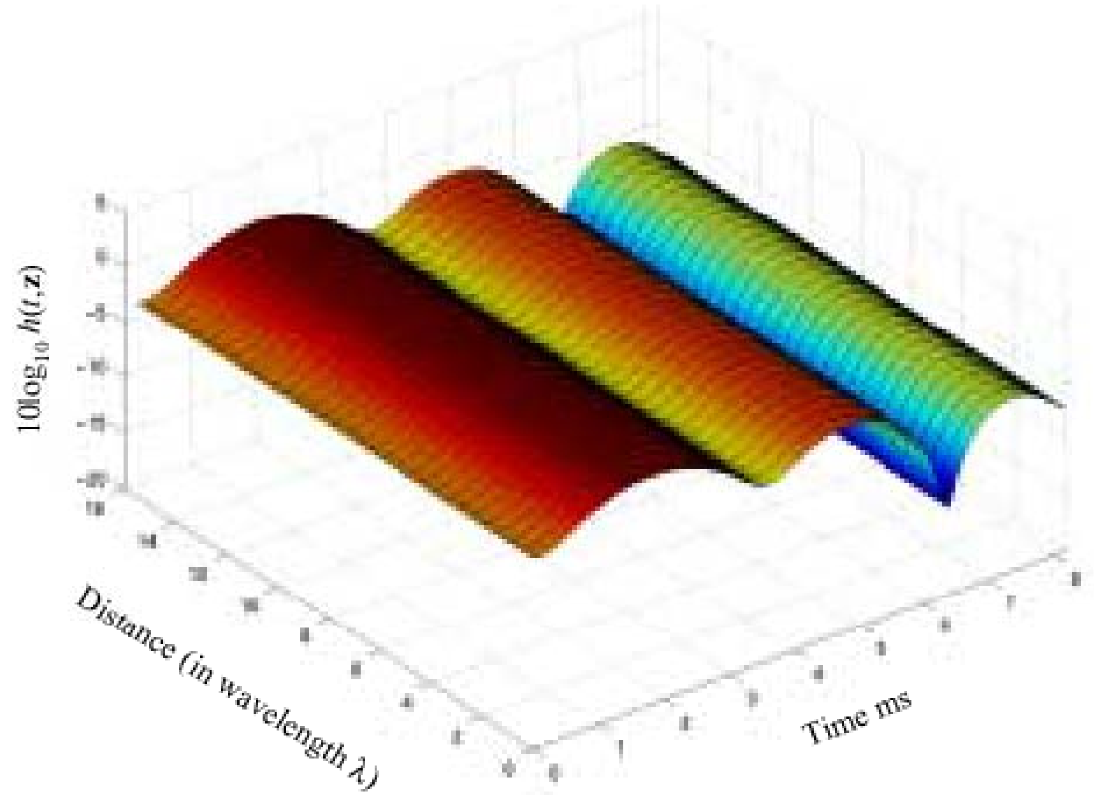
Diversity in Mobile Radio Systems

Space Time Fading: Wide Beam



Angle Spread $\Theta_d = 5°$, Doppler Spread $f_d = 200$ Hz

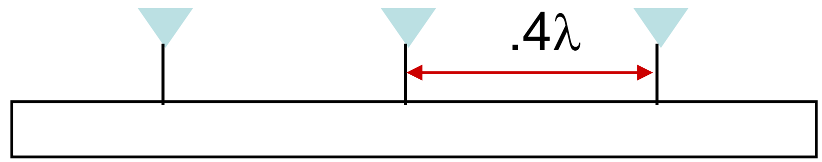
Space time Fading, narrow beam



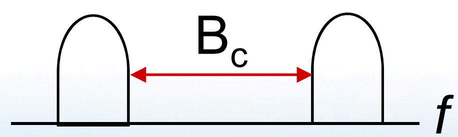
Angle Spread $\Theta_d = 0°$, Doppler Spread $f_d = 200$ Hz

Independent Paths

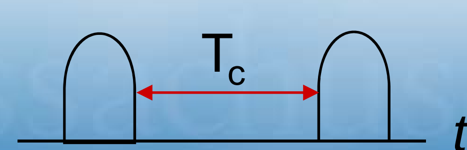
- Space Diversity



  - Multiple antenna elements separated by decorrelation distance.
- Polarization Diversity
  - Two transmit or receive antennas with different polarizations
- Frequency Diversity



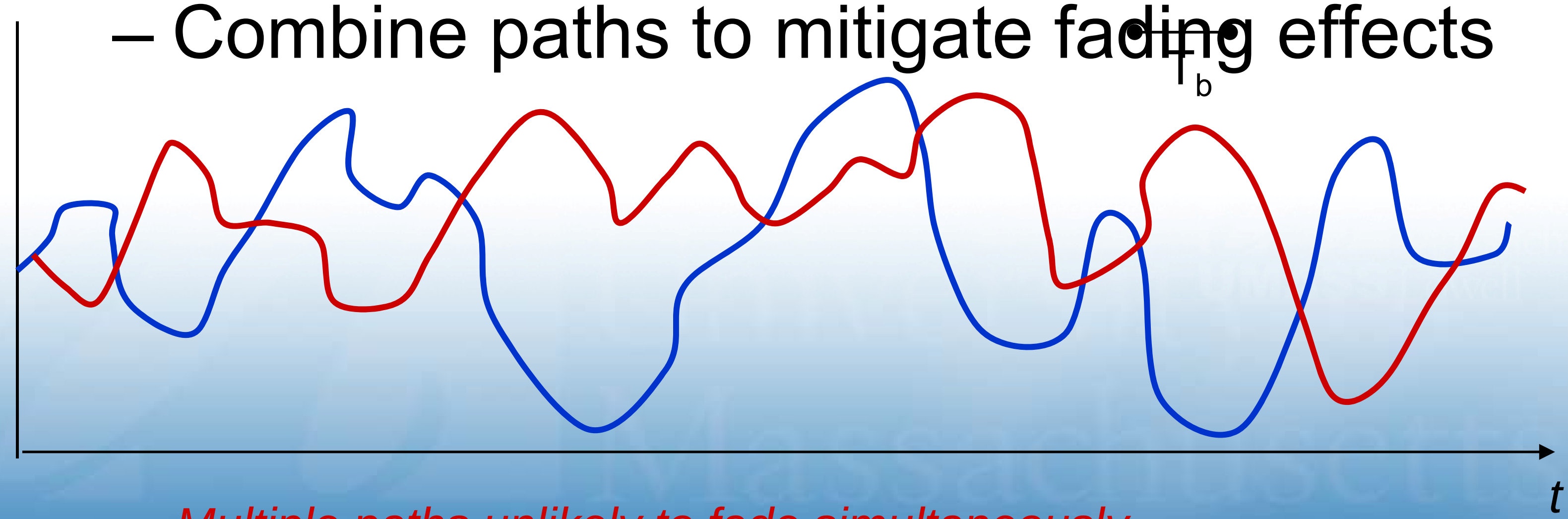
  - Multiple narrowband channels separated by channel coherence bandwidth
- Time Diversity



  - Multiple timeslots separated by channel coherence time.

Introduction to Diversity

- Basic Idea
  - Send same bits over independent fading paths
  - Combine paths to mitigate fading effects



Multiple paths unlikely to fade simultaneously

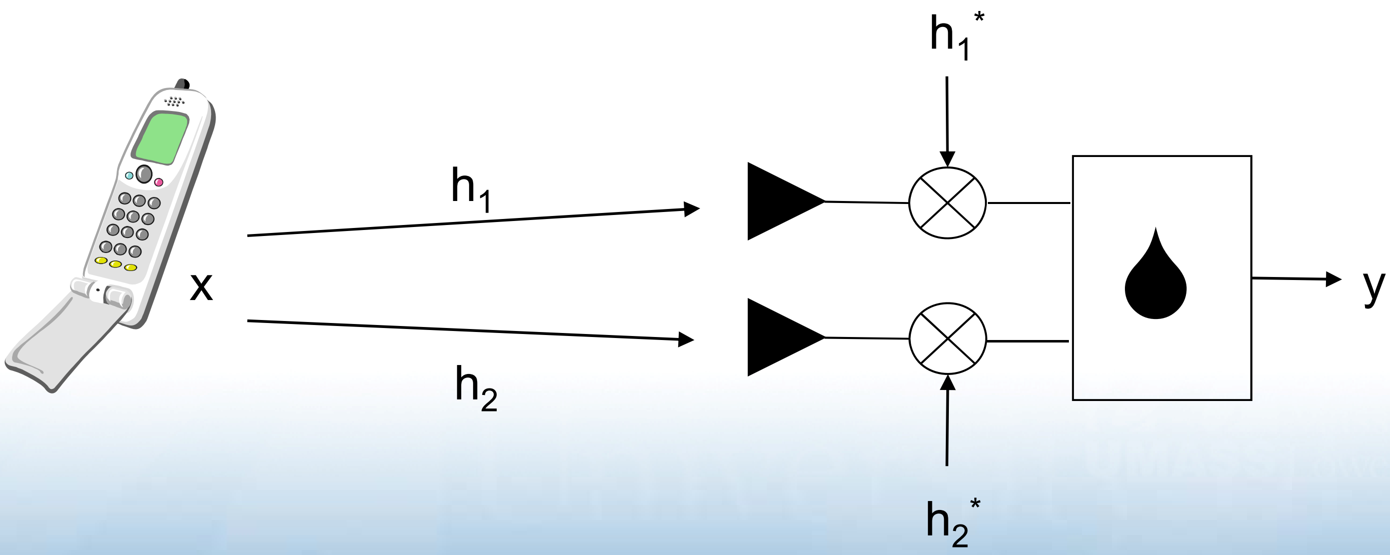
How To Maximize Diversity

- Want 2 or more signals with approximately same average power
- Want signals to be uncorrelated

Combining Techniques

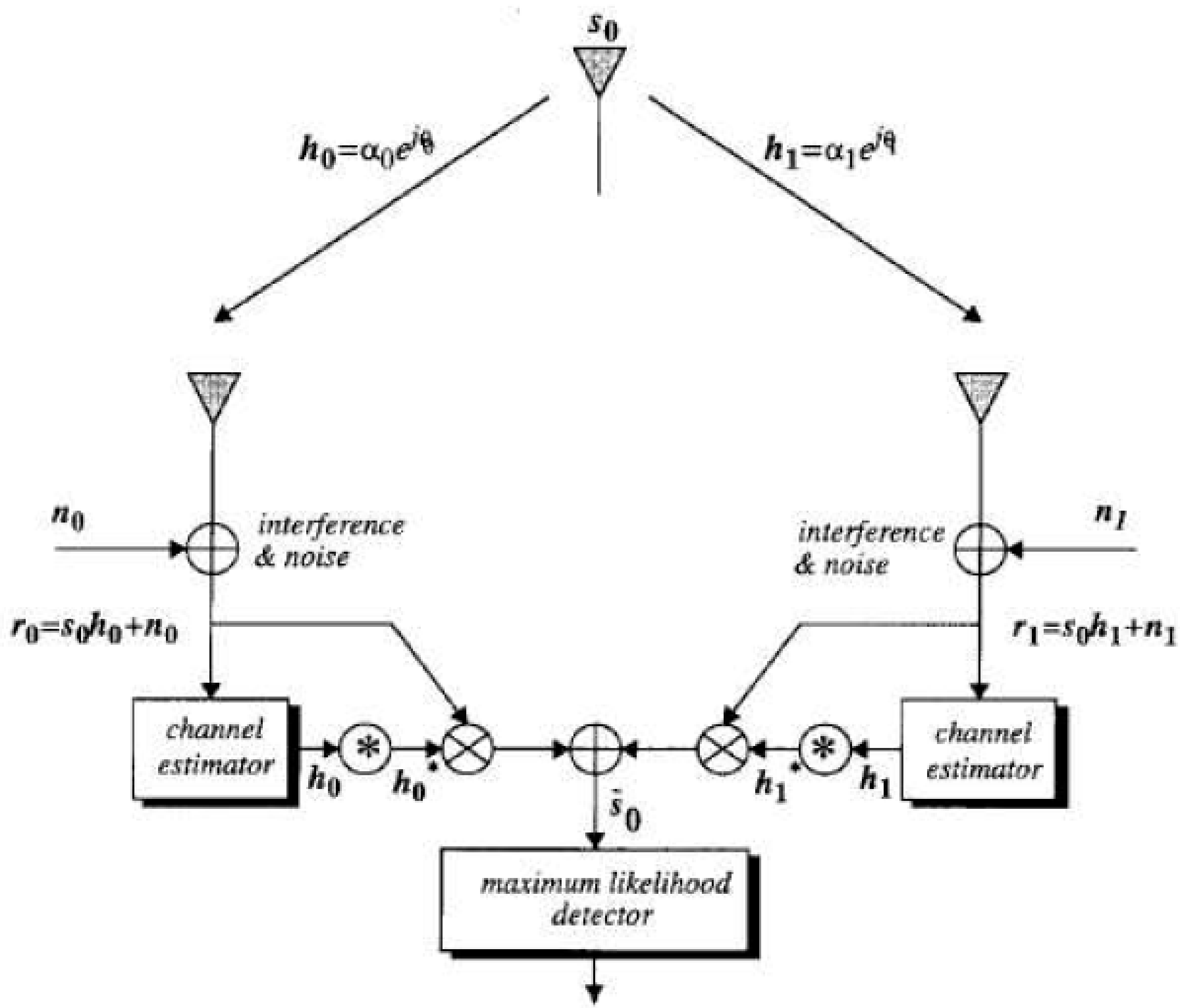
- Selection Combining
  - Fading path with highest gain used
- Equal Gain Combining
  - All paths cophased and summed with equal weighting
- Maximal Ratio Combining
  - All paths cophased and summed with optimal weighting to maximize combiner output SNR

Maximum ratio combining (MRC)

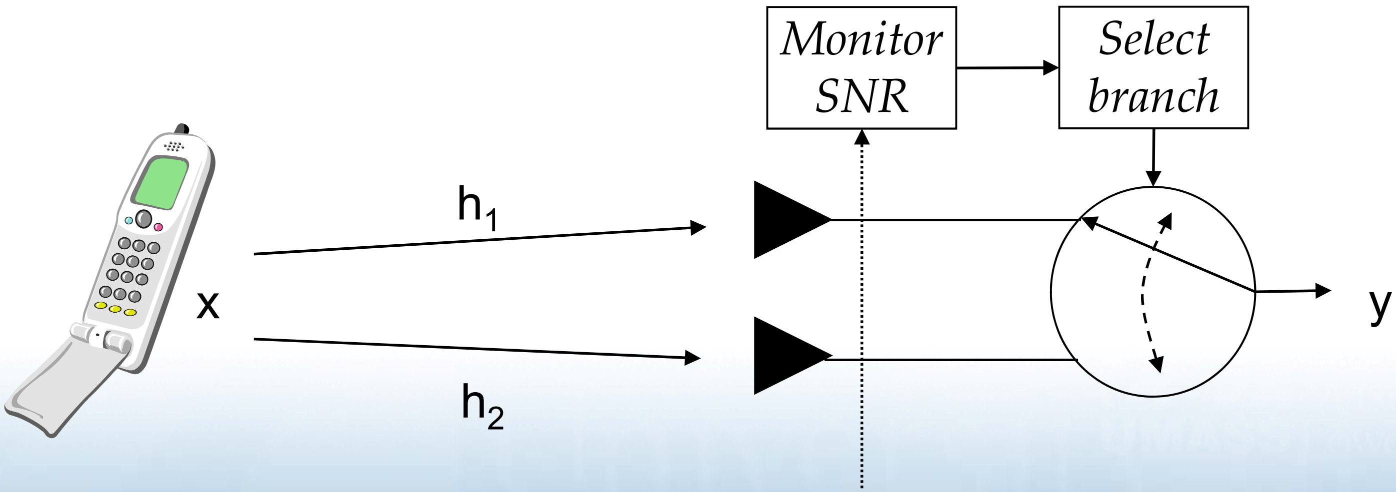


$$y = \left(|h_1|^2 + |h_2|^2\right) x$$

Maximum ratio combining (cont'd)



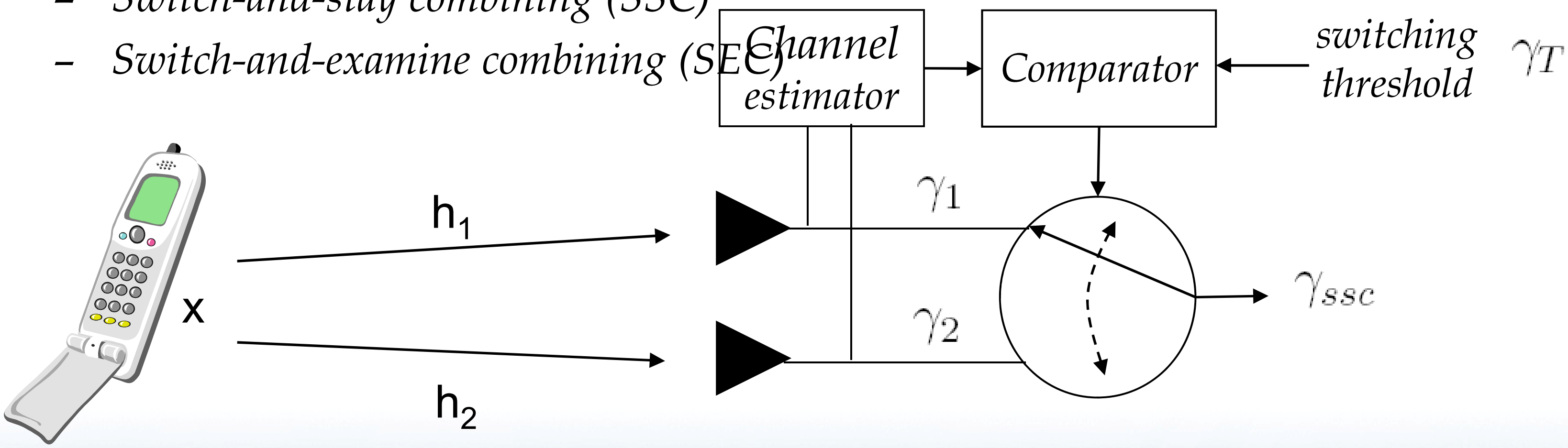
Selection combining (SC)



$$y = \max\left(|h_1|^2, |h_2|^2\right) x$$

Switched diversity

- *Switched diversity*
  - *Switch-and-stay combining (SSC)*
  - *Switch-and-examine combining (SEC)*



$$\gamma_{ssc}(n) = \gamma_1(n) \text{ iff } \begin{cases} \gamma_{ssc}(n-1) = \gamma_1(n-1) \text{ and } \gamma_1(n) \geq \gamma_T \\ \gamma_{ssc}(n-1) = \gamma_2(n-1) \text{ and } \gamma_2(n) < \gamma_T \end{cases}$$

Calculating Probability of Error

Introduction

- *Improvements related to a reduced fading level are commonly quantified by average error rate curves.*
- *The average error rate may in some cases be difficult to evaluate analytically.*

$$P_E = \int_0^\infty P_E(\gamma) p_\gamma(\gamma) d\gamma$$

Motivation

- *Quantify the severity of fading by using a measure directly related to the fading distribution.*

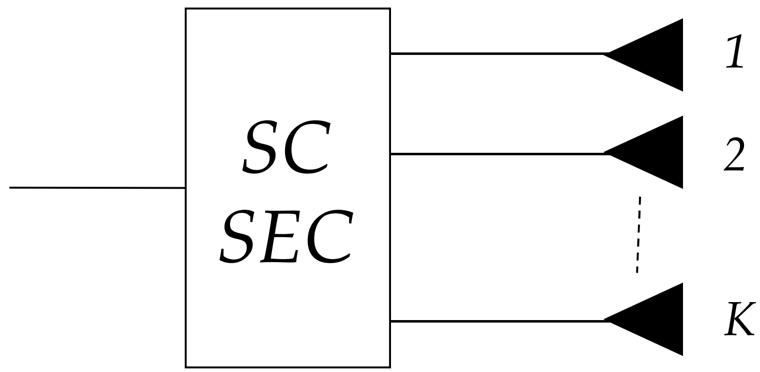
Diversity Performance

- Maximal Ratio Combining (MRC)
  – Optimal technique (maximizes output SNR)
  – Combiner SNR is the sum of the branch SNRs.
  – Distribution of SNR hard to obtain.
  – Exhibits 10-40 dB gains in Rayleigh fading.
- Selection Combining (SC)
  – Combiner SNR is the maximum of the branch SNRs.
  – Diminishing returns with # of antennas.
  – CDF easy to obtain, pdf found by differentiating.
  – Can get up to about 20 dB of gain.

Multiuser diversity Gain

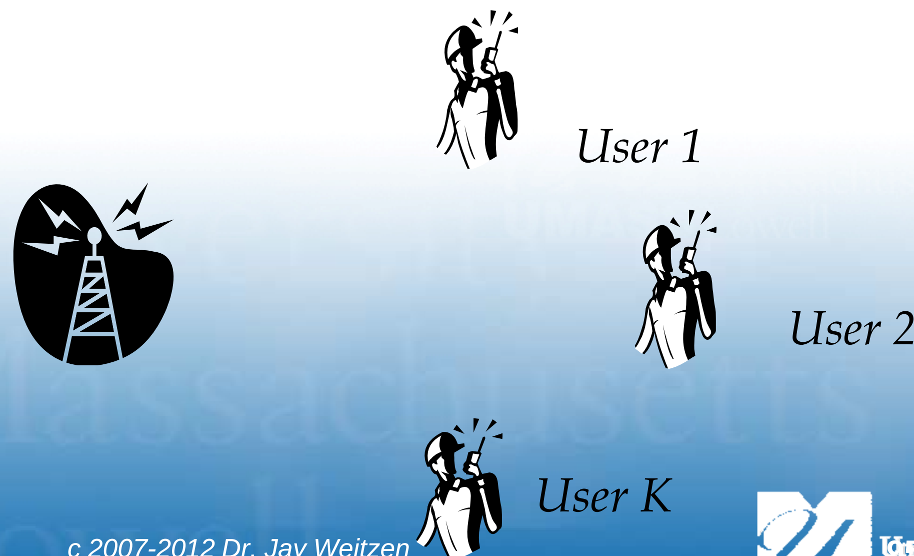
System throughput for N users > than for 1 user

Spatial diversity



Multiuser diversity

- *Combiner = Base station*
- *Antennas = Individual users*



Multi-User Diversity (cont'd)

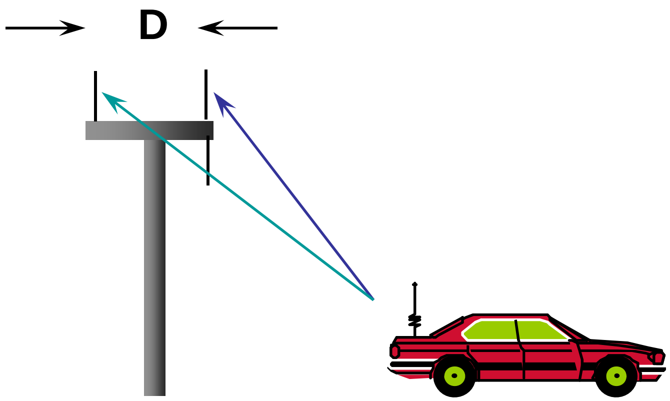
Introduction

- Always searching for the best user results in a high and determinstic feedback load.

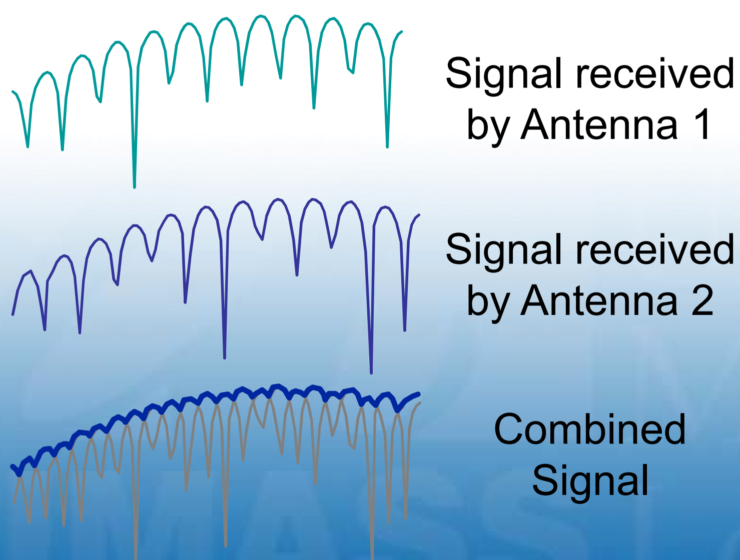
Motivation

- Utilize switched diversity algorithms reported in the literature as multiuser access schemes to reduce the average feedback load.
- The base station probes the users in a sequential manner, looking not for the best user but for an acceptable user.

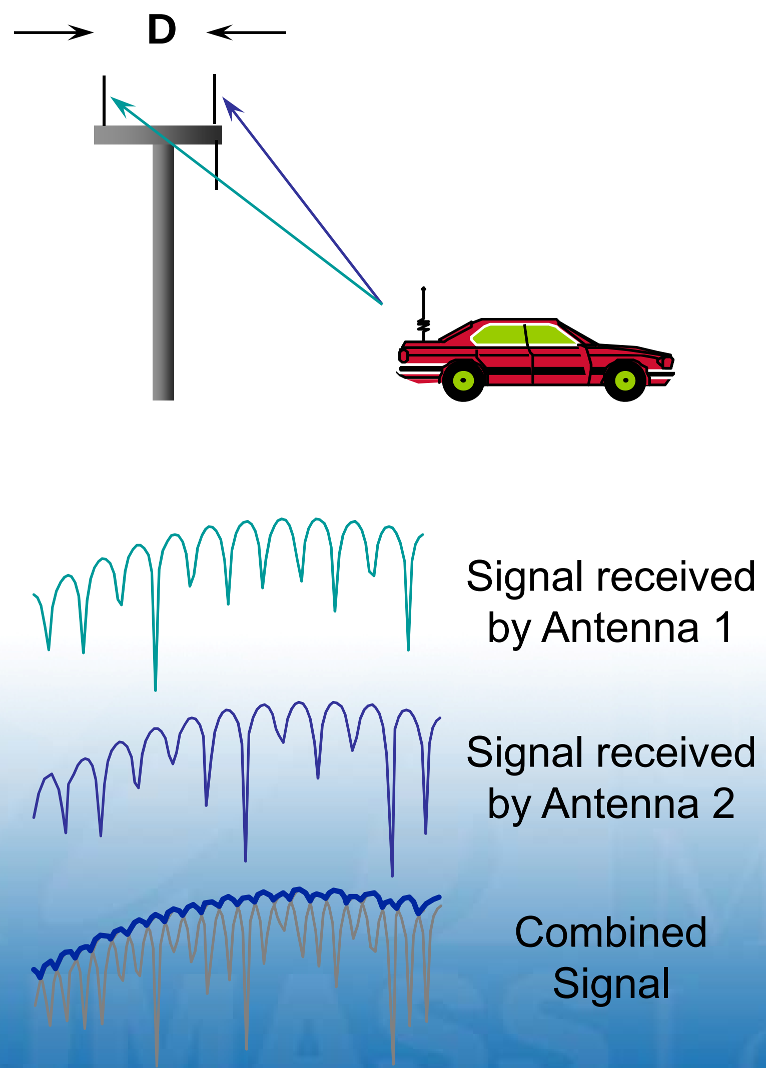
Combating Rayleigh Fading: Space Diversity



- Fortunately, Rayleigh fades are very short and last a small percentage of the time
- Two antennas separated by several wavelengths will not generally experience fades at the same time
- “Space Diversity” can be obtained by using two receiving antennas and switching instant-by-instant to whichever is best
- Required separation **D** for good decorrelation is 10-20λ
  - 12-24 ft. @ 800 MHz.
  - 5-10 ft. @ 1900 MHz.



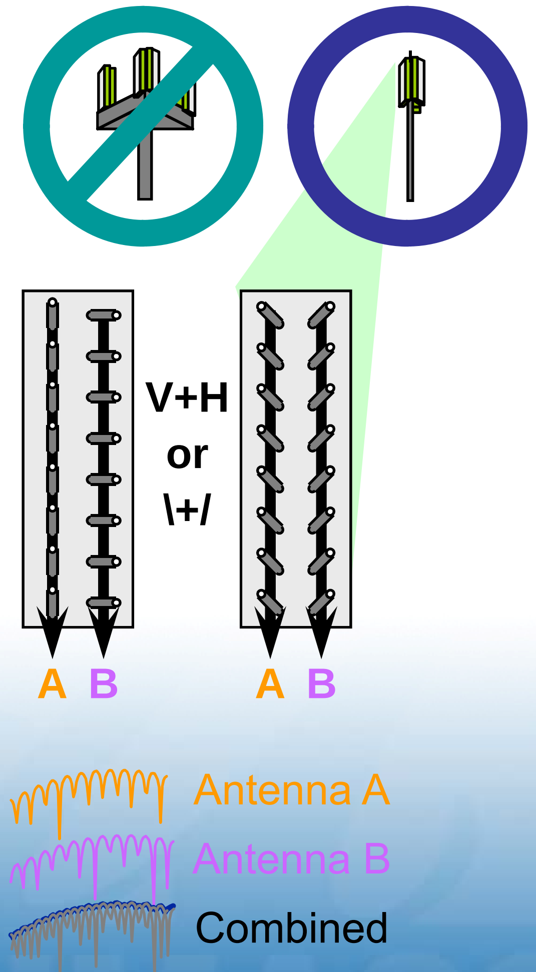
Space Diversity Application Limitations



- Space Diversity can be applied only on the receiving end of a link.
- Transmitting on two antennas would:
  - fail to produce diversity, since the two signals combine to produce only one value of signal level at a given point -- no diversity results.
  - produce objectionable nulls in the radiation at some angles
- Therefore, space diversity is applied only on the “uplink”, i.e.., reverse path
  - there isn’t room for two sufficiently separated antennas on a mobile or handheld

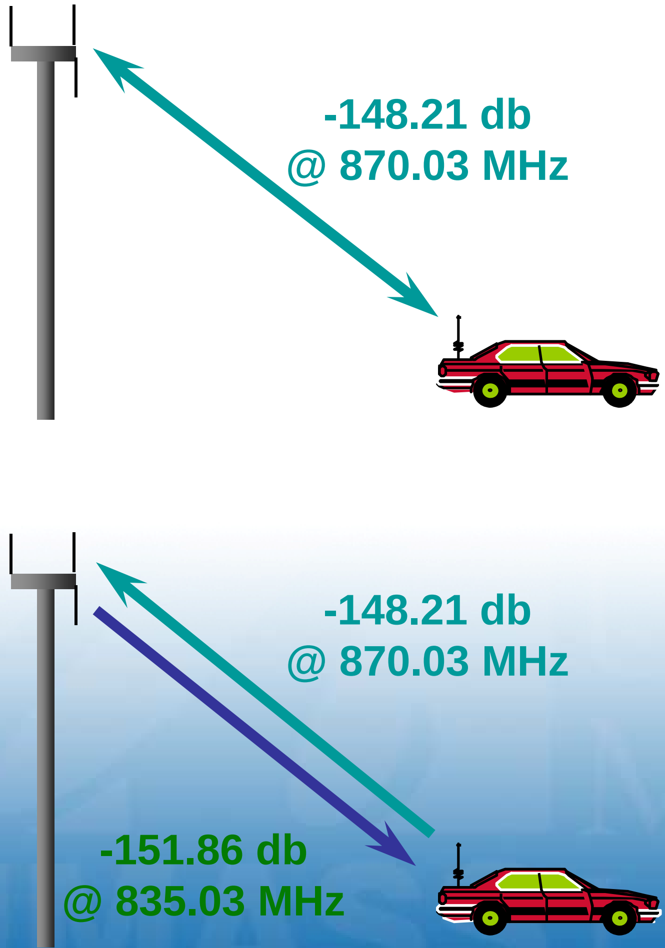
Polarization Diversity

Where Space Diversity Isn't Convenient



- Sometimes zoning considerations or aesthetics preclude using separate diversity receive antennas
- Dual-polarized antenna pairs within a single radome are becoming popular
  - Environmental clutter scatters RF energy into all possible polarizations
  - Differently polarized antennas receive signals which fade independently
  - In urban environments, this is almost as good as separate space diversity
- Antenna pair within one radome can be V-H polarized, or diagonally polarized
  - Each individual array has its own independent feedline
  - Feedlines connected to BTS diversity inputs in the conventional way; TX duplexing OK

The Reciprocity Principle
Does it apply to Wireless?

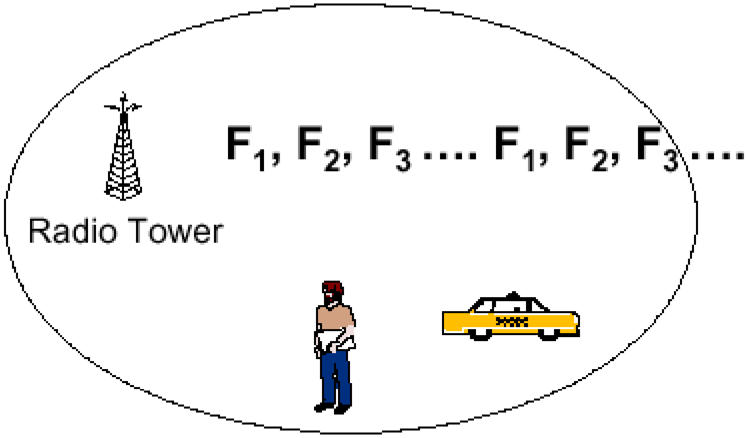


Between two antennas, on the same exact frequency, path loss is the same in both directions

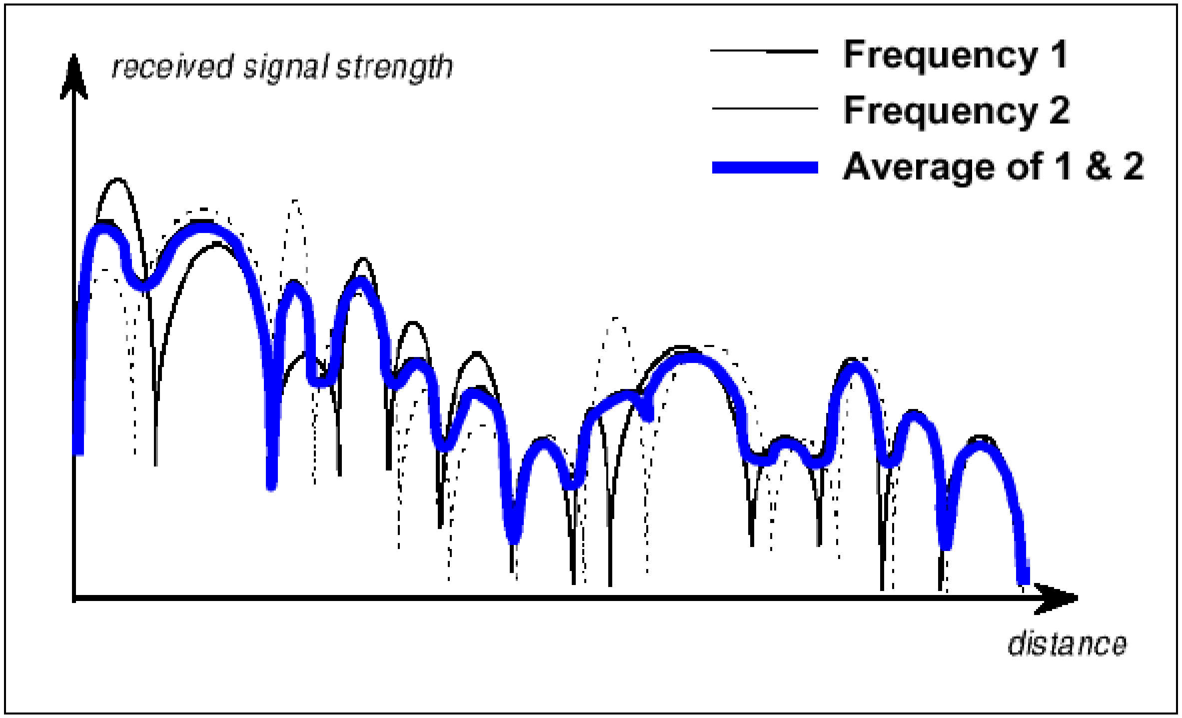
- But things aren't exactly the same in cellular --
  - transmit and receive 45 MHz. apart
  - antenna: gain/frequency slope?
  - different Rayleigh fades up/downlink
  - often, different TX & RX antennas
  - RX diversity
- Notice also the noise/interference environment may be substantially different at the two ends
- So, reciprocity holds only in a general sense for cellular

Frequency Diversity

- **Obtained by use of Frequency Hopping**
  - Frequency Hopping is used in GSM
- **If the frequencies in the hopping set fade independently, a gain can be achieved**
  - A user changes frequency on every timeslot
  - A mobile is less likely to suffer a deep fade for consecutive timeslots of information



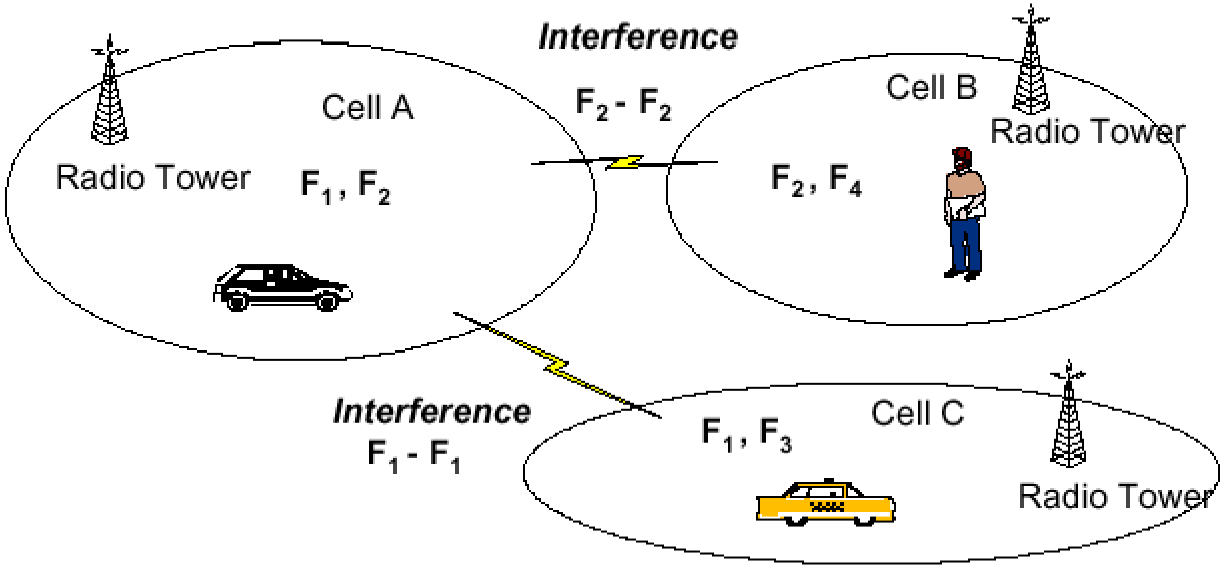
Frequency Diversity...



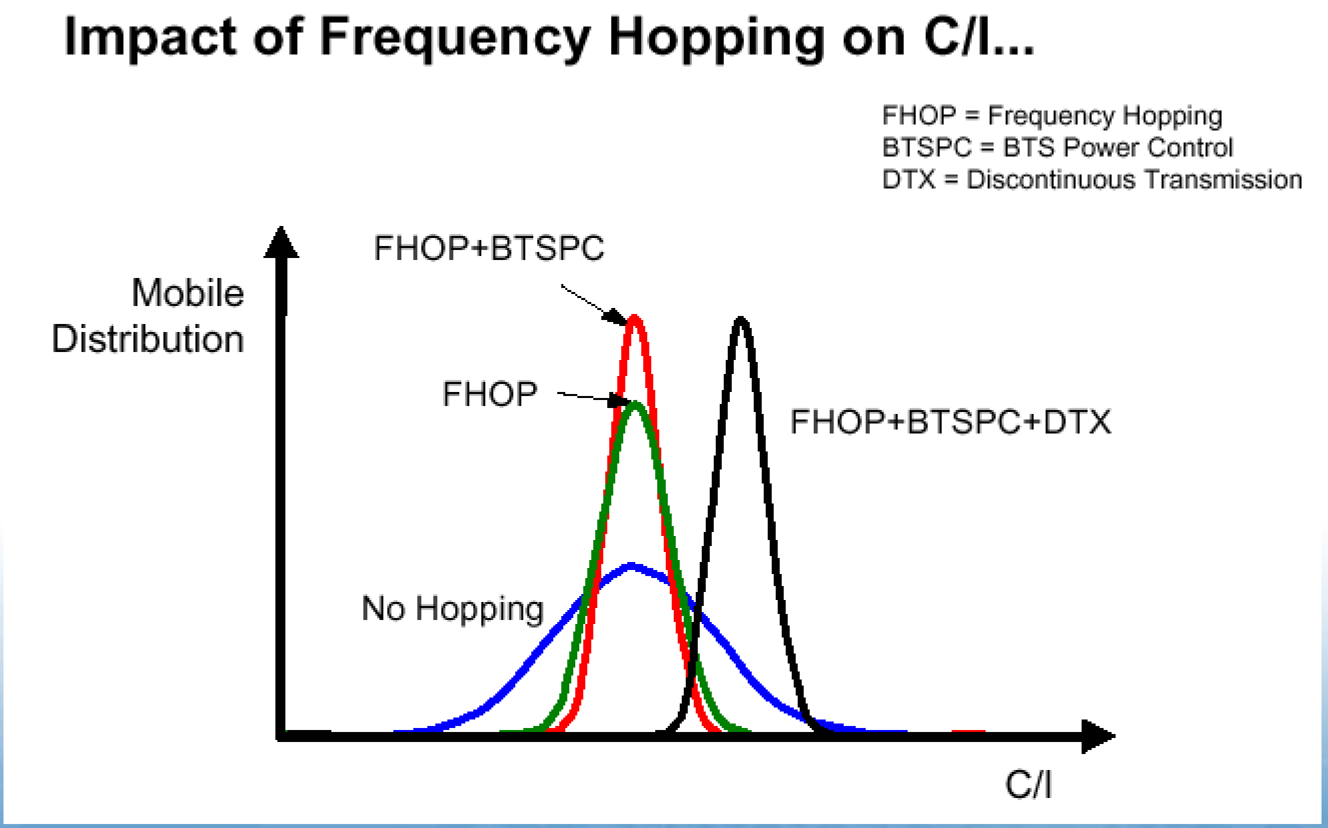
Frequency Hopping for Diversity

Frequency Hopping...

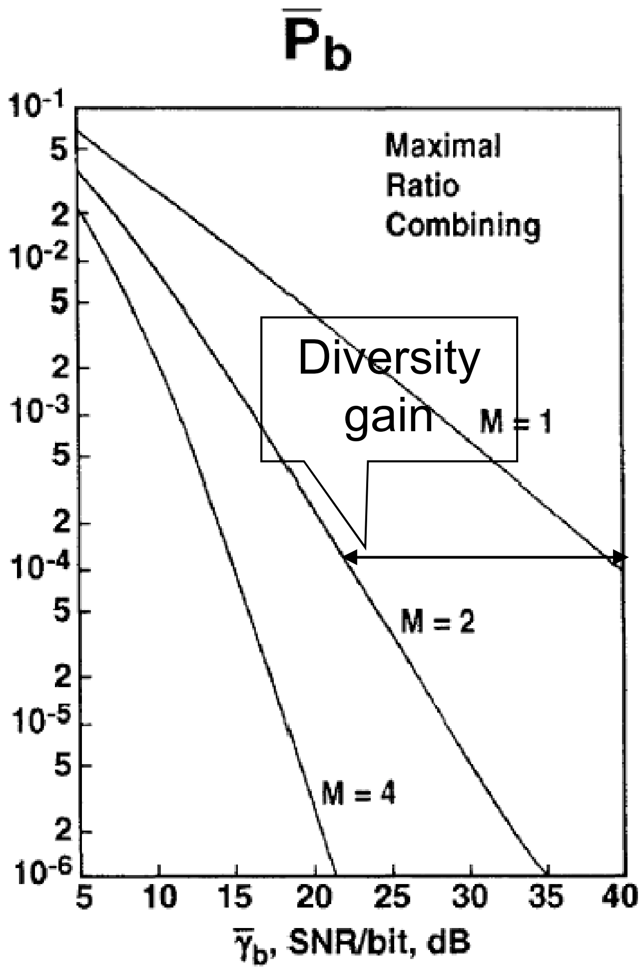
- **Important feature for interference averaging in high capacity networks**

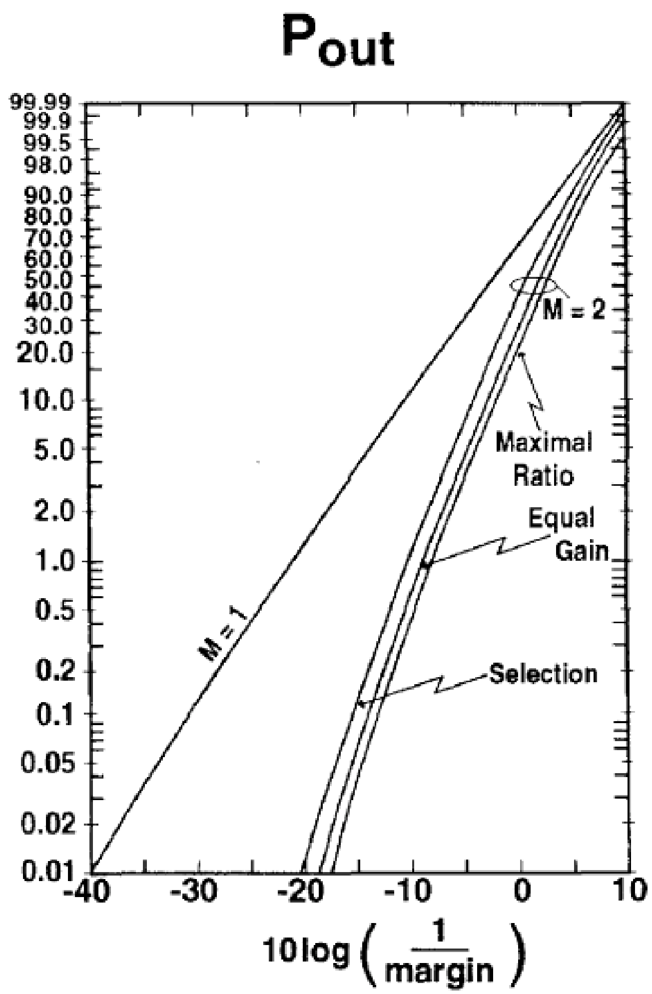


Frequency Hopping and C/I



Receive Diversity Performance





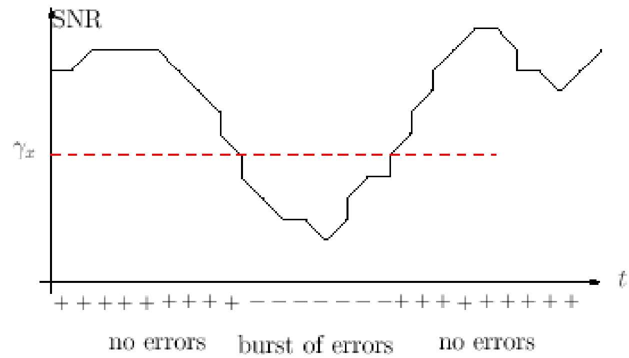
Interleaving and De-interleaving for Fading Channels

Motivation for Interleaver

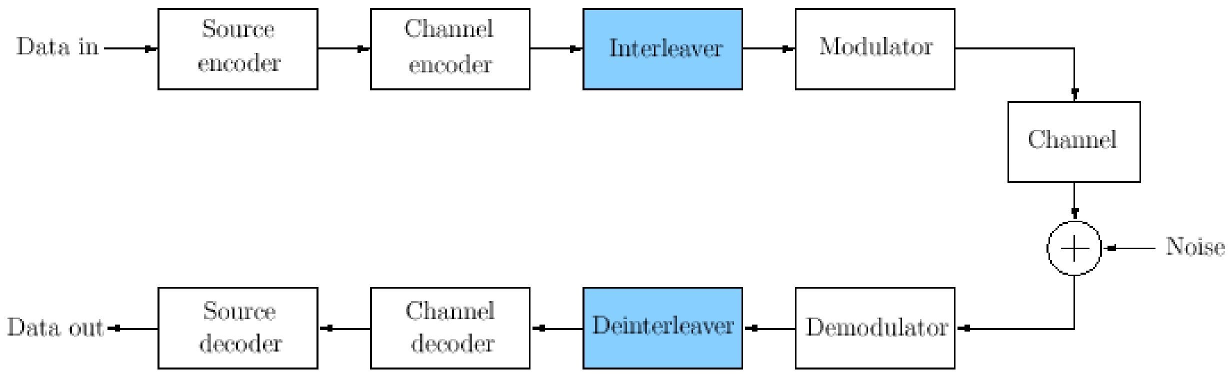
- Interleaving is a form of time diversity
  - Usually combined with coding to provide protection against burst errors caused by fading
- Viterbi Algorithm used for detection of convolutional codes is not effective against burst errors. We add interleaver to distribute burst error.

Forward Error Correction for Fading Channels

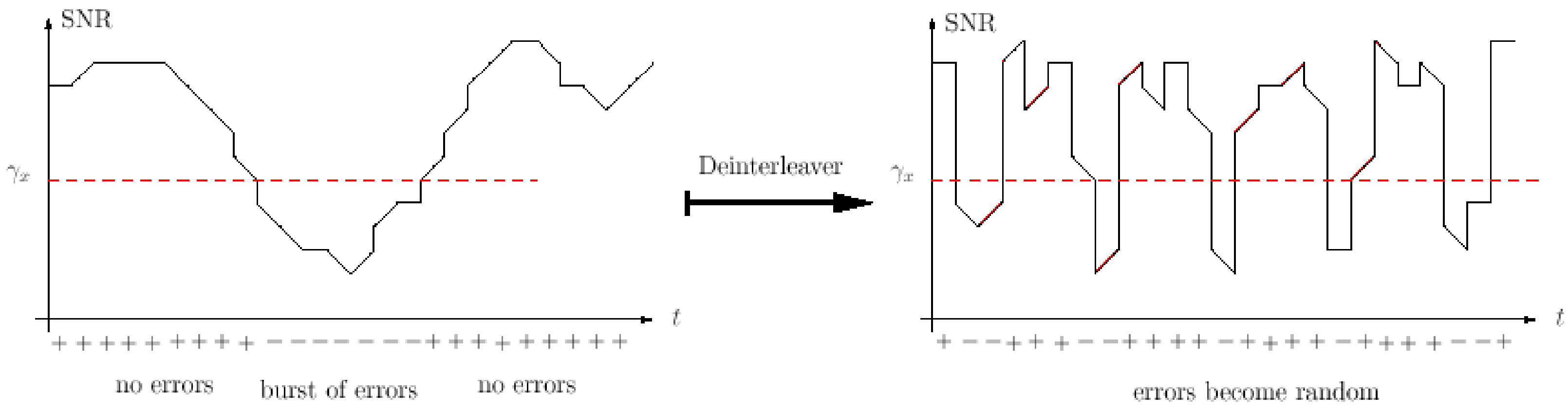
- **In fading multipath channels, errors occur in bursts.**



- **No practical FEC codes can cope with such error distribution.**
- **Randomizing these errors will make FEC efficient in fading multipath channels.**

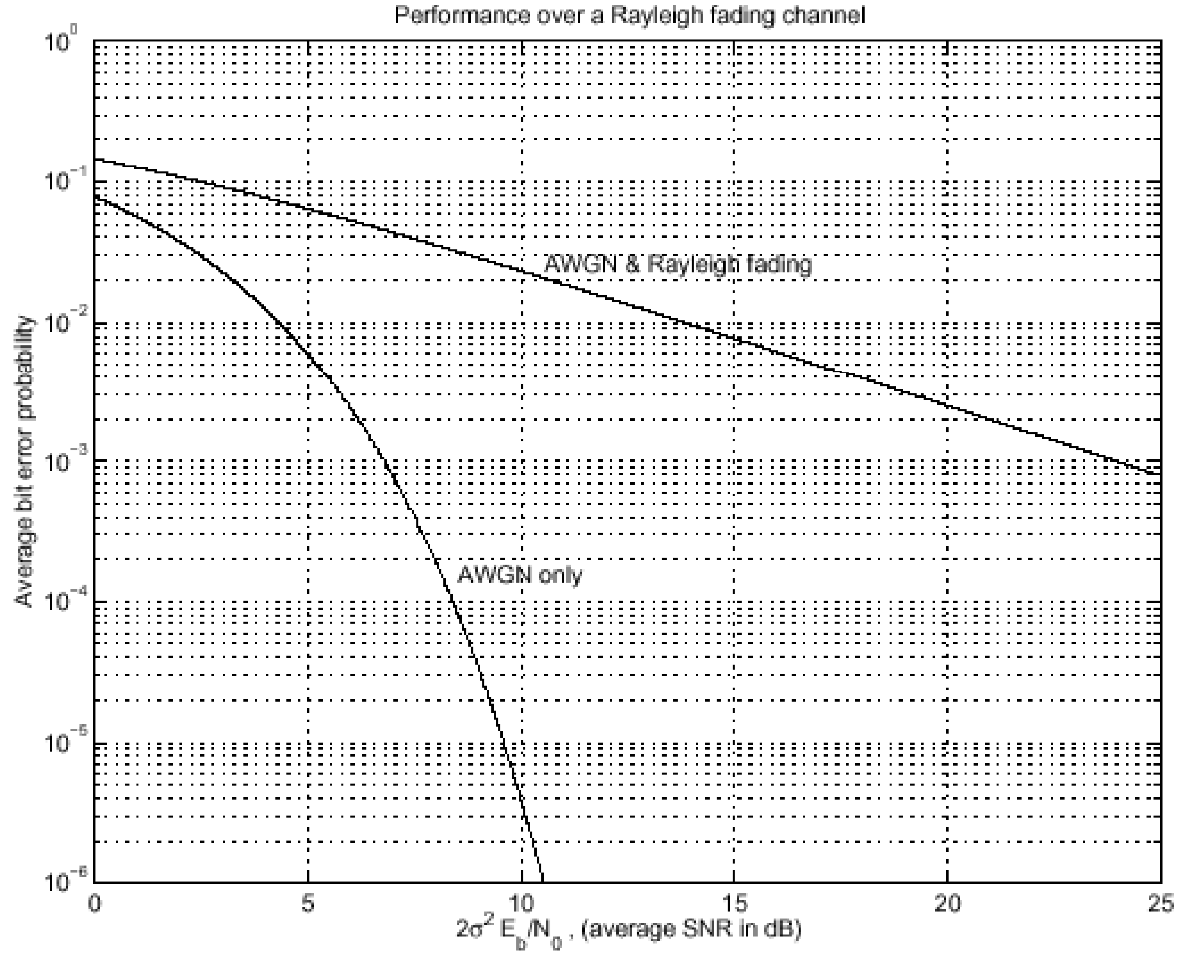


Theory of Interleaving



- Interleaving destroys correlation between consecutive symbols caused by the fading channel.
  - Block interleaving.
  - Convolutional interleaving.
- Coding/interleaving introduces a diversity gain (time diversity) into the system.
- Interleaving introduces a delay into the system.
- An interleaver is said to be ideal (full) if it makes the channel memoryless.

Error Performance on Fading Channels



'igure 1: Performance of a Coherent BPSK AWGN and Flat Rayleigh Fading Channels.

Block Interleaver

Writing →

Reading ↓

0	0	1	1
0	1	0	1
1	1	0	0
0	0	1	1
1	0	1	1

Original Message

00110101110000111011

↓ Interleaver

00101011001001111011

↓ Burst Error

00110101001001111011

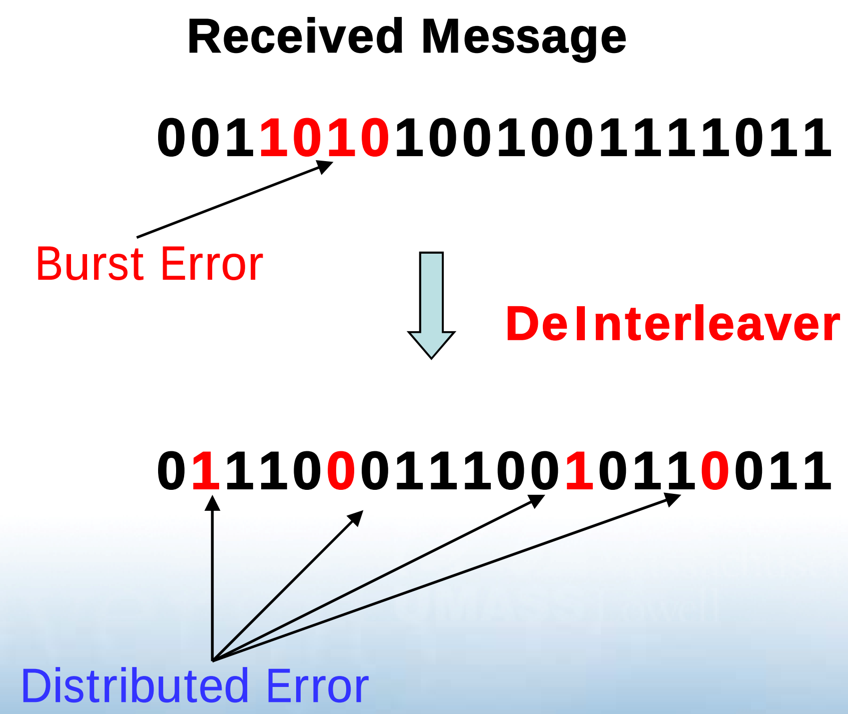
The order of original Message is changed by Block Interleaver.

Block Deinterleaver

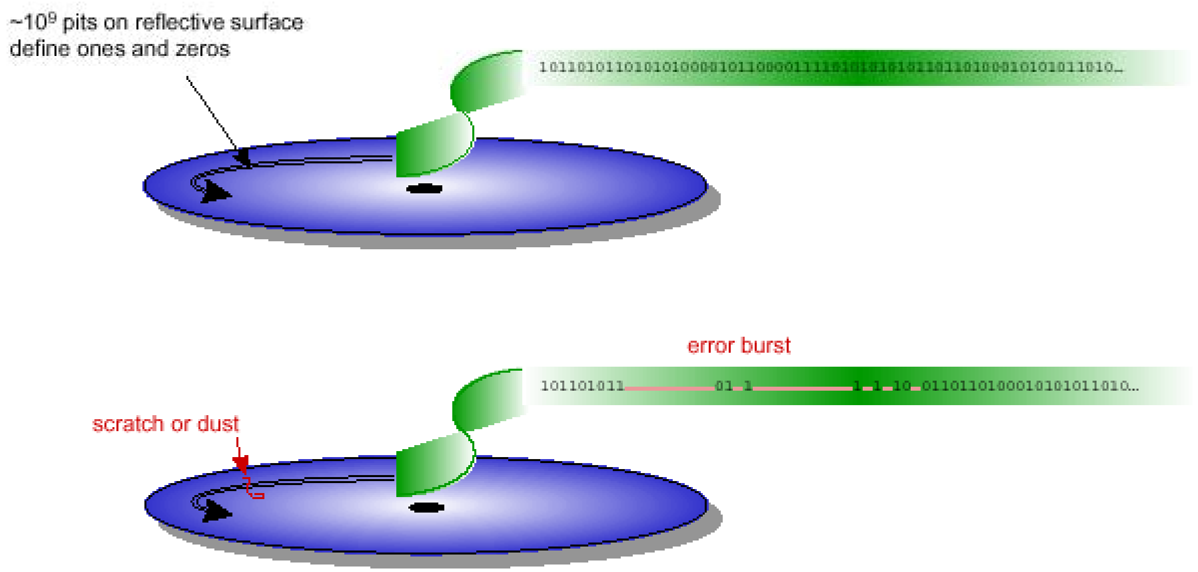
Reading →

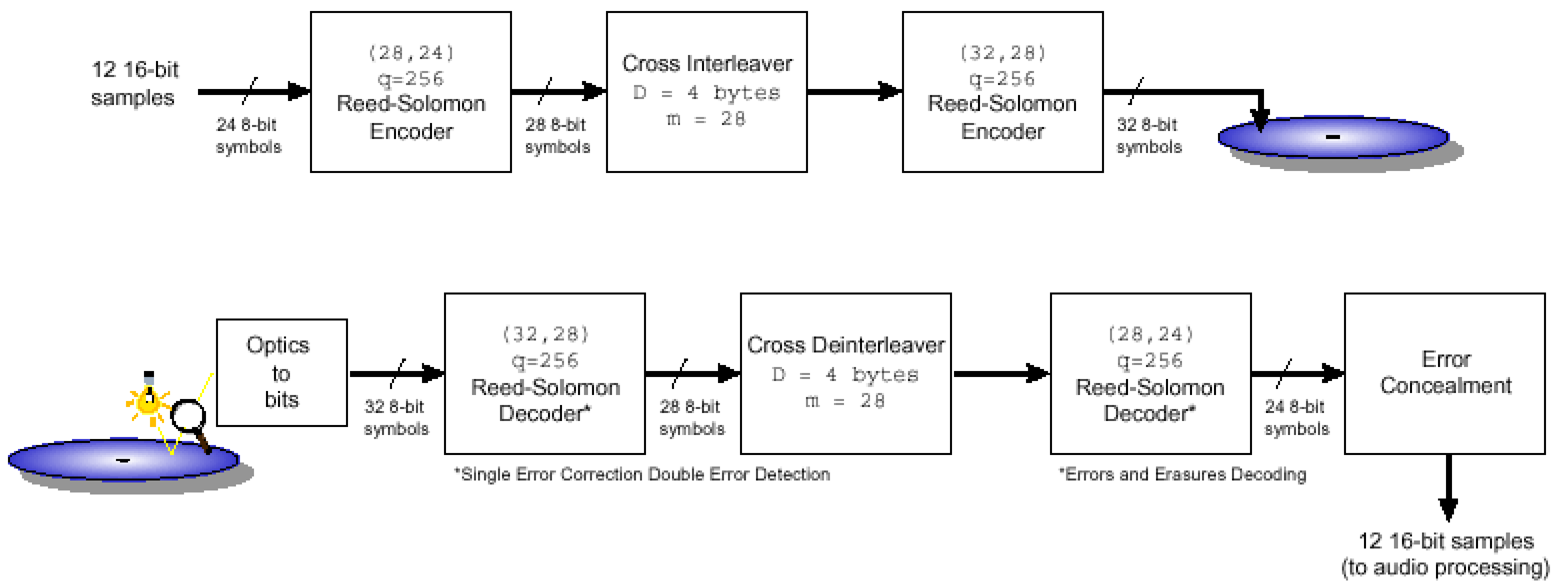
Writing ↓

0	1	1	1
0	0	0	1
1	1	0	0
1	0	1	1
0	0	1	1



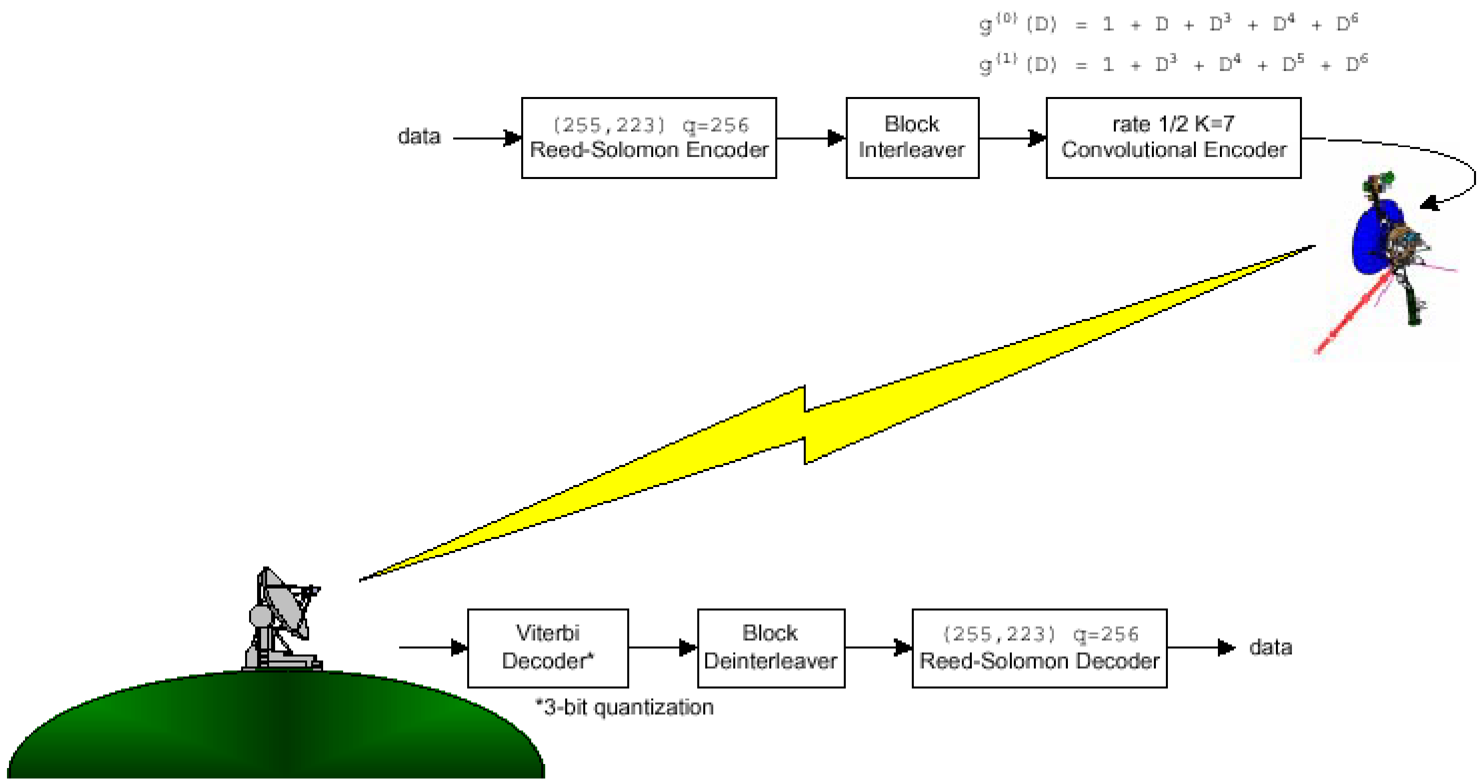
Example: CD Interleaving



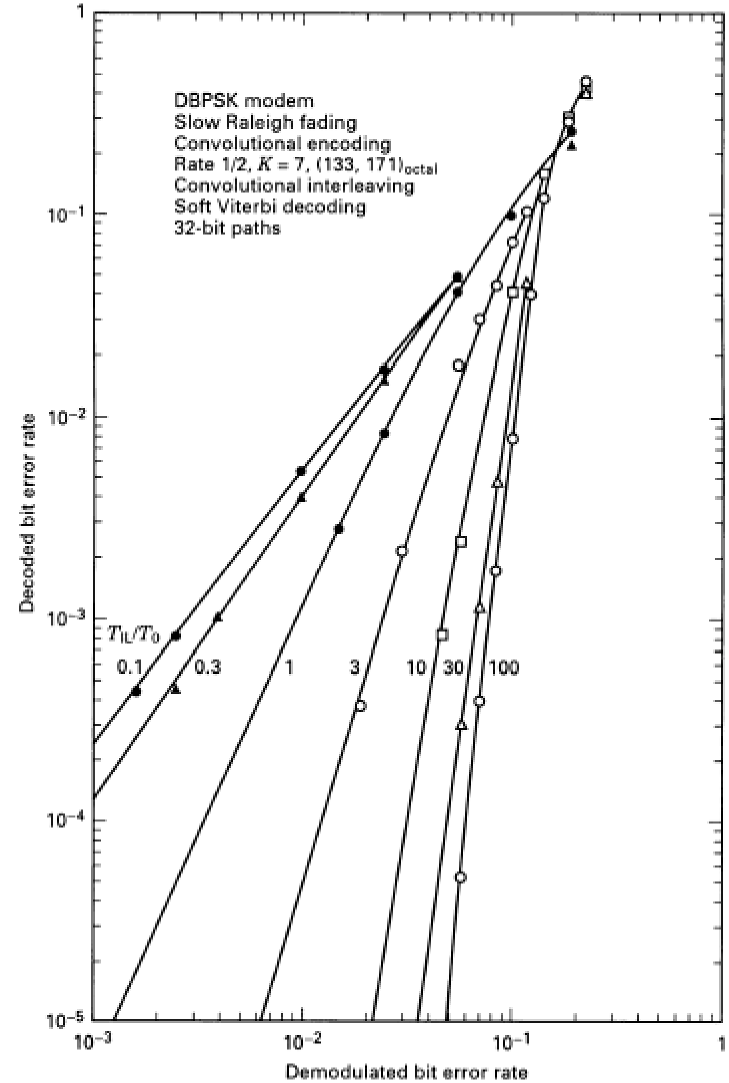


12 16-bit samples
24 8-bit symbols
(28,24) q=256 Reed-Solomon Encoder
28 8-bit symbols
Cross Interleaver D = 4 bytes m = 28
(32,28) q=256 Reed-Solomon Encoder
32 8-bit symbols
Optics to bits
32 8-bit symbols
(32,28) q=256 Reed-Solomon Decoder*
28 8-bit symbols
Cross Deinterleaver D = 4 bytes m = 28
(28,24) q=256 Reed-Solomon Decoder*
24 8-bit symbols
Error Concealment
*Single Error Correction Double Error Detection
*Errors and Erasures Decoding
12 16-bit samples
(to audio processing)

Example: Satellite Communications



Performance with Interleaving



Combating Effects of Multipath and Fading in Wireless Systems

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What to do against ISI?

- Wideband signals:
  - channel delay = many symbol periods
  - heavy distortion of the received signal.
- Several techniques can be applied to reduce or get rid of ISI in wideband signal transmission
  - Equalization (2nd gen)
  - spread-signal modulation (3rd gen)
  - OFDM (4th gen)

Equalization

- The received signal is filtered in such a way that ISI is eliminated or reduced.
  - Ideal ISI elimination is achieved when the filter is the inverse of the channel response.
  - Clearly, the channel must be known, or accurately estimated, to perform effective equalization.
  - Therefore, the equalizer needs to be trained to adapt itself to the time-varying channel in wireless systems. Usually this is achieved by transmitting a training sequence.
- Equalization of the signal results in a decrease of ISI at the cost of a lower signal-to-noise ratio (SNR)

Direct sequence spread spectrum

- In DS-SS modulation, the signal is multiplied with a code that results in a signal with a much wider bandwidth than the original information-bearing signal. In a time-dispersive multipath channel, the spread signal replicas, which travel via different paths, are un-correlated if the path delays are more than one symbol period apart from each other. After decorrelation in the receiver, the signal replicas from different paths are combined in a Rake receiver, thus all received energy is effectively used.
- A disadvantage of using DS-SS with high bit-rate signals is that to achieve a sufficiently high processing gain, a very large bandwidth is required. This is especially the case in an indoor environment, where the delay times between the paths are very short, in the order of 1 ns.

OFDM

- Symbols of high bit rate signal are distributed over a large number of subcarriers.
  - Low symbol rate per carrier.
  - Individual carrier signals see flat fading (no ISI).
- Promising technique for future high bit-rate applications.
- However, it suffers from a number of problems:
  - a very linear amplifier in the transmitter is required to prevent signal distortion,
  - accurate synchronization in the receiver is needed,
  - in the transmitter and receiver real-time discrete Fourier transform (DFT) operations have to be computed.

Improving Performance of Wireless Channels using MIMO (the next generation of diversity)

MIMO is the Next generation of Diversity Systems

- ***Single-input, single-output (SISO) channel***

  No spatial diversity

- ***Single-input, multiple-output (SIMO) channel***

  Receive diversity

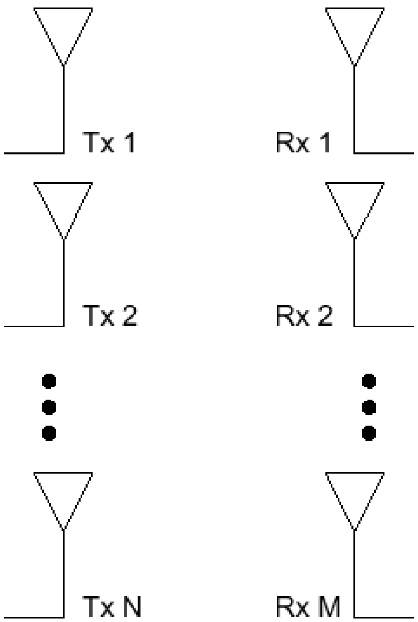
- ***Multiple-input, single-output (MISO) channel***

  Transmit diversity

- ***Multiple-input, multiple-output (MIMO) channel***

  Combined transmit and receive diversity

Introduction to the MIMO Channel



$$\mathbf{H} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1N} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{M1} & \alpha_{M2} & \cdots & \alpha_{MN} \end{bmatrix}$$

- Multiple input multiple output (MIMO) channel: N transmitters, M receiver
- α_{ij} is the complex channel gain from i-th transmit antenna to j-th receive antenna.
- $\mathbf{H}$ is the $N \times M$ channel matrix.

Capacity of MIMO Channels

- Channel capacity for SISO channel:

$$C = \log_2(1+\rho) \qquad \text{bits/sec/use}, \ \ \rho \text{ is the SNR}$$

- Channel capacity for MIMO channel:

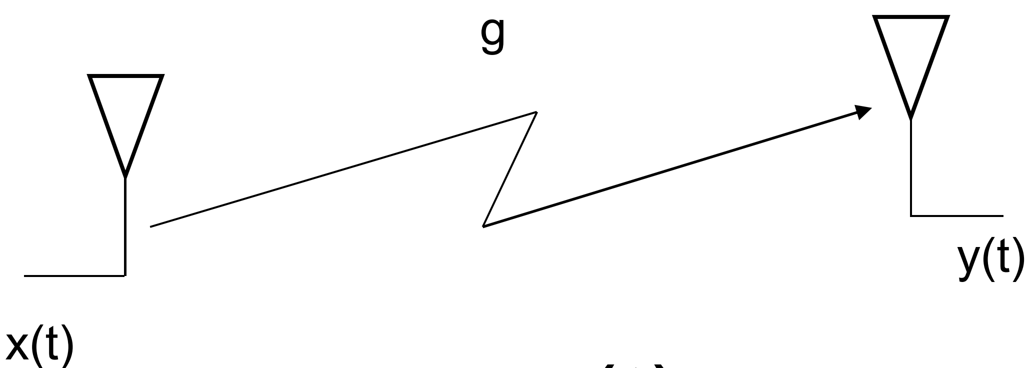
$$C = \log_2 \det\left(\mathbf{I} + \frac{\rho}{n}\mathbf{H}\mathbf{H}^*\right) \qquad \text{bits/sec/use}$$

$\mathbf{H}$ is the $n \times m$ channel matrix

- Outage capacity C_x :

$$\Pr\left(C > C_x\right) = \int_{\mathbf{H}:C(\mathbf{H})=C_x}^{\infty} C(\mathbf{H})\, f_{\mathbf{H}}(\mathbf{H})\, d\mathbf{H} = x$$

Single Input- Single Output systems (SISO)



x(t): transmitted signal
y(t): received signal
g(t): channel transfer function
n(t): noise (AWGN, σ^2)

$$y(t) = g \bullet x(t) + n(t)$$

Signal to noise ratio : $\rho = |g|^2 \frac{E_x}{\sigma^2}$

Capacity : $C = \log_2(1+\rho)$

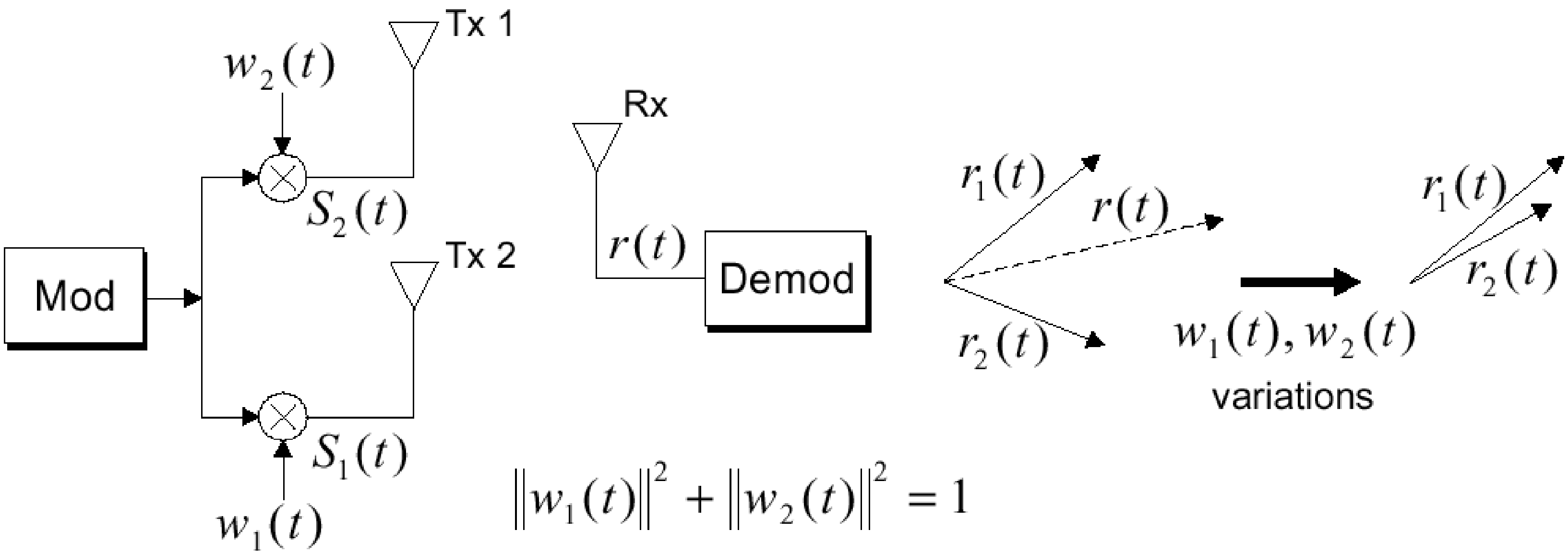
Single Input- Multiple Output (SIMC
Multiple Input- Single Output (MISC

- Principle of diversity systems (transmitter/ receiver)
- +: Higher average signal to noise ratio
  Robustness
- - : Process of diminishing return
  Benefit reduces in the presence of correlation
- Maximal ratio combining
- Equal gain combining
- Selection combining

Transmit Diversity

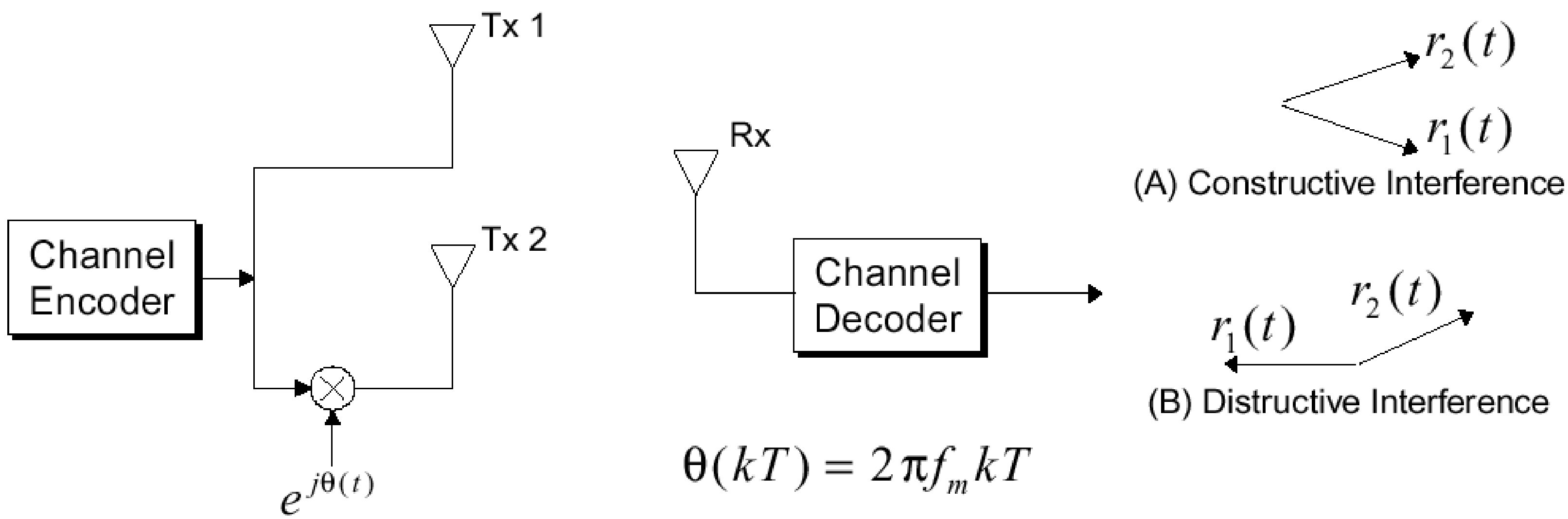
- Provide diversity benefit to a mobile using base station antenna array for frequency division duplexing (FDD) schemes. Cost is shared among different users.
- Order of diversity can be increased when used with other conventional forms of diversity.
- Two kinds of transmit diversity techniques:
  - Transmit diversity with feedback from receiver
  - Transmit diversity without feedback from receiver:
    - No training.
    - Feedforward information.

Transmit Diversity with Feedback



- $w_1(t)$ and $w_2(t)$ are varied such that $|r(t)|^2$ is maximized.
- $w_1(t)$ and $w_2(t)$ are adapted with feedback information from the receiver.

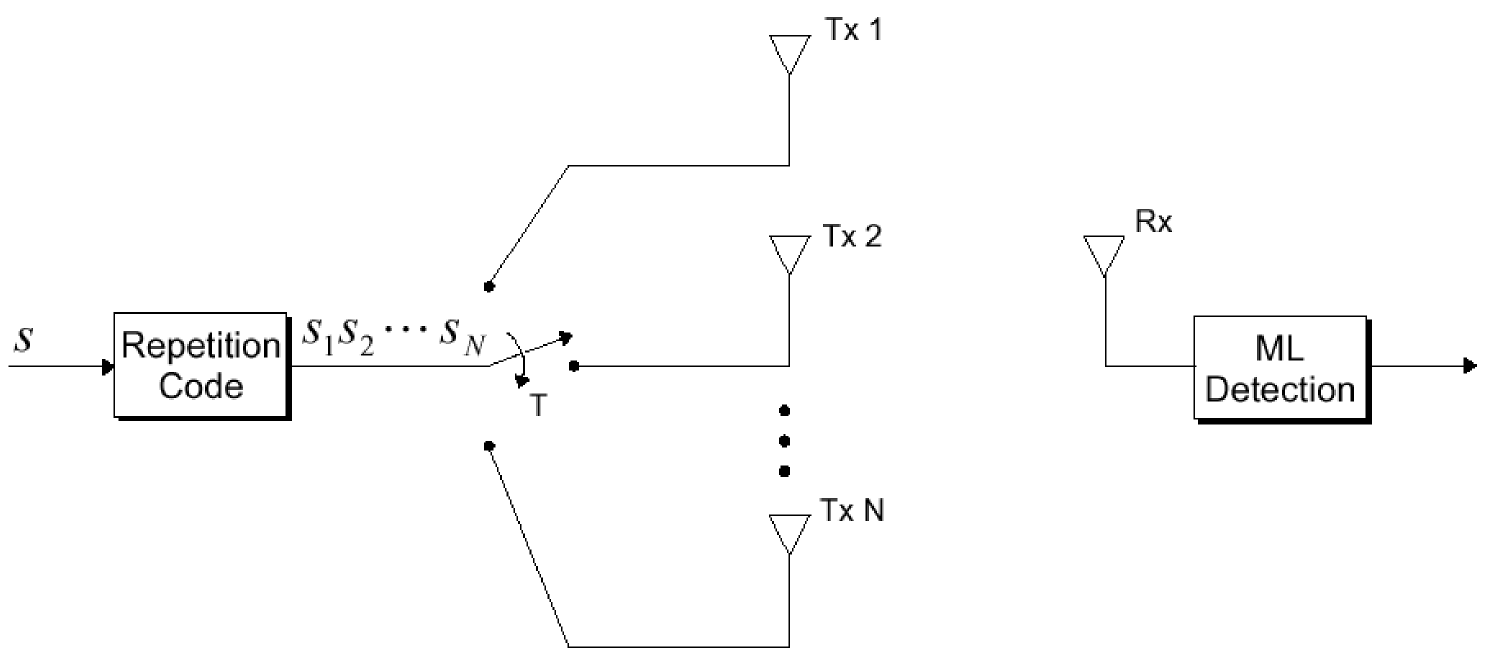
TX diversity with frequency weighting



$$\theta(kT) = 2\pi f_m kT$$

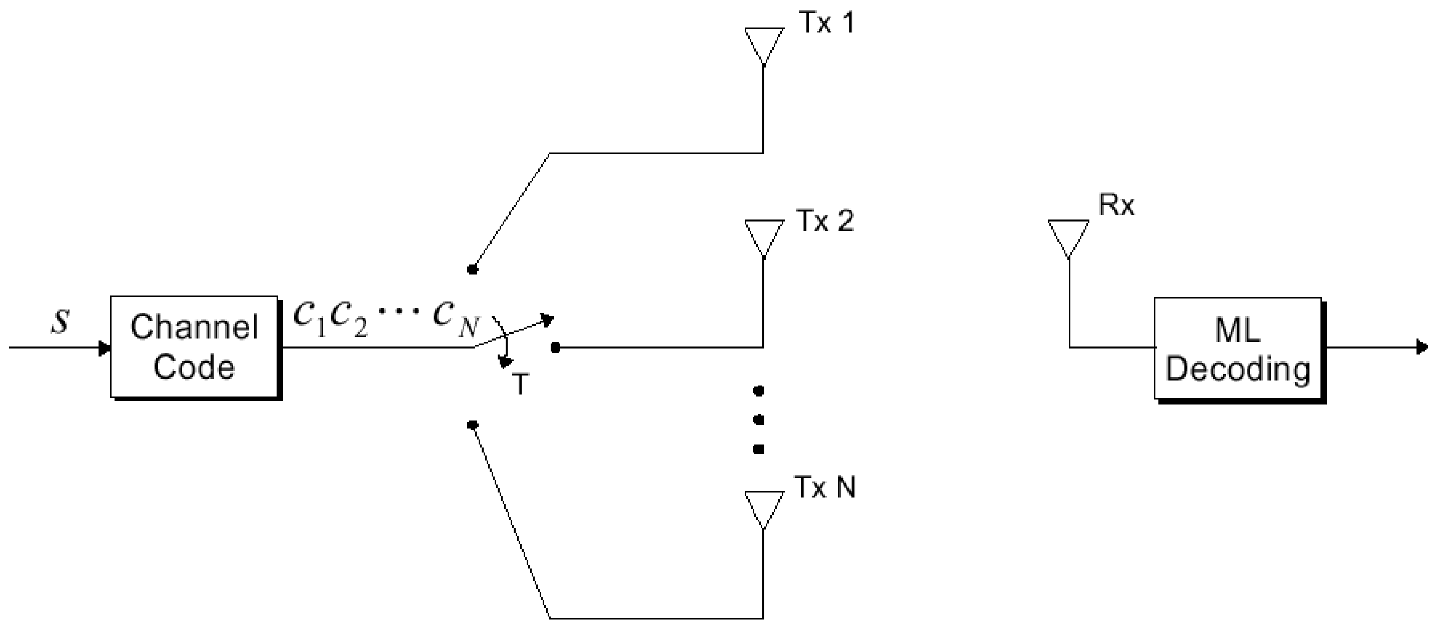
- Use frequency weighting to mitigate the harm of scenario B.
- Simulate fast fading → can use conventional channel coding and interleaving techniques.

TX Diversity with antenna hopping



- At time i, $1 \le i \le N$, transmit s from antenna i.
- Achieves a diversity order of N using ML detection or MRC at the receiver.
- Bandwidth efficiency is $1/N$.

TX Diversity with channel coding



- The channel code has a minimum Hamming distance $d_{\min} \leq N$.
- Transmit code symbol i from antenna i.
- After receiving the N symbols, the decoder performs ML decoding to decode the received codeword.

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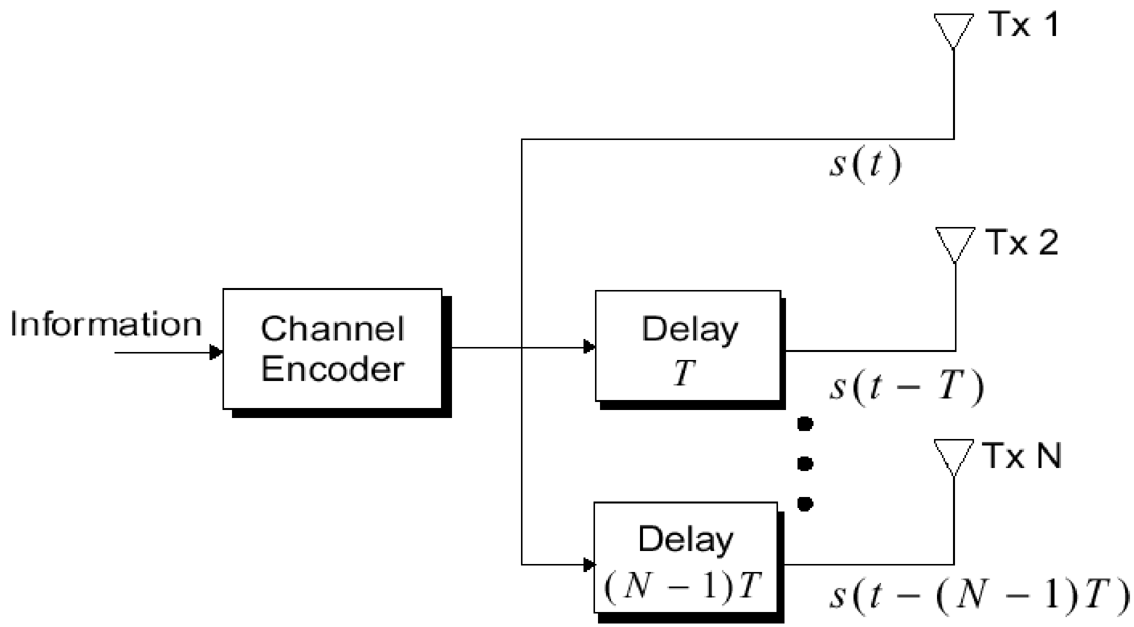
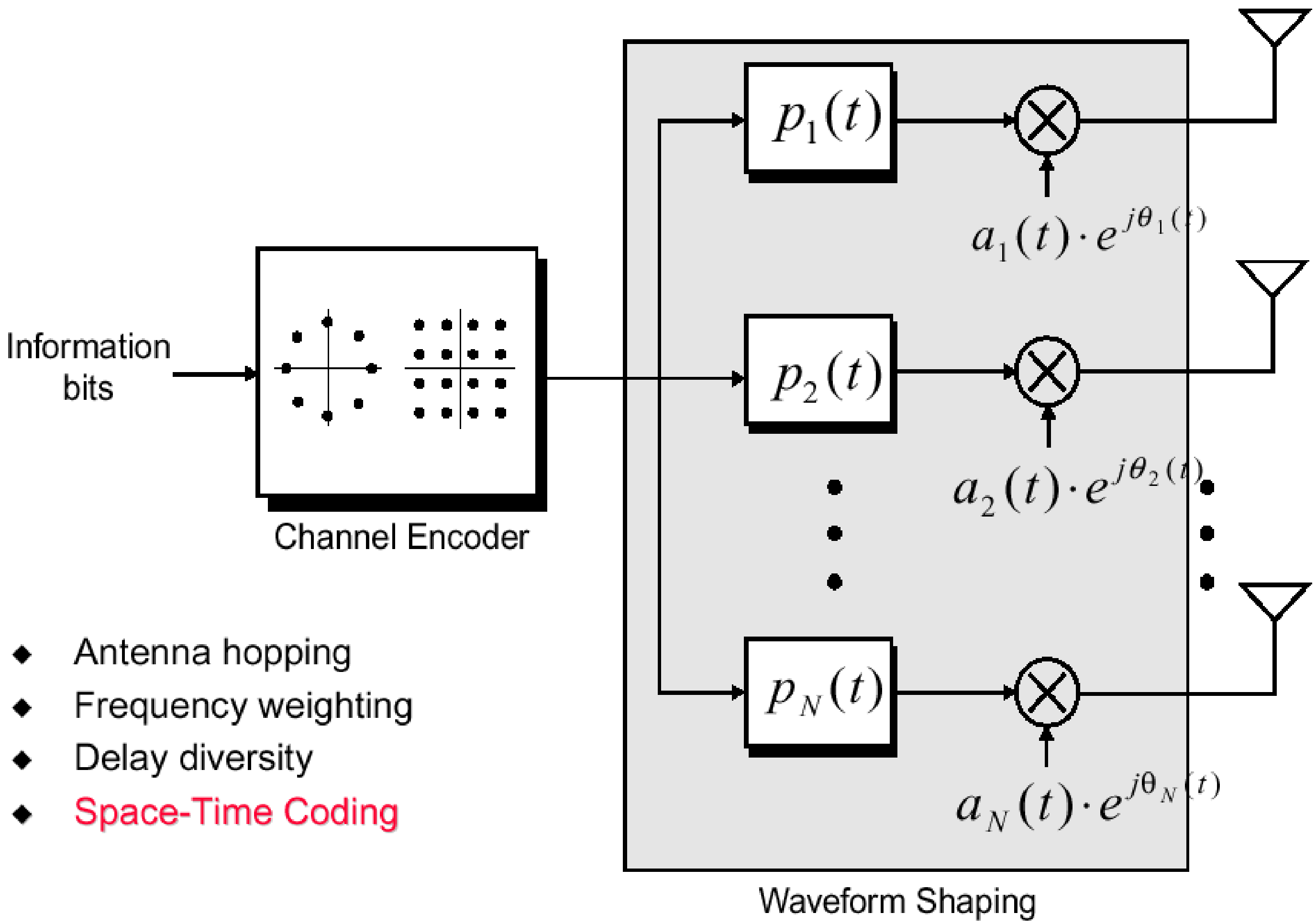
Transmit diversity via delay diversity



- Provide diversity benefit by introducing intentional multipath.
- Receiver uses an equalizer or MLSE for detection.
- Provides a diversity order of N. No loss of BW efficiency.

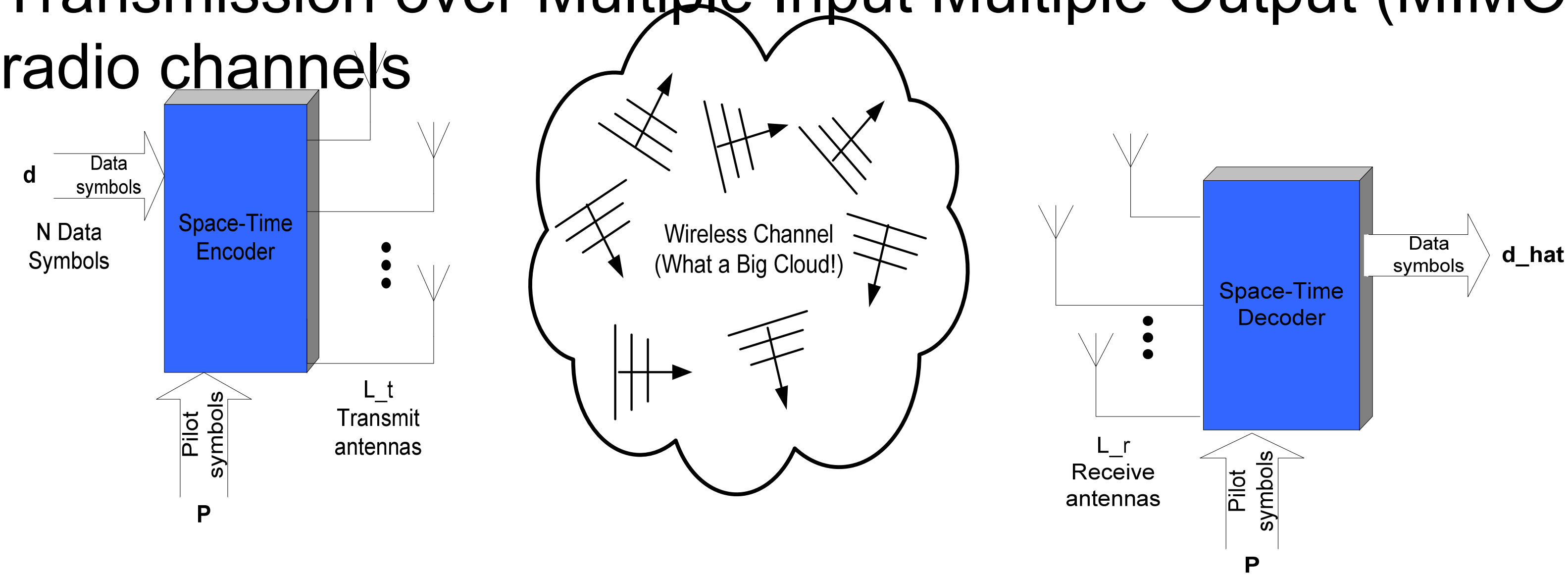
Transmit Diversity Options



- Antenna hopping
- Frequency weighting
- Delay diversity
- Space-Time Coding

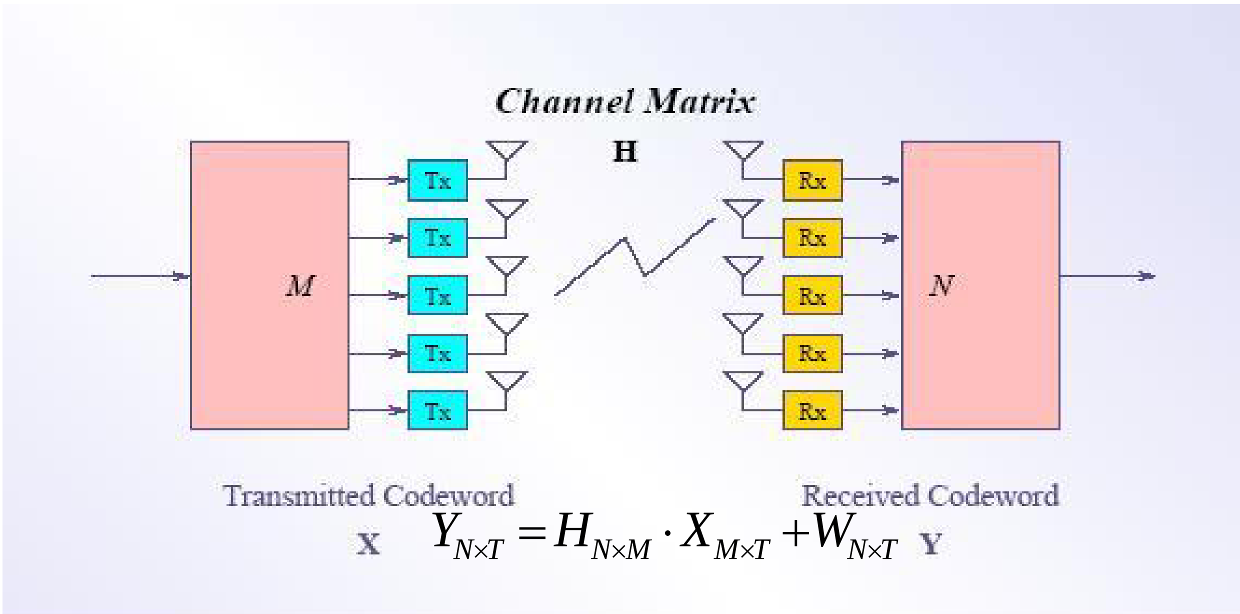
MIMO Wireless Communications: Combining TX and RX Diversity

- Transmission over Multiple Input Multiple Output (MIMO) radio channels



- Advantages: *Improved Space Diversity and Channel Capacity*
- Disadvantages: *More complex, more radio stations and required channel estimation*

MIMO Model



T: Time index

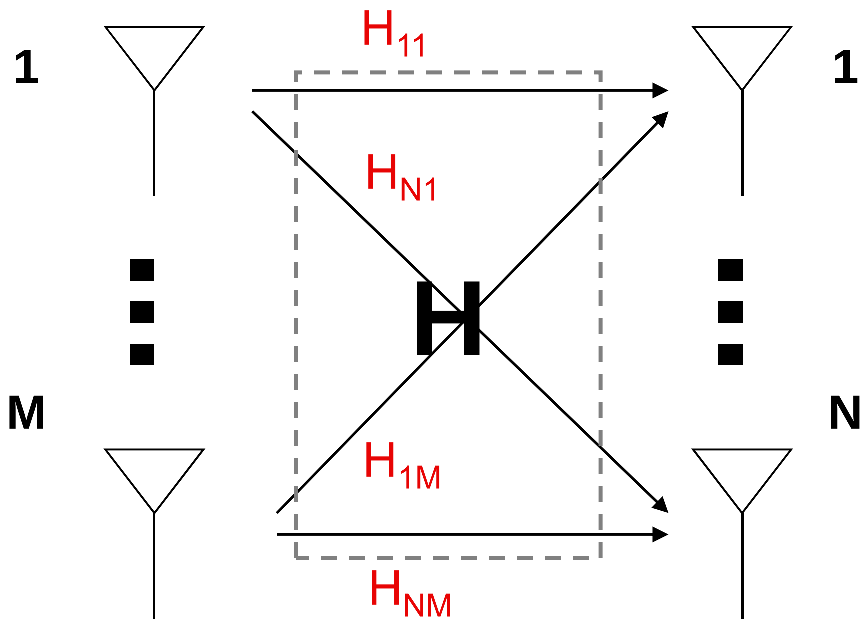
W: Noise

- Matrix Representation

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{pmatrix} = H_{N\times M} \cdot \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_M \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \\ \vdots \\ n_M \end{pmatrix}, \text{ where } H = \begin{pmatrix} H_{11} H_{12} \cdots\cdots H_{1M} \\ H_{21} \ddots \\ \vdots \quad H_{ij} \ddots \\ H_{N1} H_{N2} \cdots H_{NM} \end{pmatrix}$$

  – For a fixed T

Multiple Input- Multiple Output systems (MIMO)



$$\underline{y}_{\mathrm{Nx1}} = \mathbf{H}_{\mathrm{NxM}}\,\underline{x}_{\mathrm{Mx1}} + \underline{n}_{\mathrm{Nx1}}$$

- Average gain $\beta^2 = E\left[\left|\mathrm{H}_{ij}\right|^2\right], \overline{\mathbf{H}} = \frac{1}{\beta}\mathbf{H}$
- Average signal to noise ratio $\rho = \frac{P_{total}}{\sigma^2}\beta^2$

Shannon capacity

$$C = \log_2\left[\det\left(\mathbf{I} + \frac{E_x}{\sigma^2}\mathbf{H}\mathbf{H}^H\right)\right] = \log_2\left[\det\left(\mathbf{I} + \frac{P_T}{M\sigma^2}g^2\overline{\mathbf{H}}\overline{\mathbf{H}}^H\right)\right] =$$

$$\log_2\left[\det\left(\mathbf{I} + \frac{\rho}{M}\overline{\mathbf{H}}\overline{\mathbf{H}}^H\right)\right]$$

K= rank(**H**): what is its range of values?

Parameters that affect the system capacity

- Signal to noise ratio ρ
- Distribution of eigenvalues (u) of **H**

Interpretation I:
The parallel channels approach

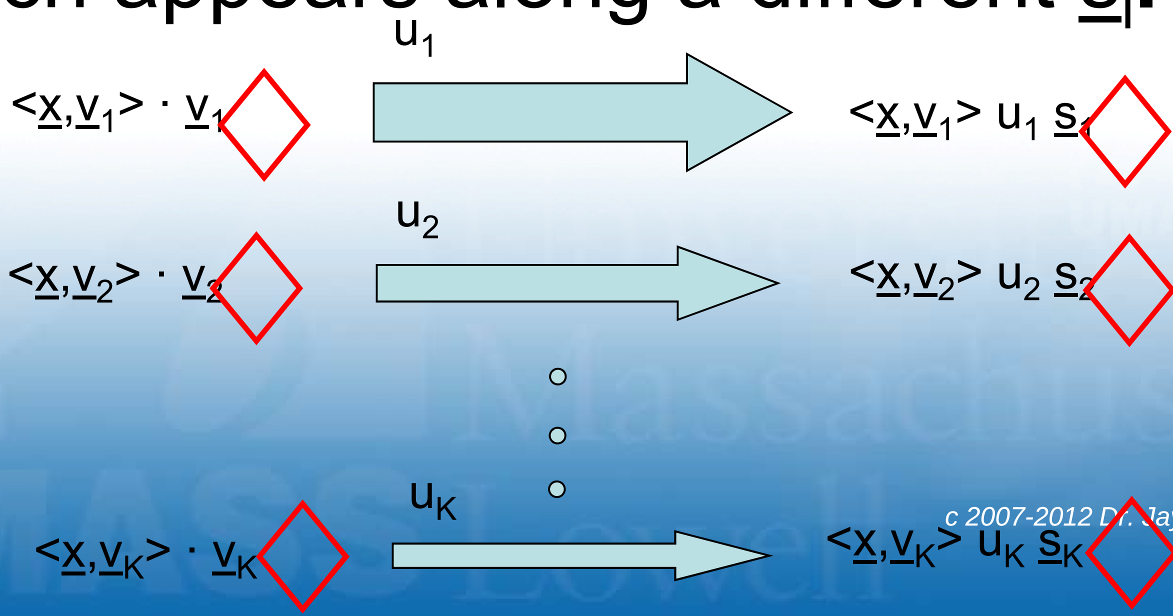
- "Proof" of capacity formula

- Singular value decomposition of **H**: $\mathbf{H} = \mathbf{S} \cdot \mathbf{U} \cdot \mathbf{V}^H$
- **S**, **V**: unitary matrices ($\mathbf{V}^H\mathbf{V}=\mathbf{I}$, $\mathbf{S}\mathbf{S}^H = \mathbf{I}$)
  U : = diag(u_k), u_k singular values of **H**

- **V**/ **S**: input/output eigenvectors of **H**
- Any input along $\underline{v}_i$ will be multiplied by u_i and will appear as an output along $\underline{s}_i$

Vector analysis of the signals

1. The input vector $\underline{x}$ gets projected onto the $\underline{v}_i$'s
2. Each projection gets multiplied by a different gain u_i.
3. Each appears along a different $\underline{s}_i$.



Capacity = sum of capacities

- The channel has been decomposed into K parallel subchannels
- Total capacity = sum of the subchannel capacities
- All transmitters send the same power: $E_x = E_k$

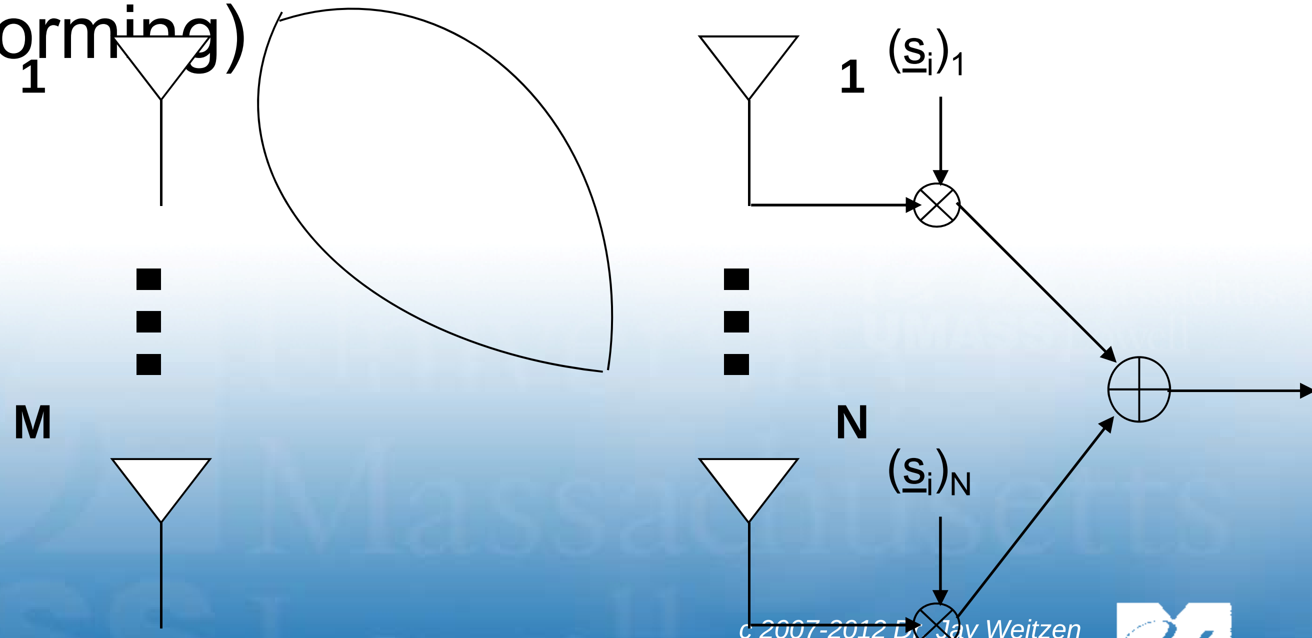
$$C = \sum_{i=1}^{K} C_k = \sum_{i=1}^{K} \log_2(1+\rho_k)$$

$$\rho_k = \frac{|u_k|^2 E\left[|\langle \underline{x}, \underline{v}_k \rangle|^2\right]}{E\left[|\langle \underline{n}, \underline{s}_k \rangle|^2\right]} = \frac{|u_k|^2 E_k}{\sigma^2}$$

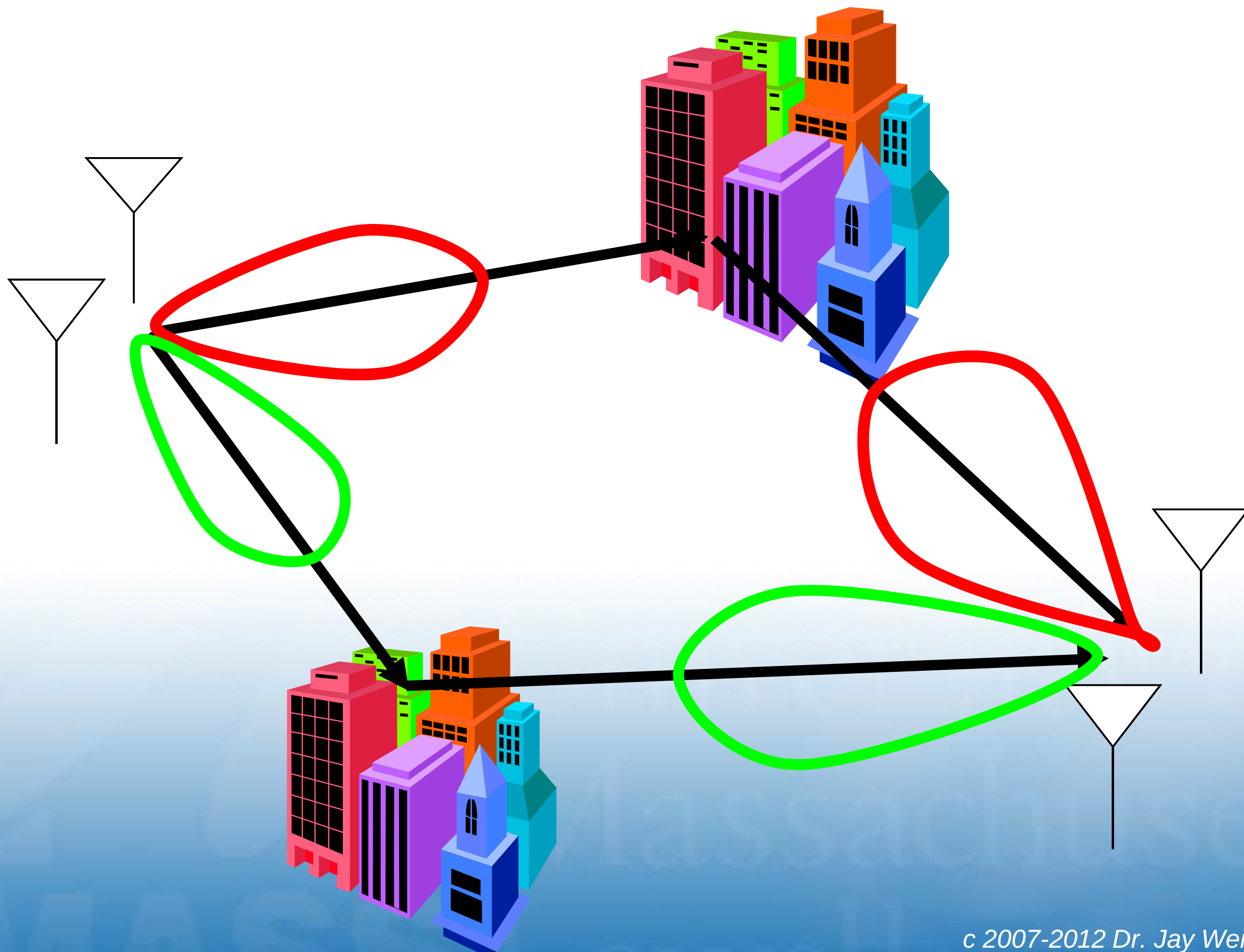
$$C = \sum_{i=1}^{K} \log_2\left(1 + \frac{E_k}{\sigma^2}|u_k|^2\right)$$

Interpretation II:
The directional approach

- Singular value decomposition of **H**: **H** = **S·U·V**^{H}
- Eigenvectors correspond to spatial directions (beamforming)



Example of directional interpretation



End of Module 7