Abstract Algebra

Assignment #1

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■ Due Sept. 26.

Instructions: Use only the definition of a group and the definition of exponentiation in groups in doing these proofs.

Definition of exponentiation in a group: If $g \in G$ and n is a positive integer,

$$g^0 = e$$
$$g^n = g^{n-1} * g$$
$$g^{-n} = (g^n)^{-1}$$

You may also assume that for all positive n, $g^n = g * g^{n-1}$. This isn't true from the definition, but can be proven by induction.

1. Prove that if n is a positive integer, then $g^{-n} = \left(g^{-1}\right)^n$. In other words, in computing g^{-n} you can invert g and then raise the result to the n^{th} power instead of raising g to the n^{th}power and then inverting the result. You get the same result either way.

2. Let a and b be elements in a group G. Prove that $a\, b^n\, a^{-1} = \left(a\, b\, a^{-1}\right)^n$ for all $n \geq 1$.

3. Let G be a finite group with identity e. Prove that if $a \in G$, there exists a positive integer m such that $a^m = e$.

■ Exploring Abstract Algebra with *Mathematica*

Home Page: http://www.central.edu/eaam/index.asp

You can download the AbstractAlgebra package from the site above, but you can also load it directly with this input:

```
Import["http://homepage.mac.com/klevasseur/Master.m"]
```

A few simple inputs.

```
Z[5]
```

Groupoid({0, 1, 2, 3, 4}, (#1 + #2) mod 5 &)

```
CayleyTable[Z[5]]
```

Group properties	Element properties	Group Calculator

Z[5]

y

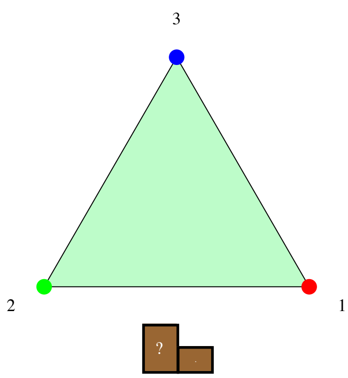
x

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

```
Dihedral[3]
```

Groupoid({1, Rot, Rot^2, Ref, Rot ** Ref, Rot^2 ** Ref}, −Operation−)

```
Dihedral[3, Mode → Visual]
```




```
SubgroupGenerated[Dihedral[3], Ref]
```

Groupoid({Ref, 1}, −Operation−)