92.421/521 Abstract Algebra

Fall 2011

Problem Set #6

Due December 10

Instructions:

Do each problem on a separate sheet of paper. On each sheet, write your name And problem statement (it can be abbreviated). Include all logical steps/observations.

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1. Let p be prime. Prove that $\mathbb{Z}_{(p)} = \left\{\frac{a}{b} : a, b \in \mathbb{Z} \text{ and } \gcd(b, p) = 1\right\}$ is a subring of the field of rational numbers. Explain why $\mathbb{Z}_{(p)}$ is not a field.

2. If R is a commutative ring with identity that isn't an integral domain, then factorization of polynomials over R isn't unique. Illustrate this fact by showing that $x^2 + x + 8$ has two different factorizations over $\mathbb{Z}_{10}[x]$

3. List all of the polynomials of degree 4 or less in $\mathbb{Z}_2[x]$ and factor them completely From among these polynomials, identify all of the irreducible polynomials of degree 2, 3 and 4 in $\mathbb{Z}_2[x]$.

Some suggested exercises to work on, but not turn in.

Chapter 15/16
1, 5, 8

Chapter 16/17
2, 3, 19

Chapter 20/21
1d, 2 a b c h , 5