92.421/521 Abstract Algebra

Fall 2011

Problem Set #5

Due November 21

Instructions:

Do the first two problems and any two of the last three problems.

Do each problem on a separate sheet of paper. On each sheet, write your name And problem statement (it can be abbreviated). Include all logical steps/observations.

1. Find all the subgroups of D_4. Which subgroups are normal? What are all the factor groups of D_4 up to isomorphism?

2. Let $\varphi : \mathbb{Z} \to \mathbb{Z}$ be given by $\varphi(n) = 7\,n$. Prove that φ is a group homomorphism. Find the kernel and the image of φ.

3. Let G be the additive group of real numbers. Let the action of $\theta \in G$ on the real plane $\mathbb{R}^2$ be given by rotating the plane counterclockwise about the origin through θ radians. Let P be a point on the plane other than the origin.
(a) Show that $\mathbb{R}^2$ is a G-set.
(b) Describe geometrically the orbit containing P.

(c) Find the group G_P.

4. Let $G = A_4$ and suppose that G acts on itself by conjugation; that is, $(g, h) \to g\,h\,g^{-1}$.

(a) Determine the conjugacy classes (orbits) of each element of G.

(b) Determine all of the stabilizer (isotropy) subgroups for each element of G_x.

5. Find the number of ways a six-sided die can be constructed if each side is marked differently with $1, 2, \ldots, 6$ dots.