

Introduction to Time and
Frequency:
Using Digital Filters
25.108 Introduction to Engineering
II
Dr. Jay Weitzen

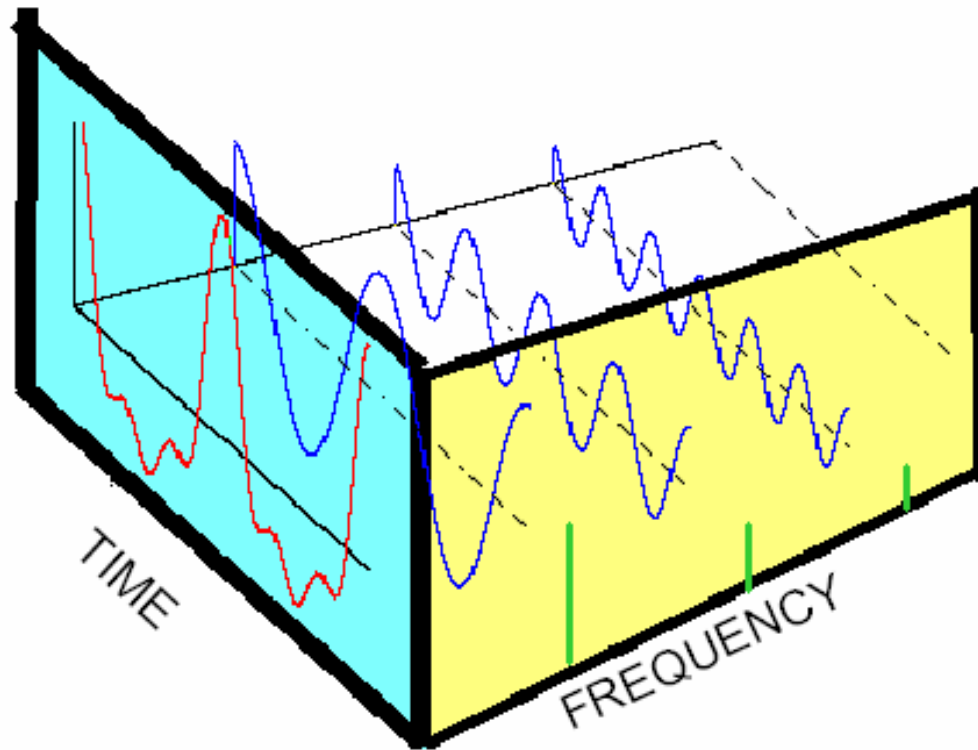
Experiment Objectives:

- Introduce concepts of time and frequency
- Build complicated waveforms from harmonics
- Introduce concepts of digital filtering
- Reduce complicated waveform to simple sinusoid

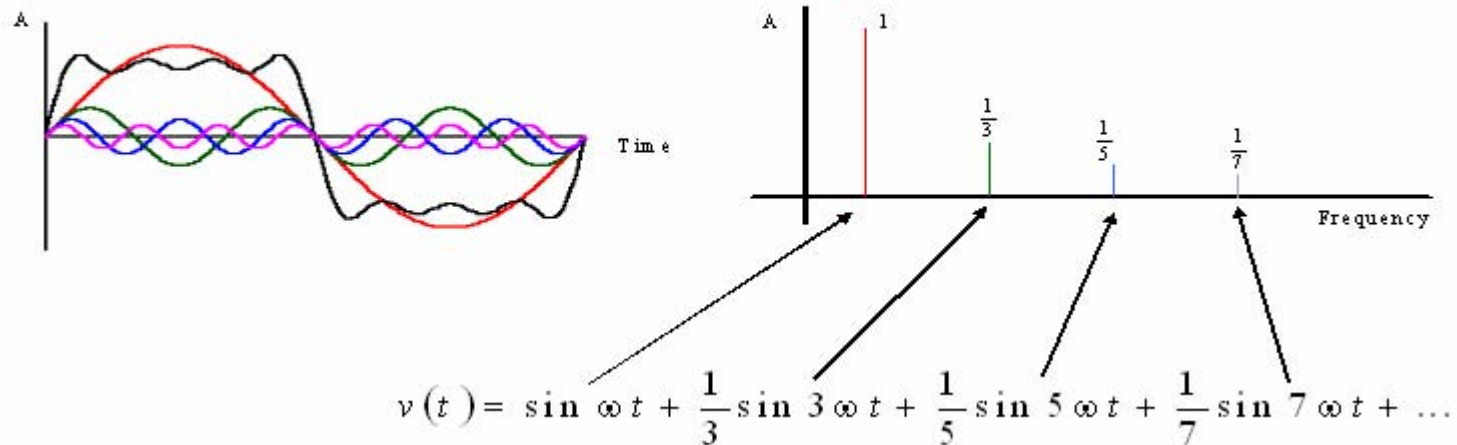
Basic Theory

- In the early 1800's Fourier showed that any periodic waveform could be expressed as an infinite sum of sine's and cosines.
 - He developed a mathematical relationship between a waveform in the time domain and it's component sines and cosines called the Fourier Series and The Fourier Transform

Time Frequency Plane

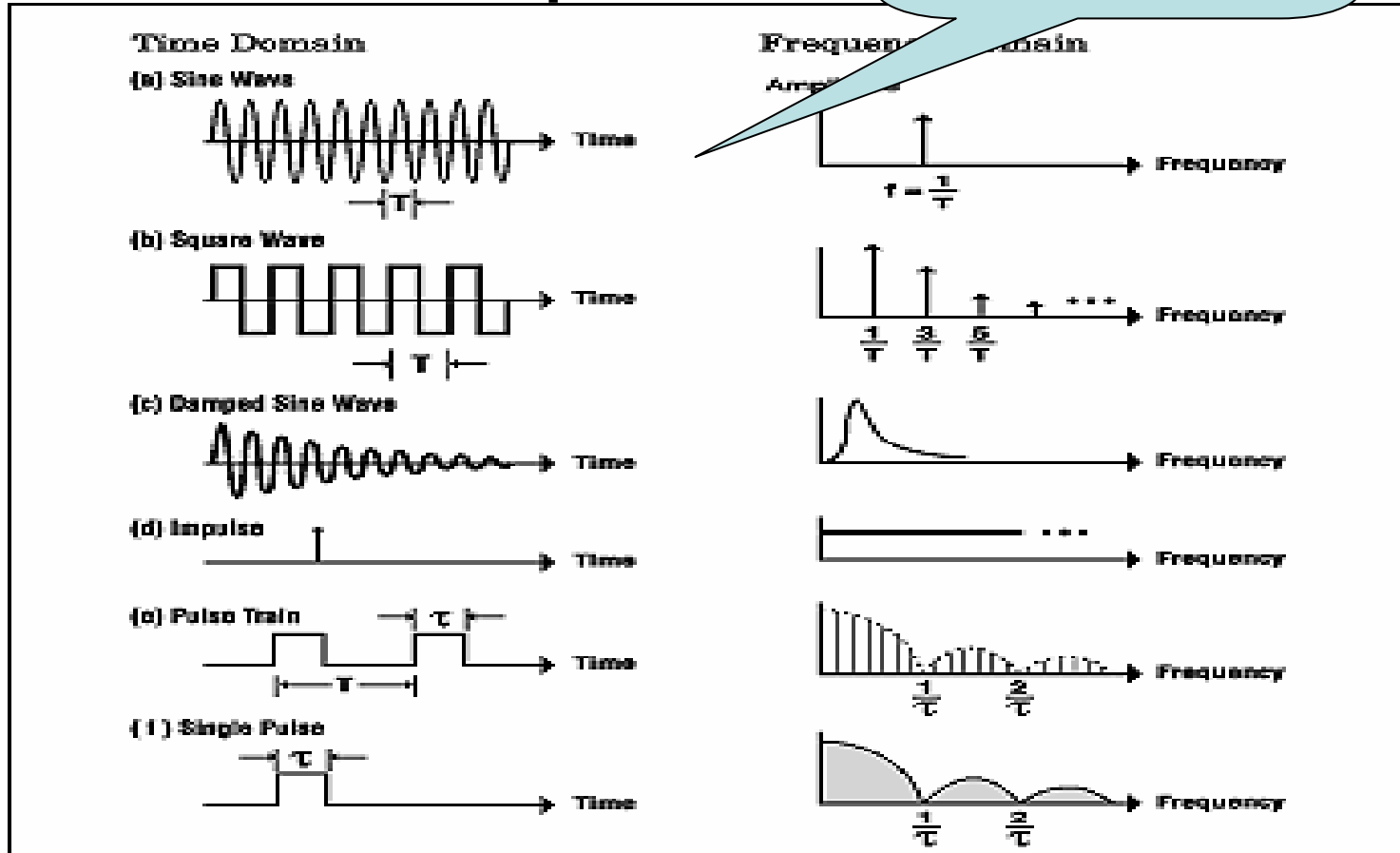


Wave Generation from Components

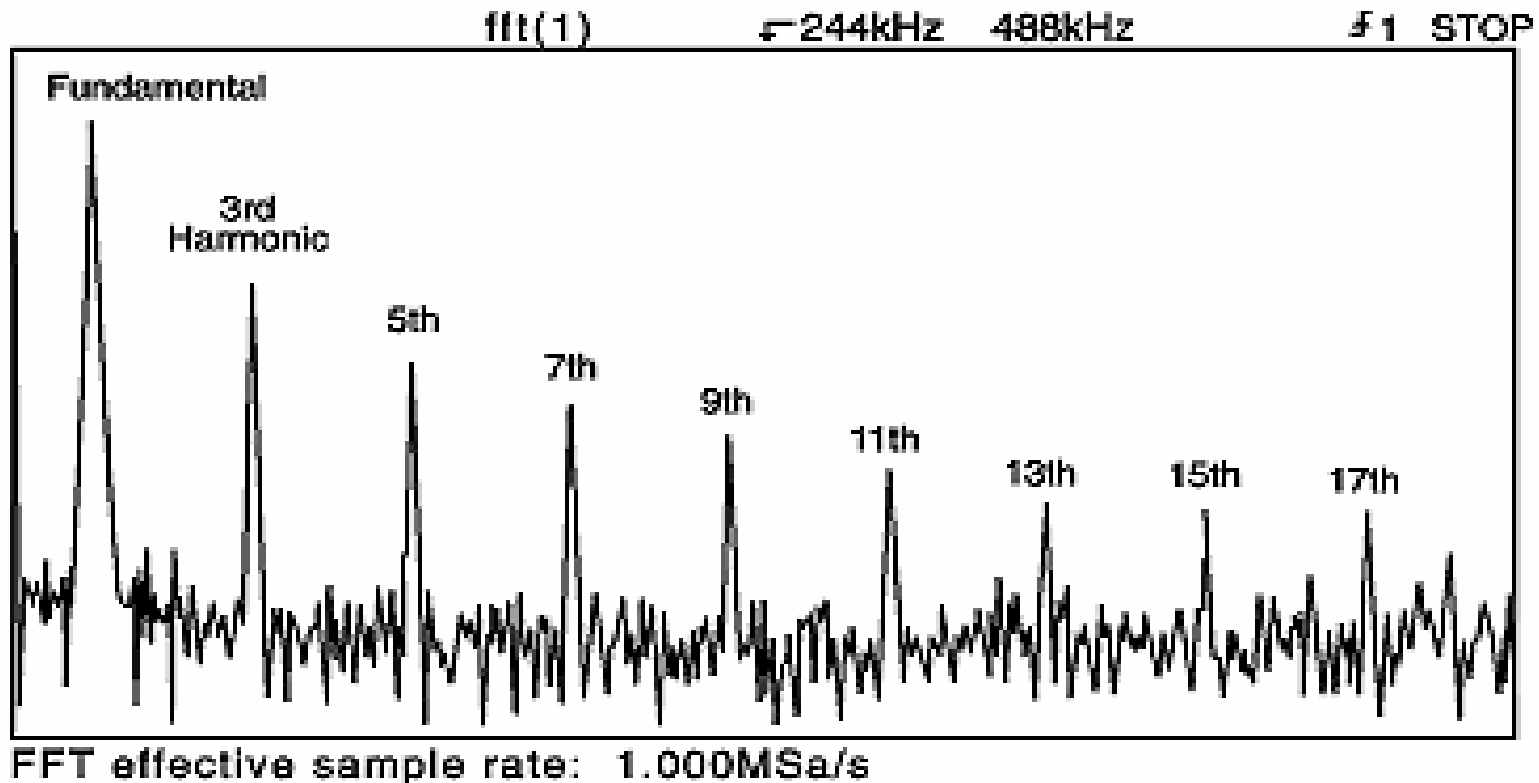


Example: Waveforms and Spectrum

A simple sinusoid in time and frequency



Example: Spectrum of a Triangle Wave



Basic Theory (You will learn this in 16.362 so do not worry now)

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos n\omega t + b_n \sin n\omega t]$$

$$\text{where } a_0 = \frac{2}{T} \int_a^{a+T} x(t) dt$$

$$a_n = \frac{2}{T} \int_a^{a+T} x(t) \cos n\omega t dt$$

$$b_n = \frac{2}{T} \int_a^{a+T} x(t) \sin n\omega t dt$$

③ Polar Fourier Series

- Another form of FS is obtained by combining the sine and cosine terms to give a single component with a phase angle

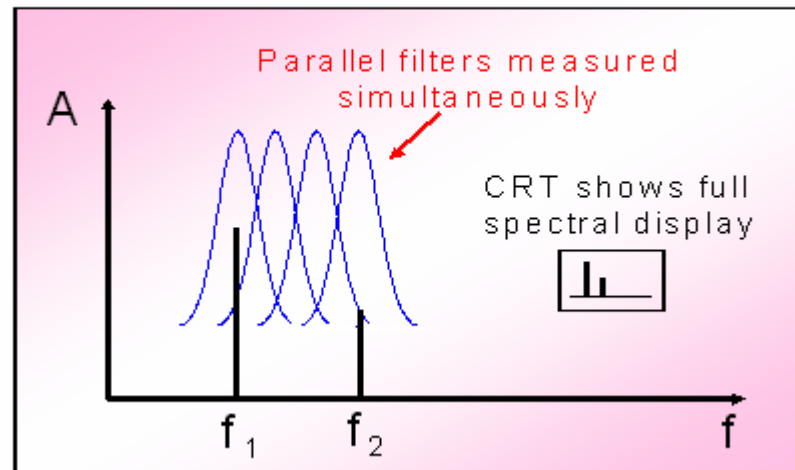
$$x(t) = d_0 + \sum_{n=1}^{\infty} [d_n \cos(n\omega t + \theta_n)]$$

$$\text{where } d_0 = a_0 = C_0$$

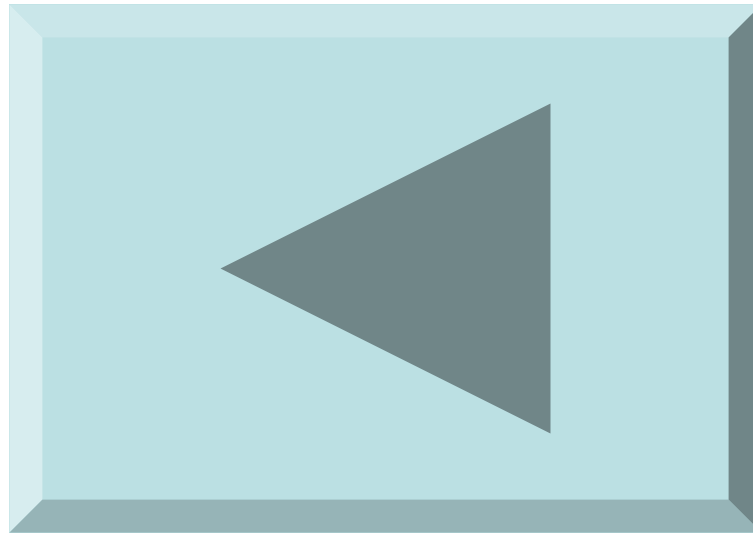
$$d_n = \sqrt{a_n^2 + b_n^2} = 2|C_n|, \quad \theta_n = -\tan^{-1}\left(\frac{b_n}{a_n}\right)$$

What is a Digital Spectrum Analyzer

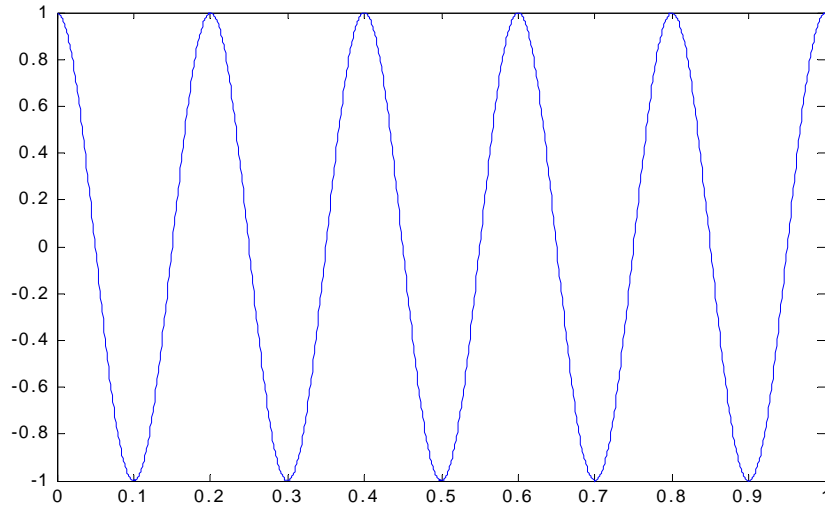
Fourier Analyzer



An Example

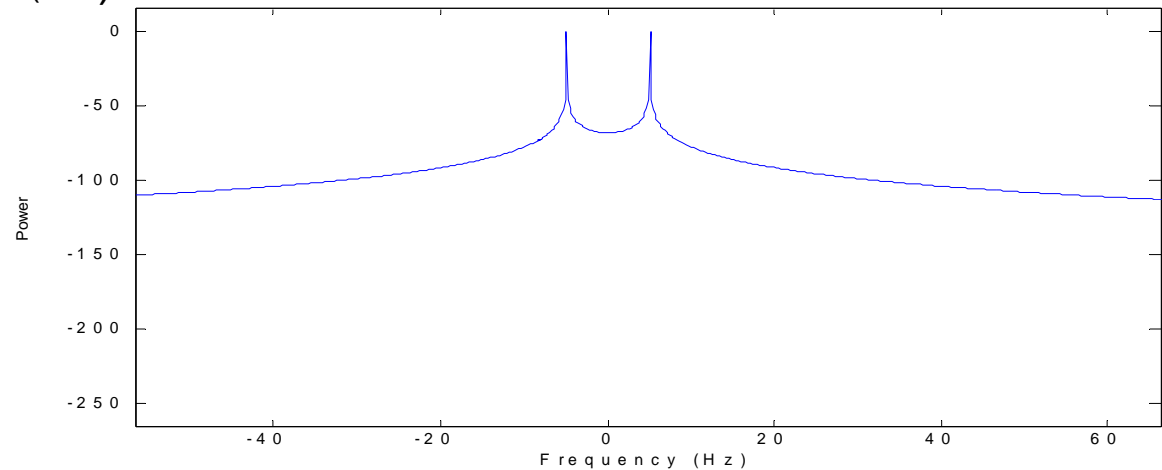


Try it Yourself

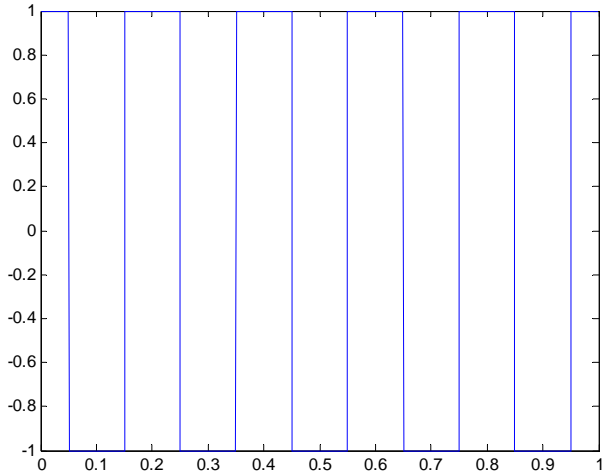


```
>> time=0:0.001:5;  
>> sin5=cos(time.*2*pi*5);  
>> plot(time,sin5)  
>> plot(time(1:1000),sin5(1:1000))  
>> SpectrumAnalyzer(sin5,1000)  
>>
```

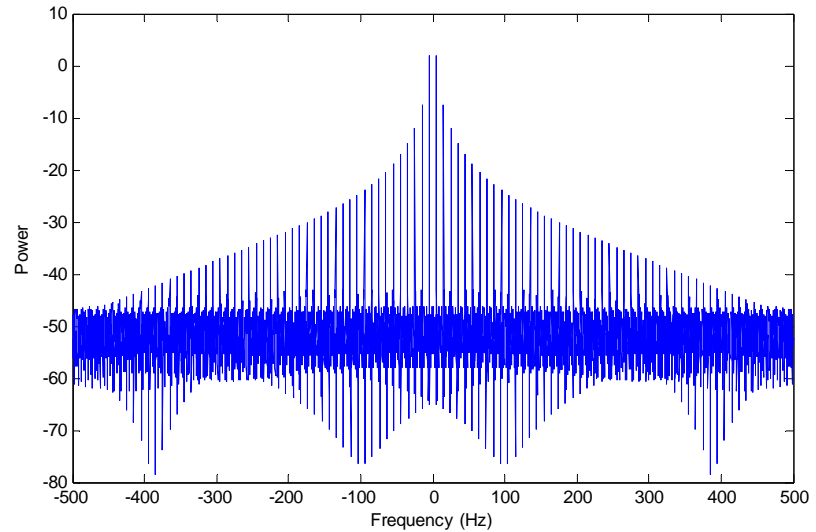
Time (sec)



Square Wave has infinite bandwidth



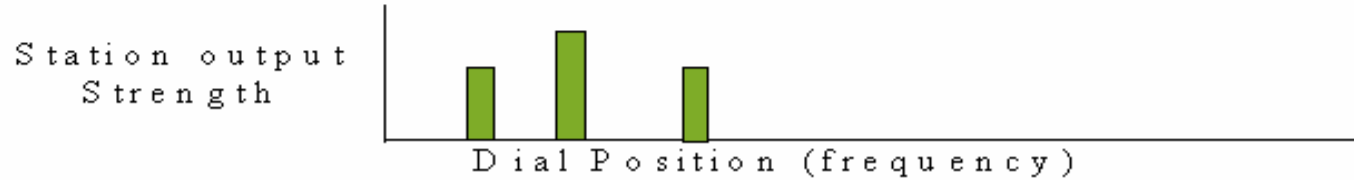
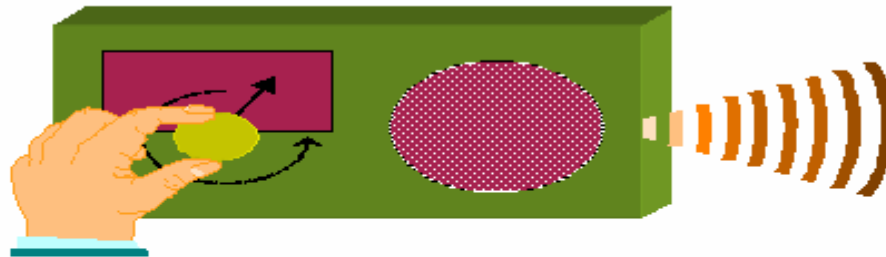
```
>> sq5=sign(sin5);  
>> plot(time(1:1000),sq5(1:1000))  
>> SpectrumAnalyzer(sq5,1000)
```



What is a Filter?

- Passes a band of frequencies and rejects other frequencies
 - Three Bands of Interest
 - Pass band, The frequencies which get through
 - Stop band, the frequencies which don't get through
 - Transition bands, the bands in which part of the frequencies get through, between stop and pass bands

Tuning a Radio, an example of Filtering



What does it do? Passes the frequencies you want and rejects those that you do not want.

Types of Digital Filters

- Low Pass: Passes low frequencies, rejects high frequencies
- High Pass: Passes high frequencies, rejects low frequencies
- Band Pass: Passes a band of frequencies
- Band Stop: Rejects a band of frequencies

Filter Implementations

- Infinite Impulse Response (IIR)
 - Feedback filter
- Finite Impulse response
 - Feed Forward
- Hybrid IIR/FIR

Key parameters in filter design

- Sampling rate
- Number of Taps
- Pass band
- Stop Band

Using FDA tool

- Type "FDATool" at command prompt

The screenshot shows the Filter Design & Analysis Tool (FDATool) interface. The window title is "Filter Design & Analysis Tool - [untitled.fda]". The interface is divided into several sections:

- Current Filter Information:** Structure: Direct form FIR, Order: 50, Sections: 1, Stable: Yes, Source: Designed.
- Filter Specifications:** A plot of Magnitude (dB) vs. Frequency (Hz). The plot shows a passband from 0 to F_{pass} and a stopband from F_{stop} to $F_s/2$. The passband ripple is A_{pass} and the stopband attenuation is A_{stop} .
- Filter Type:** Radio buttons for Lowpass, Highpass, Bandpass, and Bandstop. The "Highpass" button is selected.
- Filter Order:** Radio buttons for "Specify order" (set to 40) and "Minimum order".
- Options:** Density factor: 16.
- Design Method:** Radio buttons for IIR (Butterworth) and FIR (Equiripple). The "FIR Equiripple" button is selected.
- Frequency Specifications:** Units: Hz, F_s : 48000, F_{pass} : 9600, F_{stop} : 12000.
- Magnitude Specifications:** Units: dB, A_{pass} : 1, A_{stop} : 80.

Annotations with arrows point to various parts of the interface:

- "Pass band" points to the flat region of the magnitude plot.
- "Stop Band" points to the attenuated region of the magnitude plot.
- "Sampling Frequency" points to the F_s input field.
- "Filter Type" points to the "Highpass" radio button.
- "Compute Filter Button" points to the "Design Filter" button.
- "Don't touch this" points to the "FIR Equiripple" radio button.

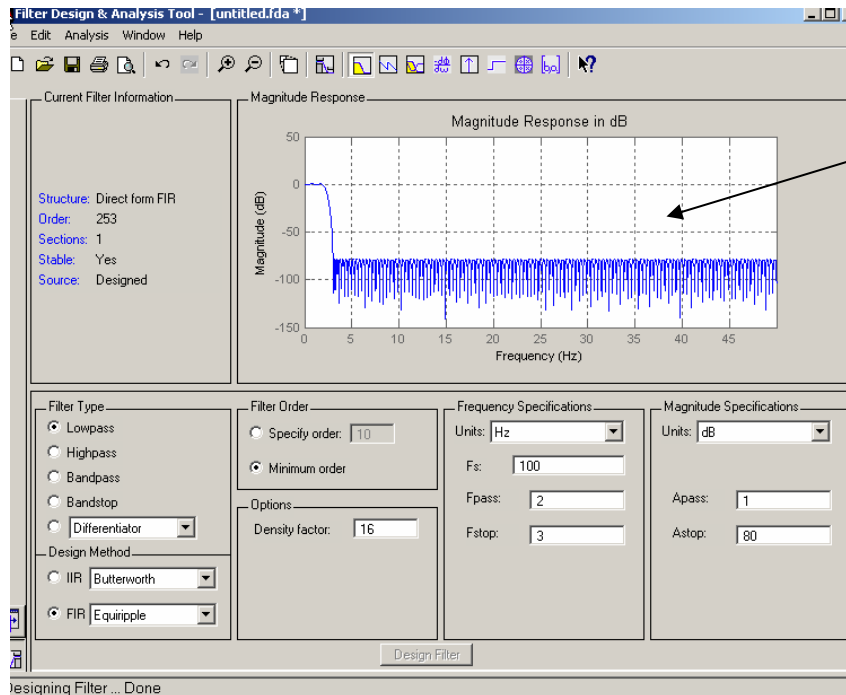
Step 2: Enter Parameters

- Enter Sampling Frequency
- Pass Band
- Stop Band
- Leave Everything else the same

Step 3: Design Filter

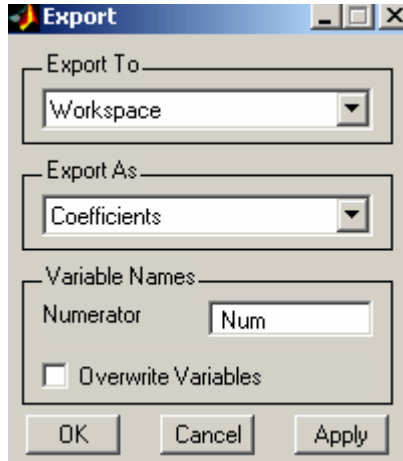
- Push “design Filter Button

Filter Response Shown



Step 4: Export Coefficients

- On “File Menu” Type Export



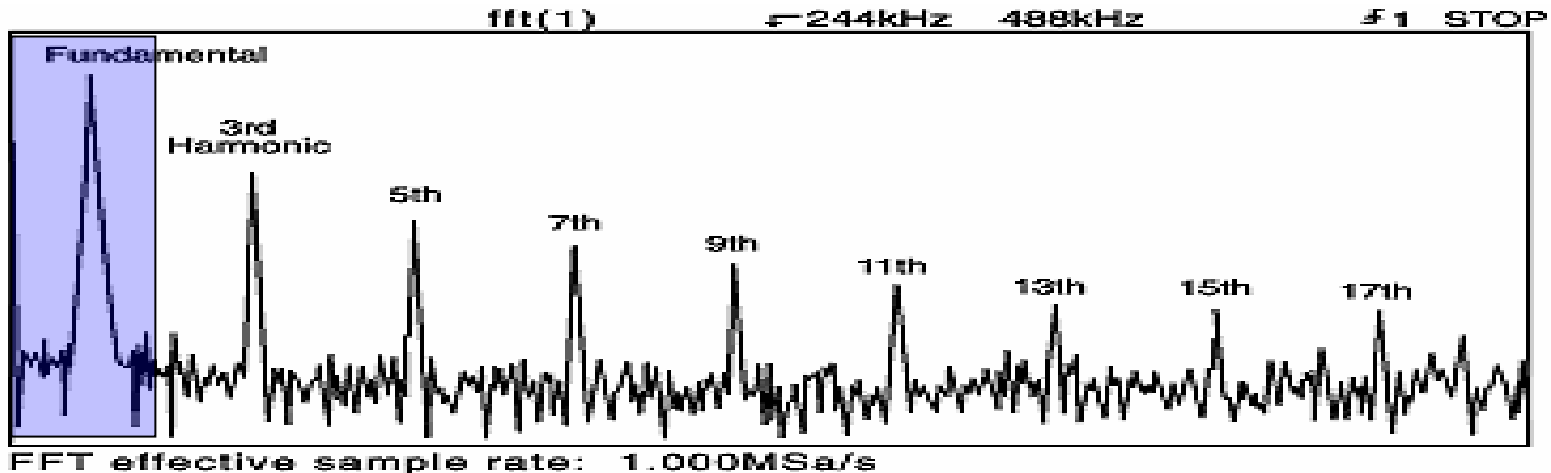
Create “Num”, and “Den”. If you have matlab 6.5, type “Den=1”

Step 5: do the filtering

- Type
- “>> Output=filter(Num,Den,Input)” to apply the filter you have created. It is simple as that.

Quick Question?

- If you take a triangle wave with spectrum shown and low pass filter remove all frequencies except the fundamental, what will you see?



Answer: A sine wave at the fundamental frequency