
Notes on Sections 6.4: Integration with Tables and Computer Algebra Systems in Stewart's *Essential Calculus*

Kenneth Levasseur
Mathematical Sciences
UMass Lowell
Kenneth_Levasseur@uml.edu

Complete coverage of this section is not part of the syllabus for Calculus II at UML. Your instructor may choose to lecture on this material or simply ask you to read these notes. Although the material will not be on any exams, it could be useful for you to know about it in the future if you need to use a Computer Algebra System (CAS)

■ What is a Computer Algebra System?

A Computer Algebra System (CAS) is a computer program that performs a wide variety of numeric, graphic, symbolic and logical operations that form the core components of algebra. Most of them also do a significant amount of the calculation that is done in calculus. The most common CAS's are *Mathematica*, *MatLab*, and *Maple*. Some Texas Instruments calculators, including the TI-89 and TI-92 have a CAS built in.

These notes were created from a *Mathematica* document, but the capabilities used here are common to all CAS's.

■ Definite and Indefinite Integrals

The same function is used for both indefinite and definite integrals... the form of the arguments provided to the function **Integrate**, are just different.

Here is an indefinite integral:

```
Integrate[1 / (x ^ 2 + 1), x]
```

$\tan^{-1}(x)$

And here is a definite integral of the same function:

```
Integrate[1 / (x ^ 2 + 1), {x, 0, 1}]
```

$\frac{\pi}{4}$

Here is how to get the general form of any function, such as **Integrate**.

? Integrate

`Integrate[f, x]` gives the indefinite integral of f with respect to x . `Integrate[f, {x, xmin, xmax}]` gives the definite integral of f with respect to x from x_{\min} to x_{\max} . `Integrate[f, {x, xmin, xmax}, {y, ymin, ymax}]` gives a multiple definite integral of f with respect to x and y . More...

■ The arbitrary constant

The typical CAS is very good at indefinite integration but does not include the familiar arbitrary constant that you are encouraged to include when you do an integral by hand. The **Integrate** function determines *an* antiderivative.

```
Integrate[x Sin[x], x]
```

```
sin(x) - x cos(x)
```

Another way to find antiderivatives of a function is to pose the problem as a *differential equation*. The following expression is finding a function $y(x)$ with the property that its derivative is $x \sin x$. Notice that the result does include an arbitrary constant.

```
DSolve[y'[x] == x Sin[x], y[x], x]
```

```
{{y(x) -> c1 - x cos(x) + sin(x)}}
```

Here is how you would find the specific antiderivative that has a desired value; e. g., the one that passes through the point $(\pi/2, 4)$.

```
DSolve[{y'[x] == x Sin[x], y[pi/2] == 4}, y[x], x]
```

```
{{y(x) -> -x cos(x) + sin(x) + 3}}
```

■ A function can have multiple forms

Functions can have different algebraic forms, particularly trigonometric functions. Since the methods used by a CAS may not match the ones you follow in integrating by hand, it is possible to get different results. It doesn't mean that you (or the CAS) is wrong. Here is the integral discussed in Example 3 on page 329 and then again in Example 4 on page 330.

```
Integrate[x  $\sqrt{x^2 + 2x + 4}$ , x]
```

$$\frac{1}{6} \left(\sqrt{x^2 + 2x + 4} (2x^2 + x + 5) - 9 \sinh^{-1} \left(\frac{x+1}{\sqrt{3}} \right) \right)$$

Here is a different, simpler, example. If you wanted to evaluate $\int \cos(x) \sin(x) dx$, you might use the substitution $u = \sin x$, in which case you would get $\frac{1}{2} \sin^2(x) + C$

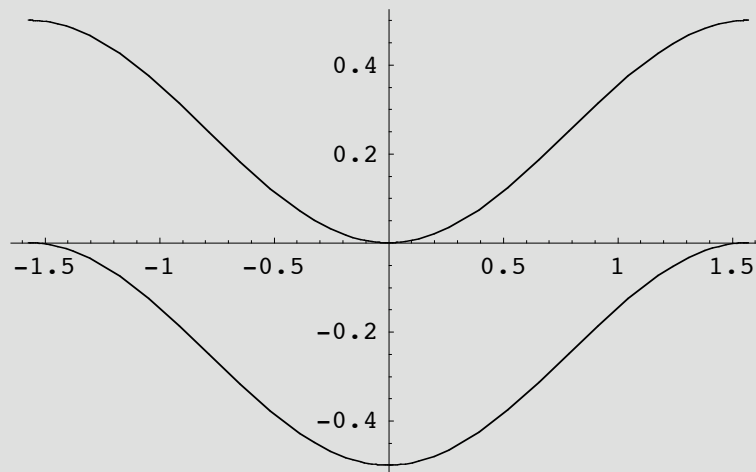
Here is what *Mathematica* gives you

```
Integrate[Cos[x] Sin[x], x]
```

$$-\frac{1}{2} \cos^2(x)$$

Plotting the two results (with $C = 0$) shows that the results are consistent since they differ by a constant (1/2).

```
Plot[{Sin[x] 2 / 2, -Cos[x] 2 / 2}, {x, -Pi / 2, Pi / 2}]
```



- Graphics -

Another example: The substitution $u = x^2 - 1$ can be employed to evaluate $\int x(x^2 - 1)^3 dx$, with the result being $\frac{1}{8} (x^2 - 1)^4 + C$. When *Mathematica* is given this integral, it apparently expands the integrand and applies the power rule to the individual powers of x .

```
Integrate[x (x ^ 2 - 1) ^ 3, x]
```

$$\frac{x^8}{8} - \frac{x^6}{2} + \frac{3x^4}{4} - \frac{x^2}{2}$$

Again, the results are consistent, as can be seen when you expand the result we might have gotten by hand with $C = 0$

```
Expand[ $\frac{1}{8}$  (x ^ 2 - 1) ^ 4]
```

$$\frac{x^8}{8} - \frac{x^6}{2} + \frac{3x^4}{4} - \frac{x^2}{2} + \frac{1}{8}$$

■ Not every function has a "nice" antiderivative

It is surprisingly difficult to find a function that can't be integrated by the typical CAS, but they do exist. Sometimes you just get a formatted version what you enter back as an output. Here is an example:

```
Integrate[Abs [x - 1] Sin [x ^ 2], x]
```

$$\int |x - 1| \sin(x^2) dx$$

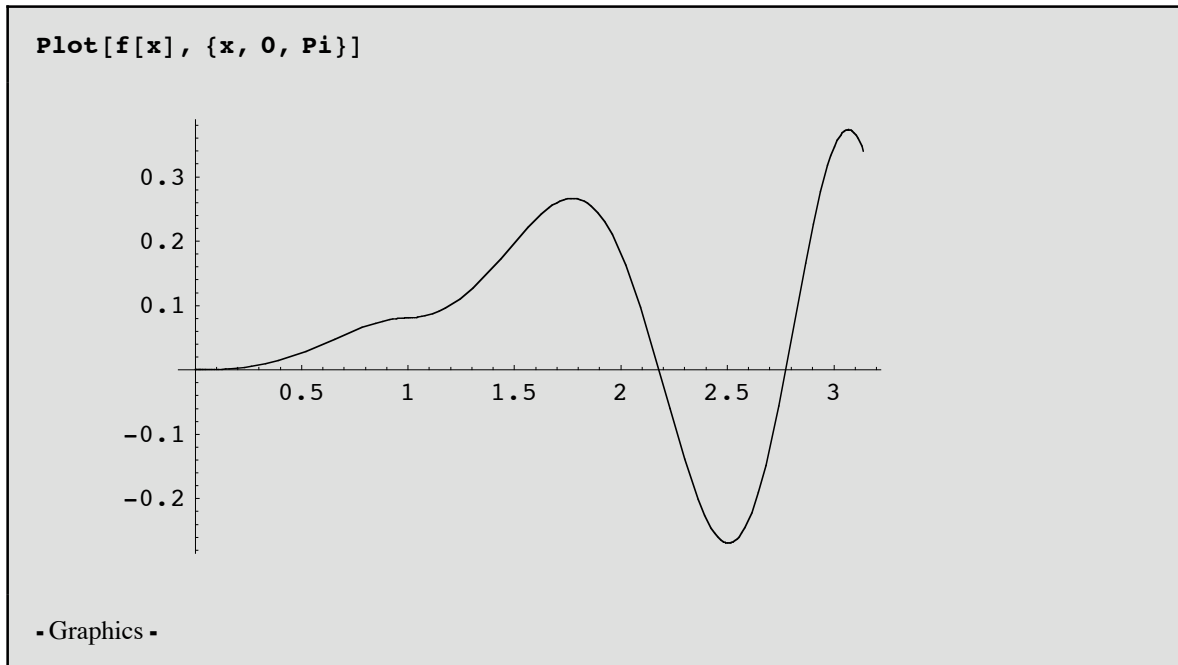
Even when you get a result it might be that it is simply a name that mathematicians have given the antiderivative. For example the Sine Integral function is defined to be $\text{Si}(x) = \int_0^x \sin(t)/t dt$

```
Integrate[Sin [x] / x, x]
```

```
Si(x)
```

Every continuous function has an antiderivative however, and there is a function whose derivative is $|x - 1| \sin(x^2)$. Here is how to define one of the antiderivatives and plot it on the interval $[0, \pi]$.

```
f [x_] := NIntegrate [Abs [t - 1] Sin [t ^ 2], {t, 0, x}]
```



■ Knowing the syntax, "oversmart" software

This final example illustrates two issues. First, any CAS will require learning some syntax. In the case of *Mathematica*, one idiosyncrasy is that the natural logarithm function is `Log` as opposed to `Ln` or `ln` as you might expect.

`D[Log[x], x]`

$$\frac{1}{x}$$

The other issue is that some seemingly simple calculations don't match what we are told in the calculus text. For example, you probably have read that $\int \frac{1}{x} dx = \ln |x| + C$. So where is the absolute value in the result below?

`Integrate[1/x, x]`

$$\text{Log}[x]$$

The reason why it doesn't appear is because *Mathematica* allows for the possibility that the variables are complex numbers, and so our customary restriction that the logarithm of a negative number is undefined is no longer valid

`Log[-2]`

$$i\pi + \text{Log}[2]$$

In Calculus II we assume that x in the expression below is positive and the value of this integral is $\ln x$. But *Mathematica* gives a result that is way too complicated for our purposes.

```
Integrate[1 / t, {t, 1, x}]
```

```
(-1 + x) If[Re[x] ≥ 0 || Im[x] ≠ 0,  $\frac{\text{Log}[x]}{-1 + x}$ , Integrate[  

 $\frac{1}{1 + t (-1 + x)}$ , {t, 0, 1}, Assumptions → ! (Re[x] ≥ 0 || Im[x] ≠ 0)]]
```

To get around the problem, we need to add some assumptions.

```
Integrate[1 / t, {t, 1, x}, Assumptions -> Im[x] = 0 ∧ Re[x] > 1]
```

```
Log[x]
```