92.132-202 Calculus II
exam 1
6/18/07

All problems are worth 10 points unless otherwise noted. Show all analytic work.

1. (15 points) Prove that the area enclosed in the circle $x^2 + y^2 = R^2$ is πR^2.

2. $f(x) = 3x^2 + 1,\ 0 \le x \le 3.$ Use the right end point and the midpoint rule for each of 3 rectangles to approximate the area bounded by this function and the x axis.

3. Velocity $v(t) = 2x - 4,\ 0 \le x \le 5.$ Find the displacement and the distance traveled.

4. Evaluate $\int_4^{12} \sqrt{(2x+1)}dx$.

5. Evaluate $\int 3x\sqrt{(x-1)}dx$.

6. Evaluate $\int x^2 \sin(3x)dx$

7. Using PFD, evaluate $\int \frac{2x+1}{x^2+5x+6}dx$

8. By a limiting sum, evaluate $\int_0^3 (2x+1)dx$

9. Evaluate each of the following:

a) (5 points) $\int_0^2 e^{-2x}dx$

b) (5 points) $\int_0^{\pi/2} \sin^3 x \cos x dx$

c) (5 points) Evaluate $\frac{d}{dx}\left(\int_{2x}^{1} t^2 \ln t dt\right)$

92.132 Calculus II
exam 2
7/20/07

All problems are worth 10 points unless otherwise noted. Show all analytic work.

1. Determine whether the improper integral $\int_1^\infty \frac{1}{x^3} dx$ is convergent or not. If convergent, evaluate it. Graph the bounded region.

2. Find the area bounded by $y = 20 - x^2$ and $y = x^2 - 12$.

3. Find the area bounded by $x = y^2$ and $x = y + 2$.

4. Find the volume of the solid obtained by rotating the line $y = \frac{2}{5}x$ about the x axis with $0 \le x \le 5$.

5. (15 points) Use shells to find the volume of the solid obtained by rotating the region bounded by $y = x \ln x$, the x axis, $1 \le x \le 4$ about the y axis.

6. (15 points) Prove that the circumference of a circle is $2\pi R$. Hint: Assume that $x^2 + y^2 = R^2$.

7. Find the center of mass for the region bounded by $y = x^3$, the x axis and $0 \le x \le 3$.

8. (20 points) Determine whether or not the following series are convergent or not. If convergent, find the sum analytically.

a) $4 + \frac{4}{5} + \frac{4}{25} + \frac{4}{125} + \ldots$

b) $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{2}{7}\right)^n$

c) $\sum_{n=1}^{\infty} \frac{3}{n^2 + 3n + 2}$

d) $\sum_{n=1}^{\infty} \frac{n}{2n+1}$

92.132 Calculus II
exam 3 part II
8/15/07
Name:_____

All problems are worth 5 points. Show all analytic work.

1. By a limiting sum, evaluate $\int_0^4 (3x-2)dx$

2. Evaluate $\int x\cos(4x)dx$

3. Evaluate $\int_0^{\pi/4} \sin^2(2x)\cos^3(2x)dx$

4. $f(x) = -3x^2 + 12$, $0 \le x \le 2$. Use the midpoint rule for each of 4 rectangles to approximate the area bounded by this function and the x axis.

5. Find the area bounded by $y = x^2 - 3$ and $y = 3x + 1$.

6. Use disks to find the volume of the solid generated when rotating the region bounded by $y = 1 - x^2$ and the x axis about the line $y = 0$.

7. Given the geometric series $\sum_{n=1}^{\infty} (-1)^n \left(\frac{2}{5}\right)^n$, show that it converges and find its sum.

8. Find the volume of the solid generated when rotating the region bounded by $x = 4 - y^2$ and the y axis about the line $x = -1$.

9. Find the interval of convergence for $\sum_{n=1}^{\infty} \frac{(x-1)^n}{3^n}$.

10. Using the alternating series test, show that $\sum_{n=1}^{\infty} \frac{(-1)^n}{n5^n}$ is convergent. How many terms are needed so that $|error| < .0001$ and what is the sum?

92.132 Calculus II
exam 1 5/30/13

Name:_____

Show all analytic work and simplify results. **An answer without work is not acceptable.** **Use 2 place accuracy wherever appropriate. There are a total of 106 possible points.**

1. (10 pts) $f(x)=16x-x^2$ and $0 \le x \le 16$. Use the midpoint rule with 4 rectangles to approximate the area bounded by f(x) and the x axis. Be sure to set up a table showing all computed values and graph the function with the approximating rectangles.

2. (8 pts) $f(x)=3x^2+\sin(2x)$. Evaluate $\int f(x)dx$ when F(0)=4.5.

3. (8 pts) $f(x)=(6x^2+2x+1)^{\frac{3}{2}}(6x+1)$. Evaluate $\int_{x=0}^{x=1} f(x)dx$.

4. (10 pts) Evaluate $\int x^3\sqrt{(x^2+2)}dx$

5. (8 pts) $f(x)=\frac{d}{dx}\int_{\sqrt{x}}^{1}(\cos(t^2)dt$. Evaluate this problem using the Fundamental Theorem of Calculus, part 1. Be sure to show all work.

6. (10 pts) Find the area bounded by $y=25-x^2$ and $y=-2x+1$. Also graph the bounded region. Hint: Integrate with respect to x.

7. (10 pts) The region in the first quadrant, bounded by $y = \sqrt{x}$ and x = 4, is revolved about the line x = -1. Find the volume generated using washers. Graph the bounded region. Hint: Integrate with respect to y and simplify results.

8. (8 pts) Use cylindrical shells to find the volume generated when the region bounded by $f(x) = 3x + \sqrt{x}$ and the x axis, $0 \le x \le 4$, is revolved about the y axis. Graph the bounded region.

9. (8 pts) Using disks, find the volume of the solid generated when $y = \sqrt{R^2 - x^2}$, $-R \le x \le R$, is revolved about the x axis. Graph the bounded region.

10. (10 pts) Given that $f(x)=\frac{4}{3}x^{\frac{3}{2}}-2$, $1\le x\le 3$, find the arc length and simplify results.

11. (10 pts) Find the surface area when the region bounded by $y=\sqrt{25-x^2}$ and the x axis, $0\le x\le 5$, is revolved about the x axis. Graph the bounded region.

12. (6 pts) We are unable at this point to integrate $\int_0^2 e^{-x^2}dx$. However we are able to approximate the area. Use left end points with 2 rectangles and set up a table showing computed values. Graph the function and approximating rectangles.

Extra work

92.132 Calculus II
exam 2 6/13/13

Name:_____

Show all analytic work and simplify results. **An answer without work is not acceptable.** **Use 2 place accuracy wherever appropriate.**

1. (10 pts) A force of 40N is required to hold a spring that has been stretched from its natural length of 10 cm to 15 cm. How much work is done in stretching the spring from 15 cm to 18 cm?

2. (10 pts) Find the center of mass of the region bounded by the line y=x and the parabola $y = x^2$. Also graph the bounded region.

3. (15 pts) Find the center of mass of the region bounded by y=sin(x) and the x axis, $0 \le x \le \pi$. (This problem requires integration by parts and a trig integral. Graph the bounded region.

4. a) (5 pts) Evaluate $\int \tan^6(x)\sec^4(x)dx$

b) (5 pts) Evaluate $\int \cos^5(3x)\sin^2(3x)dx$

5. (10 pts) $y = \frac{3}{5}\sqrt{25 - x^2}$ with $0 \le x \le 5$ represents the part of an ellipse in the first quadrant. Find the area of this part of the ellipse. Graph the bounded region.

6. (10 pts) Evaluate by trig substitution $\int \frac{1}{4 + 9x^2} dx$

7. (10 pts) Evaluate $\int \frac{x-9}{(x+5)(x-2)}dx$

8. (10 pts) Use the Trapezoidal rule and then Simpson's rule to approximate $\int_1^5 \frac{1}{x}dx$ with 4 subdivisions. Set up a table to show all computed values. Also graph the bounded region and approximating trapezoids. Use **3 place accuracy** after the decimal point.

9. a) (5 pts) Evaluate the improper integral $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$ and graph the bounded region.

b) (5 pts) Evaluate the integral or show that it is divergent $\int_{1}^{\infty} \frac{1}{(2x+1)^3} dx$.

10. Determine whether the series is convergent or divergent. If it is convergent, find its sum.

a) (5 pts) $\sum_{n=1}^{\infty} \frac{(-2)^{n-1}}{5^{n-1}}$

b) (5 pts) $\sum_{n=2}^{\infty} \frac{n^2}{n^2-1}$

Extra work

92.132 Calculus II
exam 3 6/26/13

Name:_____

Show all analytic work and simplify results. **An answer without work is not acceptable.** **Use 4 place accuracy unless otherwise requested.**

1. (10 pts) Evaluate $\int x^2 e^{-2x} dx$ using integration by parts.

2. (10 pts) Find the volume of the solid obtained by rotating the region bounded by $y = x^3$, $y=8$, and $x=0$ about the y axis. Graph the bounded region.

3. (10 pts) Use trigonometric substitution to evaluate $\int x^3\sqrt{9-x^2}\,dx$.

4. (10 pts) Find the volume of the solid obtained by rotating the region bounded by $y = x$ and $y = x^2$ about the line $y=2$.

5. (10 pts) Given the integral $\int_1^5 \frac{\cos(x)}{x} dx$. Use 4 place accuracy for all computation.

a) Use Simpson's Rule to approximate the integral with n=4. Set up a table of values used.

b) Use the Midpoint Rule to approximate this integral with n=4. Set up the table of values used.

6. (10 pts) Use series to approximate $\int_0^1 x\cos(x^3) dx$ with a max absolute error < 0.0001.

7. a) (7 pts) Show that the alternating series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^5}$ converges. Then find the partial sum so that the maximum absolute error < 0.0005.

b) (8 pts) Find the interval of convergence for $\sum_{n=1}^{\infty} (-3)^n \frac{x^n}{\sqrt{n}}$ and check end points.

8. a) (5 pts) Find the Taylor polynomial of order 4 generated by $f(x) = \sin(x)$ at $x = \frac{\pi}{4}$. Also graph $\sin(x)$ and your P_4 approximation on the interval $-3 \le x \le 3$ and indicate the x and y scales on your graph.

b) (10 pts) Use substitution to develop a polynomial of order 6, i.e. a P_6 approximation for $f(x) = x^2 e^{-\frac{x}{2}}$. Then determine for what values of *x* we can replace the P_6 approximation with a maximum absolute error < 0.001.

9. a) (8 pts) Find the area enclosed by one loop of the rose $R = \cos(2\theta)$. Also graph the rose.

b) (7 pts) Given that $R = 8\cos(\theta)$, find the arc length for one complete loop of this curve.

Extra work