Abstract Algebra, Oct. 3, 2011

Exam on Oct 17

The exam on October 17 will be on the first five chapters of Judson.

In[2]:=
```
<< AbstractAlgebra`Master`
```

Generating Groups through replacement rules

Consider the set of all strings of letters of length zero or more from some set, A. This set of strings with concatenation is monoid (associative and has an identity, the empty string e). It isn't a group. Why?

Under some conditions, replacement rules can be applied to these strings to produce a group. For example, if $A = \{a\}$, and we make a rule that any string of seven consecutive a's be replaced with e. The result is a cyclic group of order 7. For example,

$$a^4 * a^6 = a^{10} = a^7 * a^3 = e * a^3 = a^3$$

In[6]:=
```
? GenerateGroupoidByRelations
```

GenerateGroupoidByRelations[gens_List,rels_List] generates a Groupoid
from elements in gens using the relations in rels to reduce products. Relations take
the form of Mathematica equations, with == separating the two sides. The option
SizeLimit may be used to control the maximum number of elements generated. ≫

In[13]:=
```
C7 = GenerateGroupoidByRelations[{a}, {a^7 == e}]
```

Out[13]= Groupoid$\left(\{a, a^2, a^3, a^4, e, a^5, a^6\}, -\text{Operation}-\right)$

In[14]:=
```
CyclicQ[C7]
```

Out[14]= True

The dihedral group D_6 can be generated with replacement rules.

In[9]:=
```
G = GenerateGroupoidByRelations[{r, s}, {s ** r == r^5 ** s, r^6 == e, s^2 == e}]
```

Out[9]= Groupoid$\left(\{r, s, e, r^2, r ** s, r^5 ** s, r^3, r^2 ** s, r^4, r^3 ** s, r^5, r^4 ** s\}, -\text{Operation}-\right)$

In[11]:=
```
Options[CayleyTable]
```

Out[11]= {CayleyForm → StandardForm, HeadingsColored → True, BodyColored → True, KeyForm → StandardForm, Mode → Visual, ShowBodyText → True, ShowKey → True, ShowName → True, ShowOperator → True, ShowSidesText → True, TheSet → {}, VarToUse → g, UseKey → Automatic, Version → 6, TooltipInfo → Automatic, AbstractAlgebra`Master`Private`EnlargeFactor → 1.5, ShowExtraCayleyInformation → True}

In[12]:=
```
CayleyTable[G, BodyColored → False, HeadingsColored → False]
```

Group properties	Element properties	Group Calculator

TheGroup

y

Out[12]=

x

*	r	s	e	r^2	r ** s	r^5 ** s	r^3	r^2 ** s	r^4	r^3 ** s	r^5	r^4 ** s
r	r^2	r ** s	r	r^3	r^2 ** s	s	r^4	r^3 ** s	r^5	r^4 ** s	e	r^5 ** s
s	r^5 ** s	e	s	r^4 ** s	r^5	r	r^3 ** s	r^4	r^2 ** s	r^3	r ** s	r^2
e	r	s	e	r^2	r ** s	r^5 ** s	r^3	r^2 ** s	r^4	r^3 ** s	r^5	r^4 ** s
r^2	r^3	r^2 ** s	r^2	r^4	r^3 ** s	r ** s	r^5	r^4 ** s	e	r^5 ** s	r	s
r ** s	s	r	r ** s	r^5 ** s	e	r^2	r^4 ** s	r^5	r^3 ** s	r^4	r^2 ** s	r^3
r^5 ** s	r^4 ** s	r^5	r^5 ** s	r^3 ** s	r^4	e	r^2 ** s	r^3	r ** s	r^2	s	r
r^3	r^4	r^3 ** s	r^3	r^5	r^4 ** s	r^2 ** s	e	r^5 ** s	r	s	r^2	r ** s
r^2 ** s	r ** s	r^2	r^2 ** s	s	r	r^3	r^5 ** s	e	r^4 ** s	r^5	r^3 ** s	r^4
r^4	r^5	r^4 ** s	r^4	e	r^5 ** s	r^3 ** s	r	s	r^2	r ** s	r^3	r^2 ** s
r^3 ** s	r^2 ** s	r^3	r^3 ** s	r ** s	r^2	r^4	s	r	r^5 ** s	e	r^4 ** s	r^5
r^5	e	r^5 ** s	r^5	r	s	r^4 ** s	r^2	r ** s	r^3	r^2 ** s	r^4	r^3 ** s
r^4 ** s	r^3 ** s	r^4	r^4 ** s	r^2 ** s	r^3	r^5	r ** s	r^2	s	r	r^5 ** s	e