Overview

92.411 Complex Variables

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Here is an overview of what I hope to cover in the course. It's a bit ambitious, so we might scale back on some of the last topics listed here.

- **The Complex Numbers, \( \mathbb{C} \).**
  You probably know that a complex number can take the form \( a + b \, i \), where \( i^2 = -1 \), but there is more to it that that.

- **Analytic Functions**
  What functions have a derivative? In the real case, this question is surprisingly much more complicated than the complex case. The answer for functions on \( \mathbb{C} \) is surprisingly neat. The bottom line is that differentiable functions are ones that have a power series.

- **Elementary Functions**
  Polynomials, Rational functions, Exponential, Trigonometric, Hyperbolic and Logarithmic functions. You've been using the real versions of these functions for a long time, and now we extend them to domain of complex numbers.

- **Complex Integration**
  Integration of a real function is most commonly done on an interval. For complex numbers, it is most commonly done along a path. The details of how this is done takes a bit of time to explain. The bottom line is that for some functions, the result doesn't depend on the path. For some other functions, the path selection makes a big difference.

- **Series Representations for Analytic Functions**
  You probably saw power series first in Calculus II. There isn't much new when you reconsider the topic in the complex numbers, so in theory this is a lot of review. Must of us need more than one try at understanding the topic, so we'll spend some time on this topic. But, I'd like to leave time for the last topic/chapter..

- **Residue Theory**
  If you enjoy integrating complicated real functions, residue theory gives you another tool to evaluate them. I'd like to get into this topic at least enought for you to get a first look at this technique.
Topics from earlier courses for you to review

Basic trigonometric identities.

Polar coordinates.

The definition of the derivative of functions of one or more variables.

Power series