Introduction to Time and Frequency: Using Digital Filters

25.108 Introduction to Engineering II

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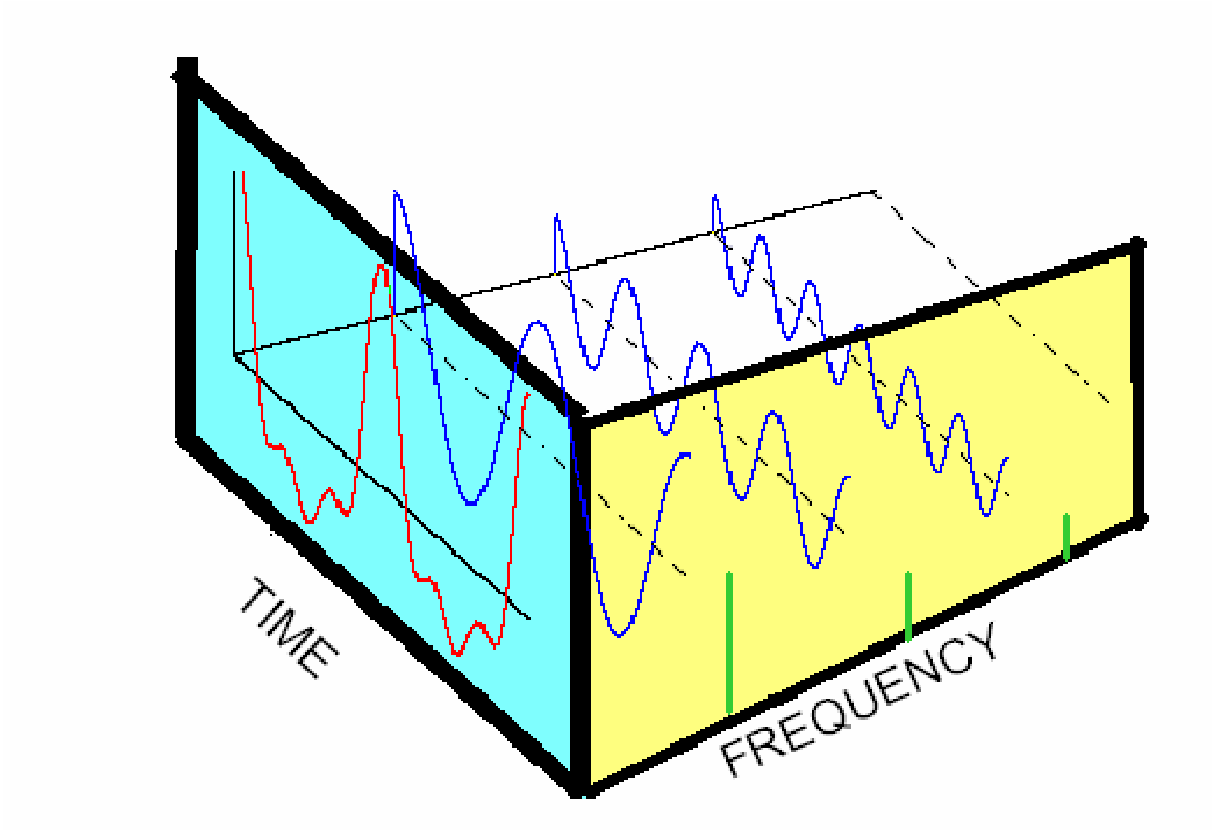
Experiment Objectives:

- Introduce concepts of time and frequency
- Build complicated waveforms from harmonics
- Introduce concepts of digital filtering
- Reduce complicated waveform to simple sinusoid

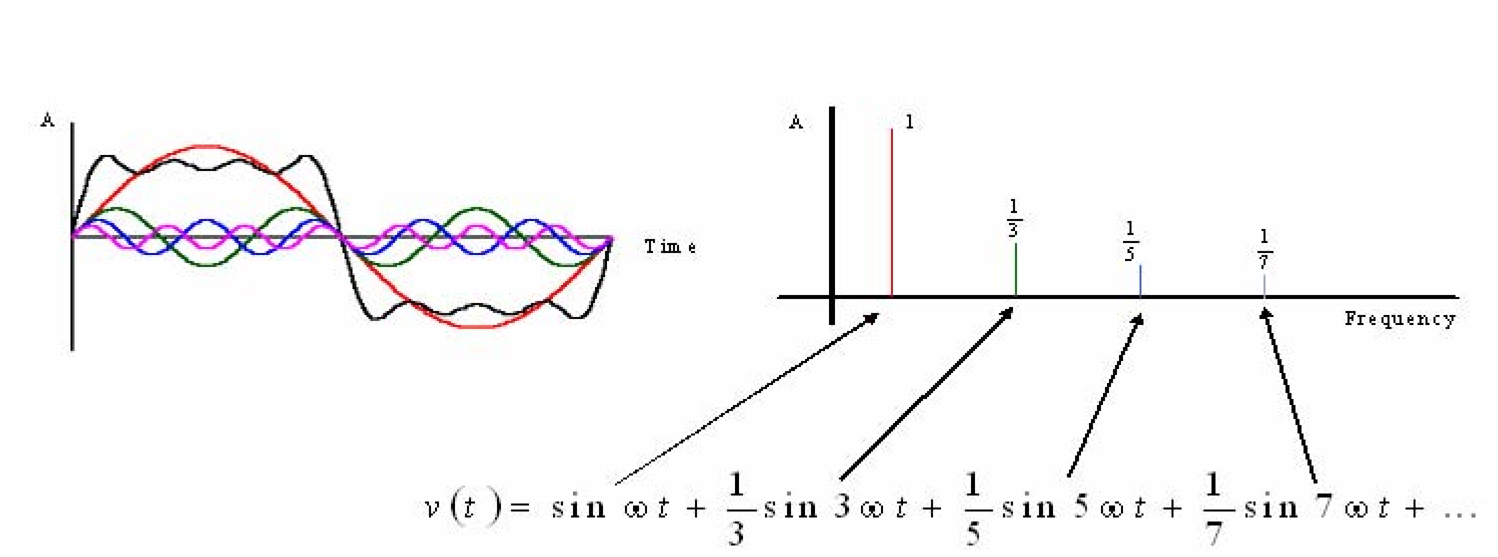
Basic Theory

- In the early 1800's Fourier showed that any periodic waveform could be expressed as an infinite sum of sine's and cosines.
  - He developed a mathematical relationship between a waveform in the time domain and it's component sines and cosines called the Fourier Series and The Fourier Transform

Time Frequency Plane

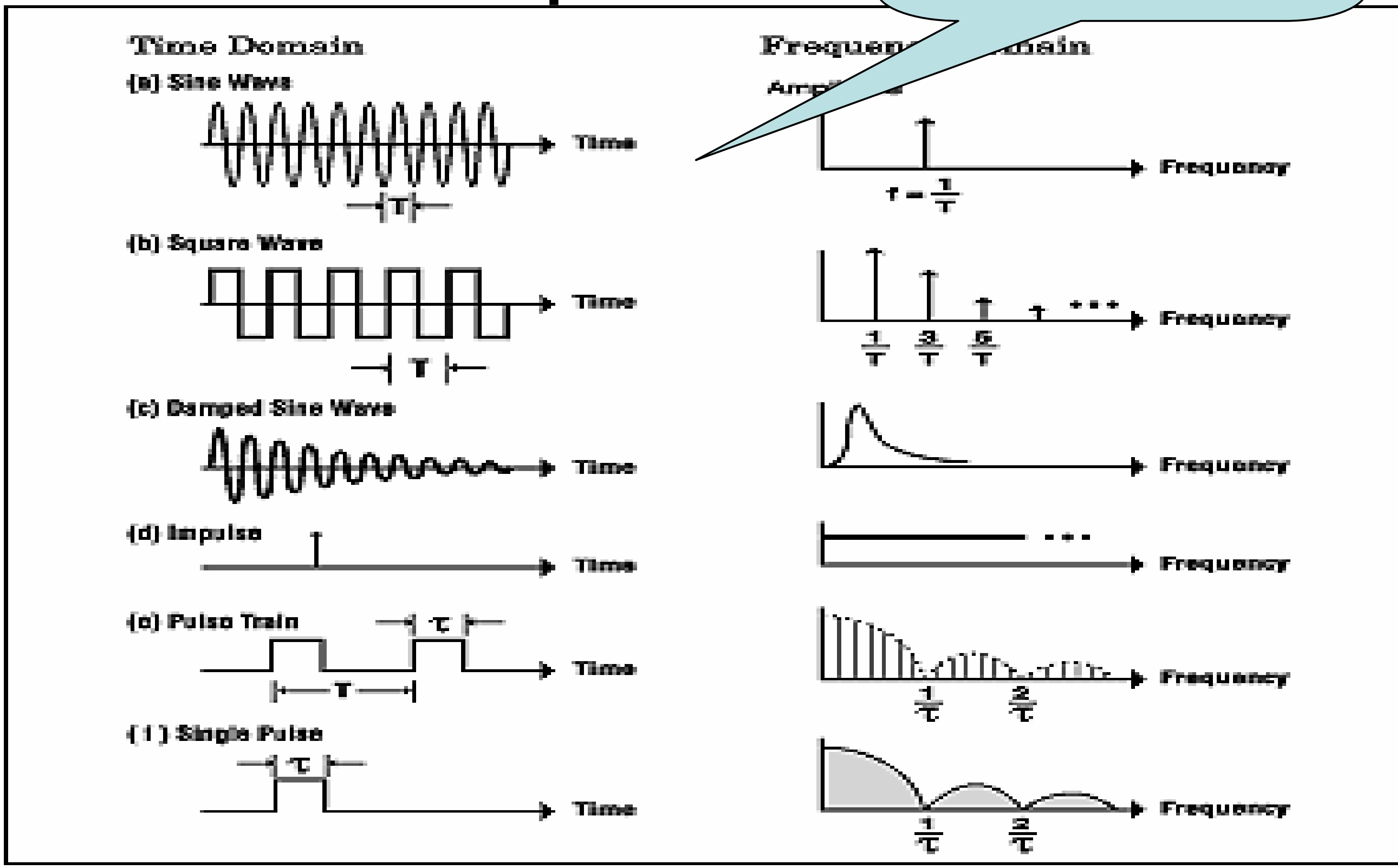


Wave Generation from Components

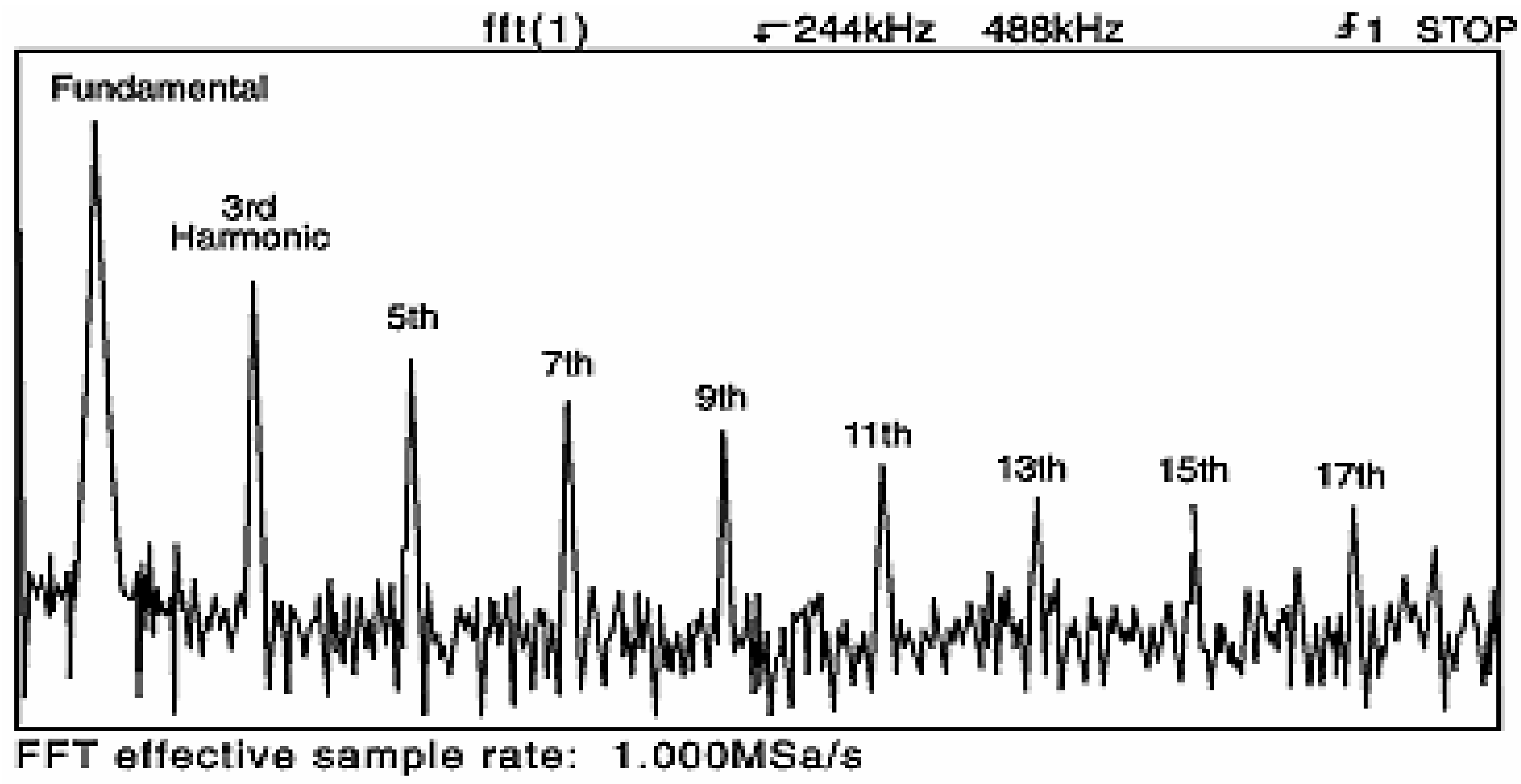


Example: Waveforms and Spectrum

A simple sinusoid in time and frequency



Example: Spectrum of a Triangle Wave



Basic Theory (You will learn this in 16.362 so do not worry now)

$$x(t) = \frac{a_o}{2} + \sum_{n=1}^{\infty}\left[a_n \cos n\omega t + b_n \sin n\omega t\right]$$

$$\text{where } a_o = \frac{2}{T}\int_a^{a+T} x(t)dt$$

$$a_n = \frac{2}{T}\int_a^{a+T} x(t)\cos n\omega t dt$$

$$b_n = \frac{2}{T}\int_a^{a+T} x(t)\sin n\omega t dt$$

③ Polar Fourier Series

- Another form of FS is obtained by combining the sine and cosine terms to give a single component with a phase angle

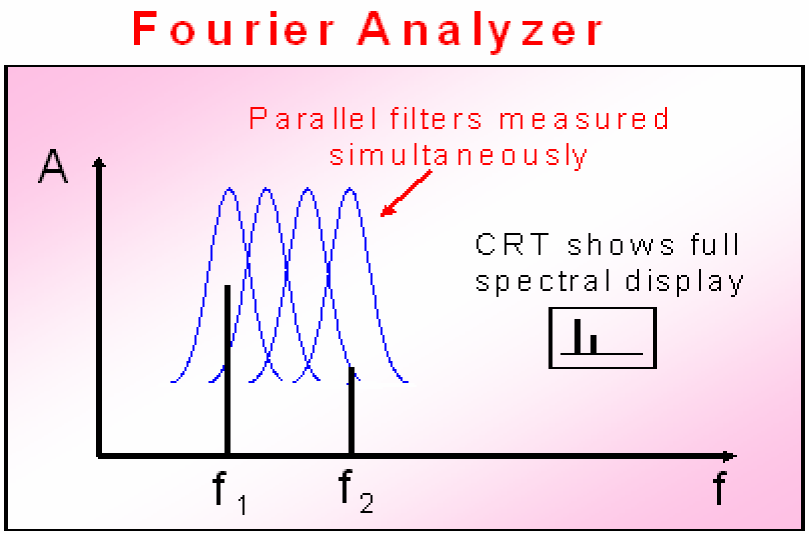
$$x(t) = d_0 + \sum_{n=1}^{\infty}\left[d_n \cos(n\omega t + \theta_n)\right]$$

$$\text{where } d_0 = a_0 = C_0$$

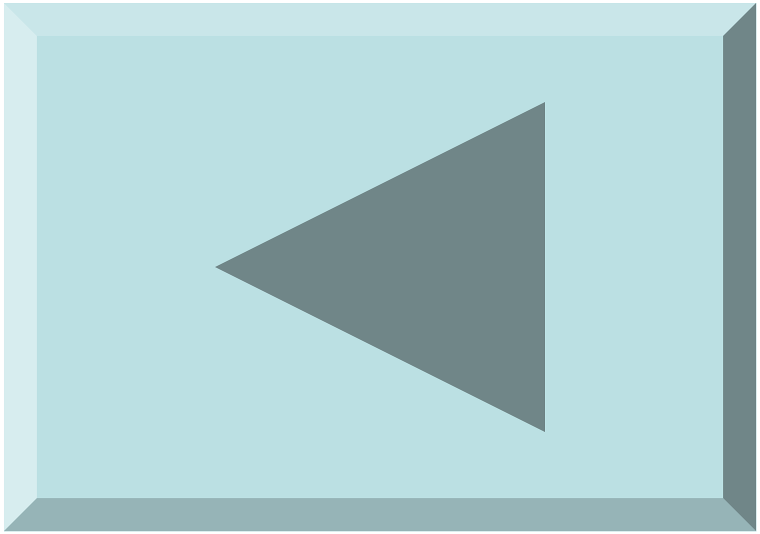
$$d_n = \sqrt{a_n^2 + b_n^2} = 2|C_n|, \quad \theta_n = -\tan^{-1}\left(\frac{b_n}{a_n}\right)$$

Analog Signals 17

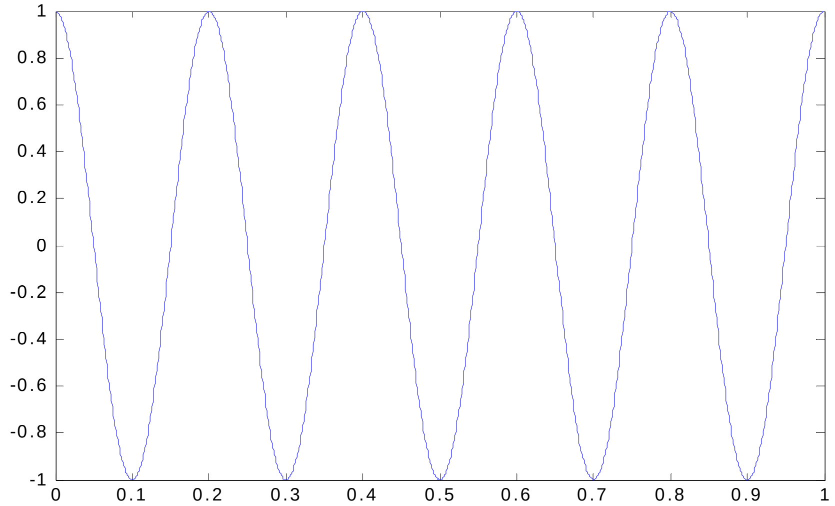
What is a Digital Spectrum Analyzer



An Example

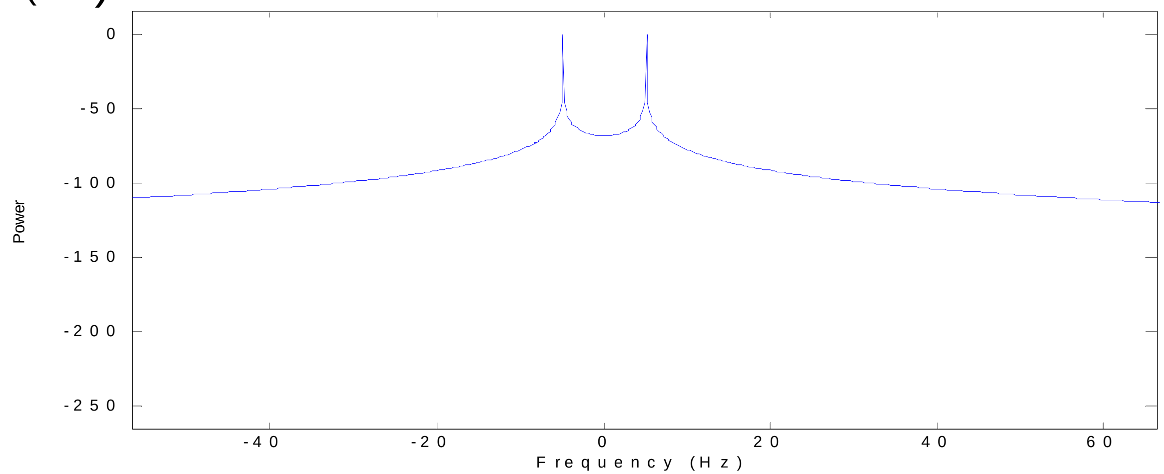


Try it Yourself

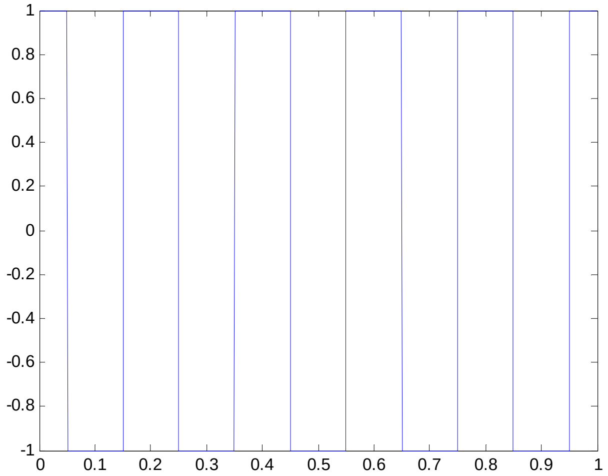



```
>> time=0:0.001:5;
>> sin5=cos(time.*2*pi*5);
>> plot(time,sin5)
>> plot(time(1:1000),sin5(1:1000))
>> SpectrumAnalyzer(sin5,1000)
>>
```

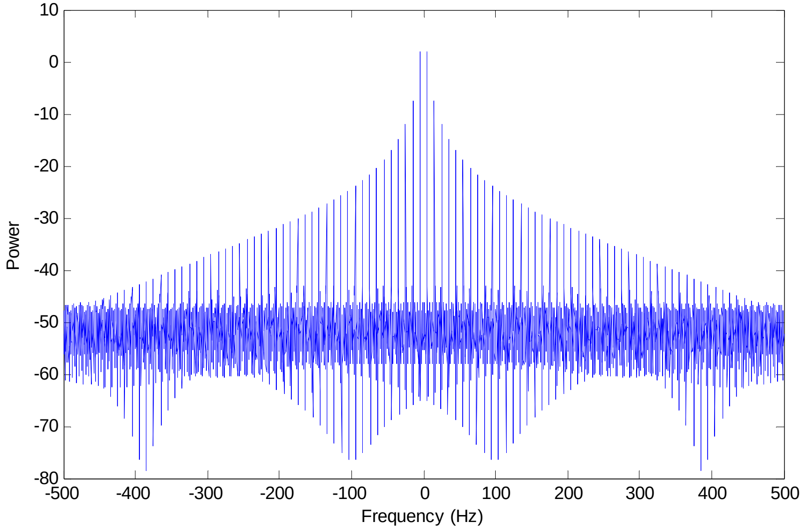
Time (sec)



Square Wave has infinite bandwidth



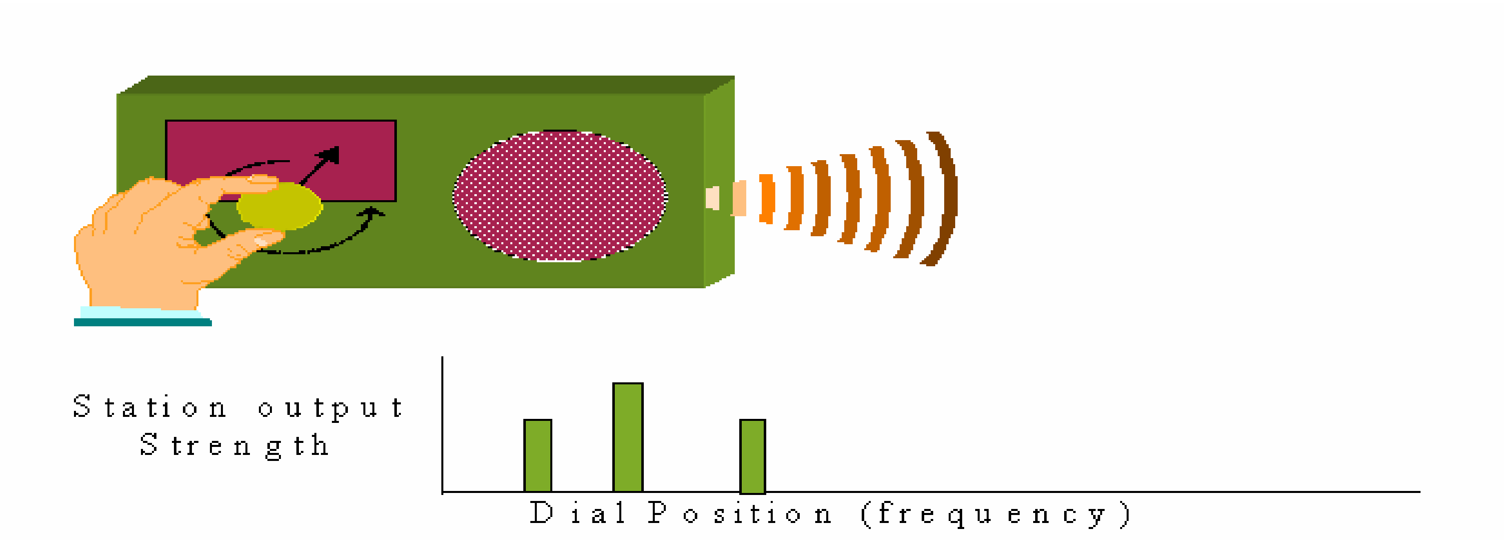

```
>> sq5=sign(sin5);
>> plot(time(1:1000),sq5(1:1000))
>> SpectrumAnalyzer(sq5,1000)
```



What is a Filter?

- Passes a band of frequencies and rejects other frequencies
  - Three Bands of Interest
    - Pass band, The frequencies which get through
    - Stop band, the frequencies which don't get through
    - Transition bands, the bands in which part of the frequencies get through, between stop and pass bands

Tuning a Radio, an example of Filtering



What does it do? Passes the frequencies you want and rejects those that you do not want.

Types of Digital Filters

- Low Pass: Passes low frequencies, rejects high frequencies
- High Pass: Passes high frequencies, rejects low frequencies
- Band Pass: Passes a band of frequencies
- Band Stop: Rejects a band of frequencies

Filter Implementations

- Infinite Impulse Response (IIR)
  - Feedback filter
- Finite Impulse response
  - Feed Forward
- Hybrid IIR/FIR

Key parameters in filter design

- Sampling rate
- Number of Taps
- Pass band
- Stop Band

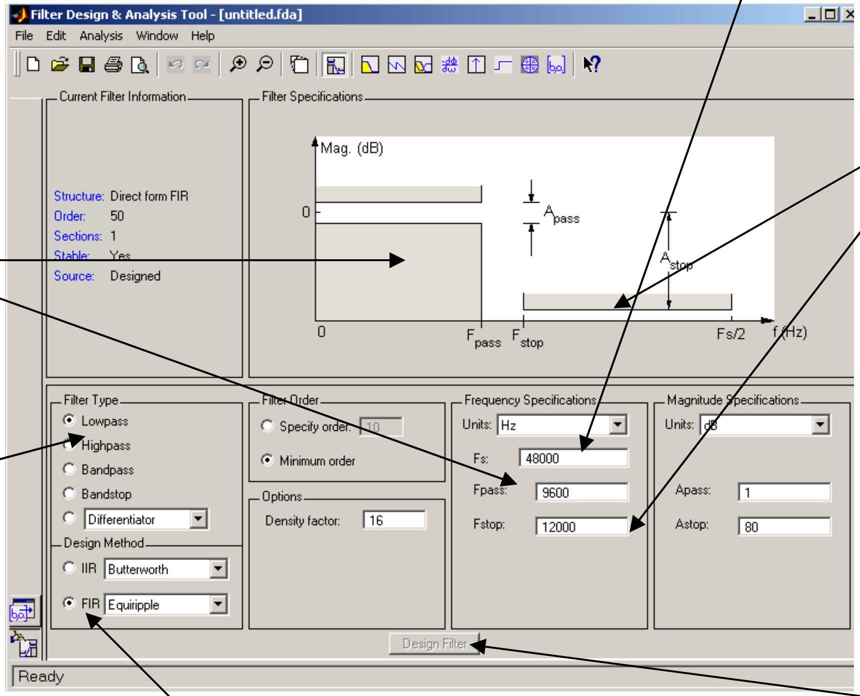
Using FDA tool

- Type “FDATOOL” at command prompt

Sampling Frequency

Stop Band

Pass band



Filter Type

Compute Filter Button

Don’t touch this

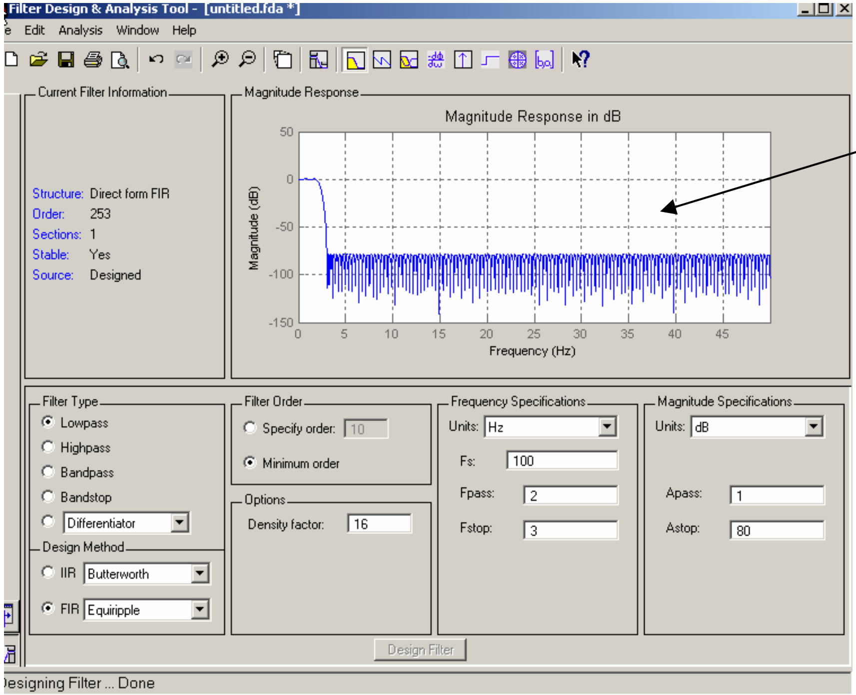
Step 2: Enter Parameters

- Enter Sampling Frequency
- Pass Band
- Stop Band
- Leave Everything else the same

Step 3: Design Filter

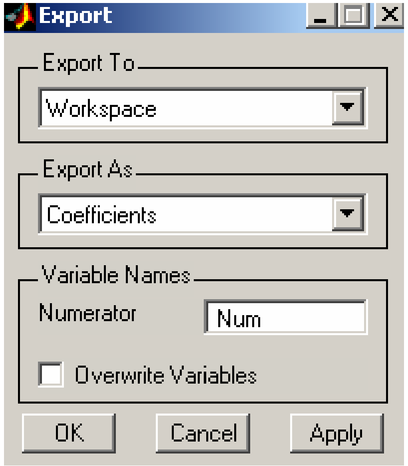
- Push “design Filter Button

Filter Response Shown



Step 4: Export Coefficients

- On “File Menu” Type Export



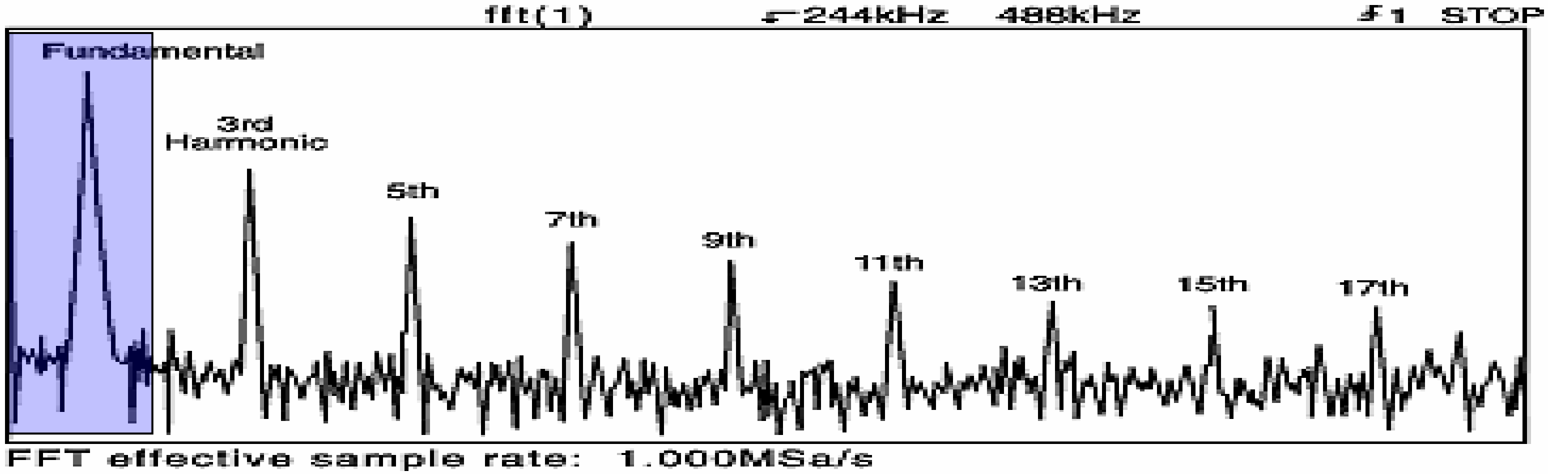
Create “Num”, and “Den”. If you have matlab 6.5, type “Den=1”

Step 5: do the filtering

- Type
- “>> Output=filter(Num,Den,Input)” to apply the filter you have created. It is simple as that.

Quick Question?

- If you take a triangle wave with spectrum shown and low pass filter remove all frequencies except the fundamental, what will you see?



Answer: A sine wave at the fundamental frequency