

MATH-92.564 Numerical Algebra / Computational Mathematics II

by Konstantin Rybnikov

Syllabus

Textbooks:

- 1) Chee Yap, Fundamental Problems of Algorithmic Algebra, any edition
- 2) Timothy A. Davis, Direct Methods for Sparse Linear Systems, any edition.

Time and place:

Wednesday 6–9 PM, North Campus

The course will cover some important methods and algorithms used for solving problems involving functions of many variables and large systems of linear equations. The course will have two major conceptual parts, exact computing and computing with floating point arithmetic. In general, exact computing is much slower than computing with floating point arithmetic. Most problems in classical areas of applications of mathematics, especially in physics, data processing, and engineering, etc. where exact computing is not warranted because the input data are physical measurements or just because exact computing is infeasible. Yet, there are equally important problems in the real world where we have to compute exactly as in, e.g., cryptography and error-correcting codes.

The course will begin from a review of linear algebra (vectors, dot products, matrices) and complex numbers. In the area of floating point computations we will cover accumulation of errors, numerical stability, algorithms for Gauss-Jordan elimination, discrete Fourier transform, methods for sparse systems, fast matrix multiplication, and some other topics. On the engineering side we will cover problems of storage, properties of floating point arithmetic, and good practices of implementation of floating point arithmetic computations.

On the side of exact computing we will cover fast integer multiplication, LLL-reduction for lattices, Korkine-Zolotareff reduction for quadratic forms, Hermit and Smith normal forms for integral matrices, and some fundamental inequalities. The reduction methods are important in error correcting codes, cryptography, and other areas. If time permits we will also cover some basics of quantifier elimination, an important part of computational real algebraic geometry.

There are also areas of applied mathematics / computer science where floating point and exact computing paradigms are interwoven. Such are computational geometry and mesh generation of finite elements method. For example, although the finite elements method is run in floating point arithmetic, the combinatorial and geometric structure of a mesh depends on accurate evaluation of many determinants. We will discuss Sylvester's method for exact determinant evaluation.

Grading: Homework (20%), midterm (20%), class participation (10%), project (20%), take home final (30%). The midterm can be substituted with another project.