■ Ring stuff

Since the focus of the talk was on the labs for group theory, we do not illustrate any of the ring labs here. If interested, on the homepage of Al Hibbard (http://www.central.edu/homepages/hibbarda/hibbard.html) you can find a reference to the Exploring Abstract Algebra with Mathematica page, on which you can find illustrations of ring labs.

What follows gives just a brief look at what can be done on the ring side.

```
R=ZR[6]
Ringoid[{0, 1, 2, 3, 4, 5}, Mod[#1 + #2, 6] & , M
CayleyTables[R,Mode -> Visual];
```

Addition Table

+	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

Multiplication Table

.	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	0	2
3	0	3	0	3	0
4	0	4	2	0	4
5	0	5	4	3	2

```
P = PolynomialRing[ZR[2]]

ExtRing[PolyRing,
 Ringoid[{0, 1}, Mod[#1 + #2, 2] & ,
 Mod[#1 #2, 2] & ], {},
 Head[#1] == Poly &&
 Apply[And, (MemberQ[Ringoid[{0, 1},
 Mod[#1 + #2, 2] & , Mod[#1 #2, 2] & ][
 [1]], #1] & ) /@ #1] & ]

m = Poly[1, 0, 1]

1 + X^2

Q = QuotientRing[m, ZR[2]];
RingDomain[Q]

{0, X, 1, 1 + X}

DivisionAlgorithm[Poly[0, 1, 1], m, P,
 Mode -> Textual]
```

a (X) = b (X) q (X) + r (X) where

a (X) = $X + X^2$

b (X) = $1 + X^2$

q (X) = 1 and

r (X) = 1 + X

Notice that either r (X) = 0 or deg r < deg b

{1, 1 + X}

```
RingDomain[Q = QuotientRing[Poly[1, 0, 1],
 ZR[3]]]
DisplayPowers[Q, Poly[0, 1], 8]
```

```
{0, X, 2 X, 1, 1 + X, 1 + 2 X, 2,
 2 + X, 2 + 2 X}
```

```
X^1 =  X
X^2 =  2
X^3 =  2 X
X^4 =  1
X^5 =  X
X^6 =  2
X^7 =  2 X
X^8 =  1
```

```
MultiplicationTable[BooleanRing[
 {1, 2}], Mode -> Visual]
```

Multiplication Table

Key:	.	g1	g2	g3	g4
{}<-	g1	g1	g1	g1	g1
{2}<-	g2	g1	g2	g1	g2
{1}<-	g3	g1	g1	g3	g3
{1,2}<-	g4	g1	g2	g3	g4

```
{{{}, {}, {}, {}}, {{}, {2}, {}, {2}},
 {{}, {}, {1}, {1}},
 {{}, {2}, {1}, {1, 2}}}
```

**CayleyTable[MGroupoid[ZR[15]],
 Mode -> Visual]**

*	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
2	0	2	4	6	8	10	12	14	1	3	5	7	9	11	13
3	0	3	6	9	12	0	3	6	9	12	0	3	6	9	12
4	0	4	8	12	1	5	9	13	2	6	10	14	3	7	11
5	0	5	10	0	5	10	0	5	10	0	5	10	0	5	10
6	0	6	12	3	9	0	6	12	3	9	0	6	12	3	9
7	0	7	14	6	13	5	12	4	11	3	10	2	9	1	8
8	0	8	1	9	2	10	3	11	4	12	5	13	6	14	7
9	0	9	3	12	6	0	9	3	12	6	0	9	3	12	6
10	0	10	5	0	10	5	0	10	5	0	10	5	0	10	5
11	0	11	7	3	14	10	6	2	13	9	5	1	12	8	4
12	0	12	9	6	3	0	12	9	6	3	0	12	9	6	3
13	0	13	11	9	7	5	3	1	14	12	10	8	6	4	2
14	0	14	13	12	11	10	9	8	7	6	5	4	3	2	1

BooleanRing[2]

```
Ringoid[{{}, {2}, {1}, {1, 2}},
 Union[Complement[#1, #2],
 Complement[#2, #1]] &,
 Intersection[#1, #2] & ]
```

IntegralDomainQ[BooleanRing[2]]

```
False
```

```
M = Matrices[ZR[5], 2]
```

```
ExtRing[MatRing,
 Ringoid[{0, 1, 2, 3, 4},
 Mod[#1 + #2, 5] &, Mod[#1 #2, 5] & ],
 2, Apply[And, (MemberQ[Ringoid[{0, 1,
 2, 3, 4}, Mod[#1 + #2, 5] &,
 Mod[#1 #2, 5] & ][[1]], #1] & ) /@
 Flatten[#1, 1]] &&
 (Take[#1, 2] & )[Dimensions[#1]] ==
 {2, 2} & ]
```

```
BaseRing[M]
```

```
Ringoid[{0, 1, 2, 3, 4},
 Mod[#1 + #2, 5] &, Mod[#1 #2, 5] & ]
```

```
M = Matrices[ZR[11], 2];
(mA = {{4, 10}, {6, 10}}) // MatrixForm
RDet[mA, M]
```

```
4  10
6  10
```

```
2
```

```
M = Matrices[ZR[8], 2];
{W = {{3, 4}, {7, 5}},
 "Determinant = "<> ToString[RDet
 [W, M]]} // TableForm
```

```
3  7
4  5

Determinant = 3
```

```
RMatrixInverse[W, M]
{{7, 4}, {3, 1}}
Multiplication[M][W, %]
{{1, 0}, {0, 1}}
RMatrixInverse[{{6, 1}, {0, 4}}, M]
Matrix argument is not a unit
{{0, 0}, {0, 0}}
```