1) From p. 156 of the textbook. “Expanding (3.3.1) to its composite form in the same way as (3.1.6) was extended to (3.1.8), we obtain for \( g \in C^{2m+2}[0, N] \) ...”

Do the expanding and demonstrate that (3.3.1) is, indeed, equivalent to the formula given in the sentence, partially quoted above.

2) Let \( b \in \mathbb{R}^n \) and let \( A \) be an \( n \times n \) matrix of full rank. Suppose that each entry \( a \) of \( A \) and \( b \) is represented in the computer in the form \( a = (-1)^s \cdot m \cdot 2^l \), where \( s \in \{0, 1\} \), \( m \in \mathbb{N} \) (\( \mathbb{N} = \{0, 1, 2, \ldots\} \)), and \( l \in \mathbb{Z} \).

The binary size of \( a \) is defined as \( 1 + \max\{1, \lceil \log_2 m \rceil\} + \max\{1, \lceil \log_2 |l| \rceil\} \) (this is just the minimal number of bits required to store \( a \) in the above form). The binary size of mantissa, \( m \), is defined as \( \max\{1, \lceil \log_2 m \rceil\} \).

You are solving the system \( Ax = b \) by standard Gaussian elimination and your computations need to be as exact as possible. Since during the computation the binary size of numbers involved grows, you have to make reservations for the number of bits available for the entries of the solution \( x \) and the numbers appearing during the intermediate calculations.

Suppose the binary size of each entry of \( A|b \) is at most \( S \), while the binary size of mantissa of each entry is at most \( M \) \((M < S)\). Suppose that:
1) additions and subtractions are exact (figure out what it takes);
2) multiplications are exact (figure out what it takes);
3) when a division is performed, the operations on the logarithm and sign are exact, but the number of bits allocated for the mantissa of the ratio is sum of the numbers of bits allocated for the mantissas of the numerator and the denominator.

What is the largest number of bits that you need to allocate for a number appearing (in the final answer or in any of the intermediate calculations) during this computation?

Your answer need not be absolutely exact, that is you are allowed and encouraged to make simplifying assumptions, but it should be asymptotically exact, i.e.

\[
\lim_{{(n,S,M) \to \infty}} \frac{\text{Exact Answer}}{\text{Your answer}} = 1
\]