92.421/521 Abstract Algebra

Fall 2011

Problem Set #3

Due October 31

Instructions: Do each problem on a separate sheet of paper. On each sheet, write your name And problem statement (it can be abbreviated). Include all logical steps/observations.

1. List the left and right cosets of the subgroups in each of the following.

   (a) A_n in S_n (b) $\langle 5 \rangle$ in $\mathbb{Z}_{20}$

2. Show that the additive group of real numbers has infinite index in the additive group of the complex numbers.

3. Suppose that $[G : H] = 2$. If a and b are not in H, show that $a\,b \in H$.

4. Prove that $\mathbb{Q}$ is not isomorphic to $\mathbb{Z}$.

5. Prove that $G \times H$ is isomorphic to $H \times G$.

6. Let H_1 and H_2 be subgroups of G_1 and G_2, respectively. Prove that $H_1 \times H_2$ is a subgroup of $G_1 \times G_2$.