92.421/521 Abstract Algebra

Fall 2011

Problem Set #4

Due November 14

Instructions: Do each problem on a separate sheet of paper. On each sheet, write your name And problem statement (it can be abbreviated). Include all logical steps/observations.

1. Which of the following subgroups of D_6 are normal?

   (a) $\langle s \rangle$ (b) $\langle r \rangle$ (c) $\langle r^2 \rangle$

   Use the representation of D_6 where $|s| = 2$, $|r| = 6$, and $s\,r = r^5\,s$.

2. If a group G has exactly one subgroup H of order k, prove that H is normal in G.

3. (a) Show that a homomorphism defined on a cyclic group is completely determined by its action on the generator of the group.

   (b) Describe all homomorphisms from

   (i) $\mathbb{Z}_{20}$ into $\mathbb{Z}_{18}$

   (i) $\mathbb{Z}$ into $\mathbb{Z}_{12}$.

We'll do this in class tonight

Define the centralizer of an element g in a group G to be the set

$$C(g) = \{x \in G \mid x\,g = g\,x\}$$

Show that $C(g)$ is a subgroup of G. If g generates a normal subgroup of G, prove that $C(g)$ is normal in G