

Discrete Structures II. Fall of 2008.
For the Final
14 December 2008

Instructions:

The final may include questions on all topics discussed in class in reasonable detail.

Combinatorics

Skills: Counting permutations and subsets, Binomial formula, Inclusion-Exclusion, Counting gift distributions (another name: combinations with repetitions)

Arithmetic

Skills: Euclidean Algorithm, Modular Arithmetic, Solving simple equations in modular arithmetic, Fast Modular Exponentiation, Euler's φ function, Chinese Remainder Theorem.

Typical Arithmetic Problems:

Problem 1

The least common multiple (LCM) of natural numbers n and m is the smallest natural number divisible by both n and m . Find $LCM(3399, 2729)$.

Problem 2

Solve $18x \equiv 1 \pmod{2003}$.

Problem 3

Compute $7^{160} \pmod{16}$.

Problem 4

Solve the system of congruences.

$$\begin{cases} x \equiv 3 \pmod{15} \\ x \equiv 5 \pmod{11} \\ x \equiv 2 \pmod{7} \end{cases}$$

Graph Theory

Topics: Data structures for storing graphs, Euler walks (Euler's theorem with proofs), Hamilton walks, Prüfer code of a labeled tree, planar code of a tree ("fence code"), Notion of graph minor, Kuratowski's theorem (without proof), Notion of graph rigidity, Euler's theorem ($V - E + F = \chi$, where χ is 2 for a spherical map), Various counting exercises, Greedy algorithm for finding the cheapest spanning tree, Traveling salesman problem and its reduction to the cheapest spanning tree problem (in the case where the triangle inequality holds). The notion of error-correcting code. The cube code. The Fano code.

Typical problems: problems assigned in the last two midterms and the practice problems for the second midterm.