Mechanical Vibrations

Chapter 4

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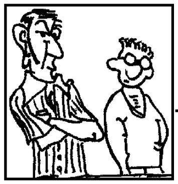


Impulse Excitation

Impulsive excitations are generally considered to be a large magnitude force that acts over a very short duration time

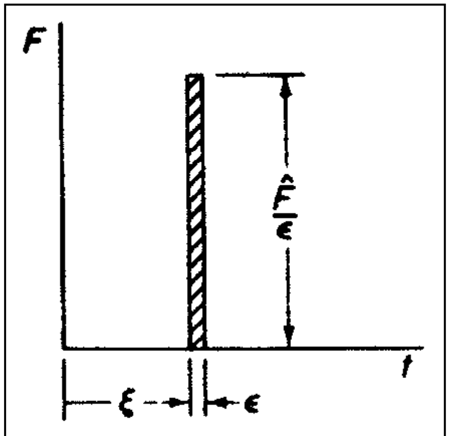


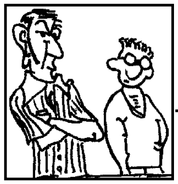
FIGURE 4.1.1.

The time integral of the force is

$$\hat{F} = \int F(t)dt \qquad (4.1.1)$$

When the force is equal to unity and the time approaches zero then the unit impulse exists and the delta function has the property of

$$\delta(t-\xi) = 0 \quad \text{for} \quad t \neq \xi$$



Impulse Excitation

Integrated over all time, the delta function is

$$\int_0^\infty \delta(t-\xi)\,dt = 1 \qquad 0<\xi<\infty \tag{4.1.2}$$

If this function is multiplied times any forcing function then the product will result in only one value at t=ξ and zero elsewhere

$$\int_0^\infty f(t)\,\delta(t-\xi)\,dt = f(\xi) \qquad 0<\xi<\infty \tag{4.1.3}$$

Impulse Excitation

Considering impact-momentum on the system, a sudden change in velocity is equal to the actual applied input divided by the force.

Recall that the free response due to initial conditions is given by

$$x = \frac{\dot{x}(0)}{\omega_n} \sin \omega_n t + x(0) \cos \omega_n t$$

Impulse Excitation

Then the velocity initial condition yields

$$x = \frac{\hat{F}}{m\omega_n}\sin\omega_n t = \hat{F}h(t) \qquad (4.1.4)$$

and it can be seen that the solution includes h(t)

$$h(t) = \frac{1}{m\omega_n}\sin\omega_n t \qquad (4.1.5)$$

Impulse Excitation

When damping is considered in the solution, the free response is given as (x(0)=0)

$$x = \frac{\dot{x}(0)e^{-\zeta\omega_n t}}{\omega_n\sqrt{1-\zeta^2}}\sin\omega_n\sqrt{1-\zeta^2}\,t = \frac{\dot{x}(0)e^{-\sigma t}}{\omega_d}\sin\omega_d t$$

which can be written as

$$x = \frac{\hat{F}}{m\omega_n\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\omega_n\sqrt{1-\zeta^2}\,t \qquad (4.16)$$

or as

$$x = \frac{\hat{F}}{m\omega_d}e^{-\sigma t}\sin\omega_d t = \hat{F}h(t)$$

Arbitrary Excitation

Using the unit impulse response function, the response due to arbitrary loadings can be determined.

The arbitrary force is considered to be a series of impulses

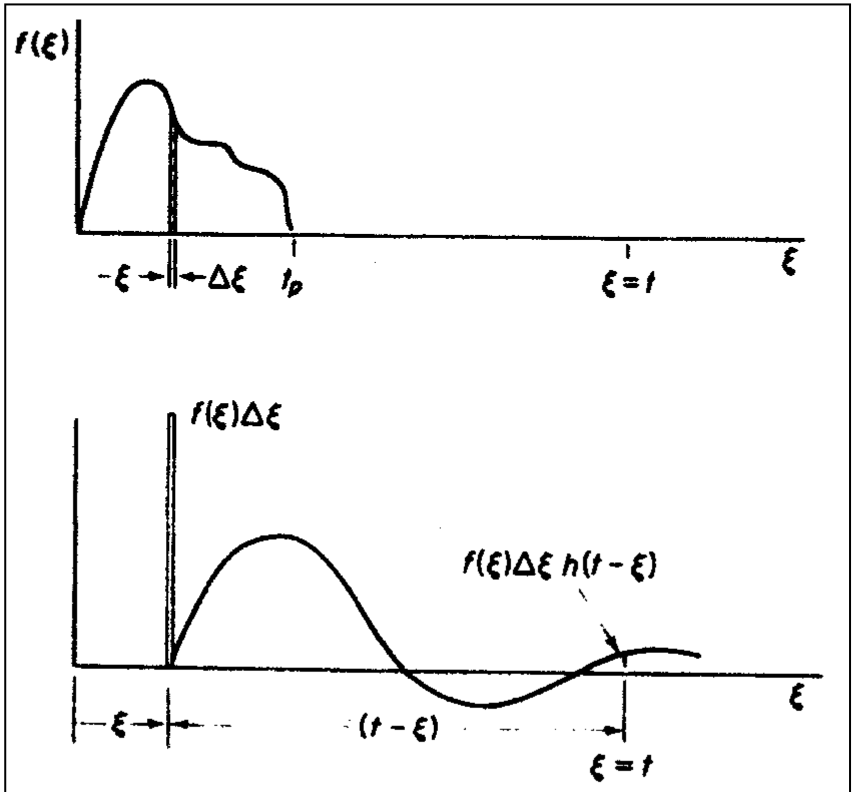
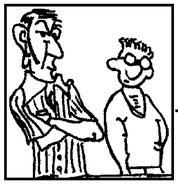


FIGURE 4.2.1.



Arbitrary Excitation

Since the system is considered linear, then the superposition of the responses of each individual impulse can be obtained through numerical integration

$$x(t) = \int_0^t f(\xi)h(t-\xi)d\xi \qquad (4.2.1)$$

This is called the superposition integral. But it is also referred to as the

Convolution Integral
or
Duhammel's Integral

Step Excitation

Determine the indamped response due to a step.

For the undamped system,

$$h(t) = \frac{1}{m\omega_n} \sin \omega_n t$$

which is substituted into (4.2.1) to give

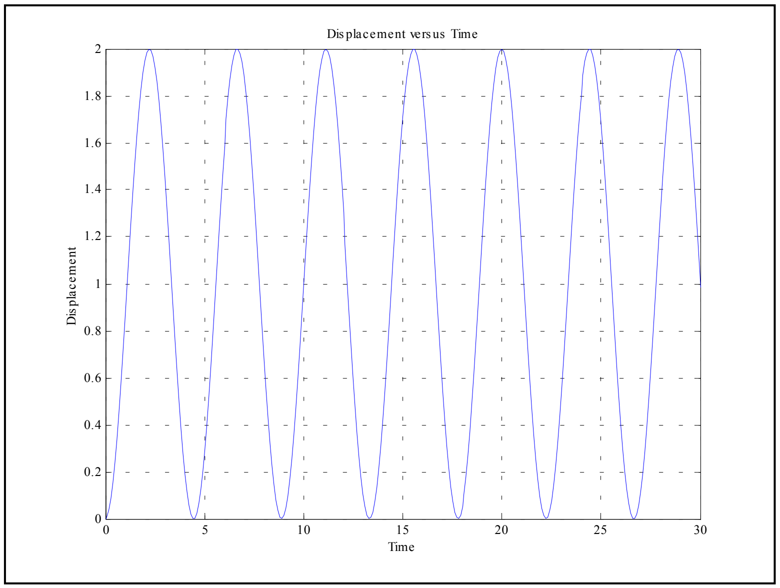
$$x(t) = \frac{F_0}{m\omega_n} \int_0^t \sin \omega_n (t - \xi) d\xi \qquad (4.2.2)$$

$$x(t) = \frac{F_0}{k} (1 - \cos \omega_n t)$$

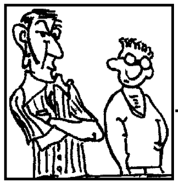
Step Excitation

This implies that the peak response is twice the statical displacement

$$x(t) = \frac{F_0}{k}\left(1 - \cos\omega_n t\right) \tag{4.2.2}$$



Note: Force selected such that F/k ratio is 1.0



Step Excitation

When damping is included in the equation, then

$$h(t) = \frac{e^{-\zeta\omega_n t}}{m\omega_n\sqrt{1-\zeta^2}} \sin \omega_n \sqrt{1-\zeta^2}\, t \tag{4.2.2}$$

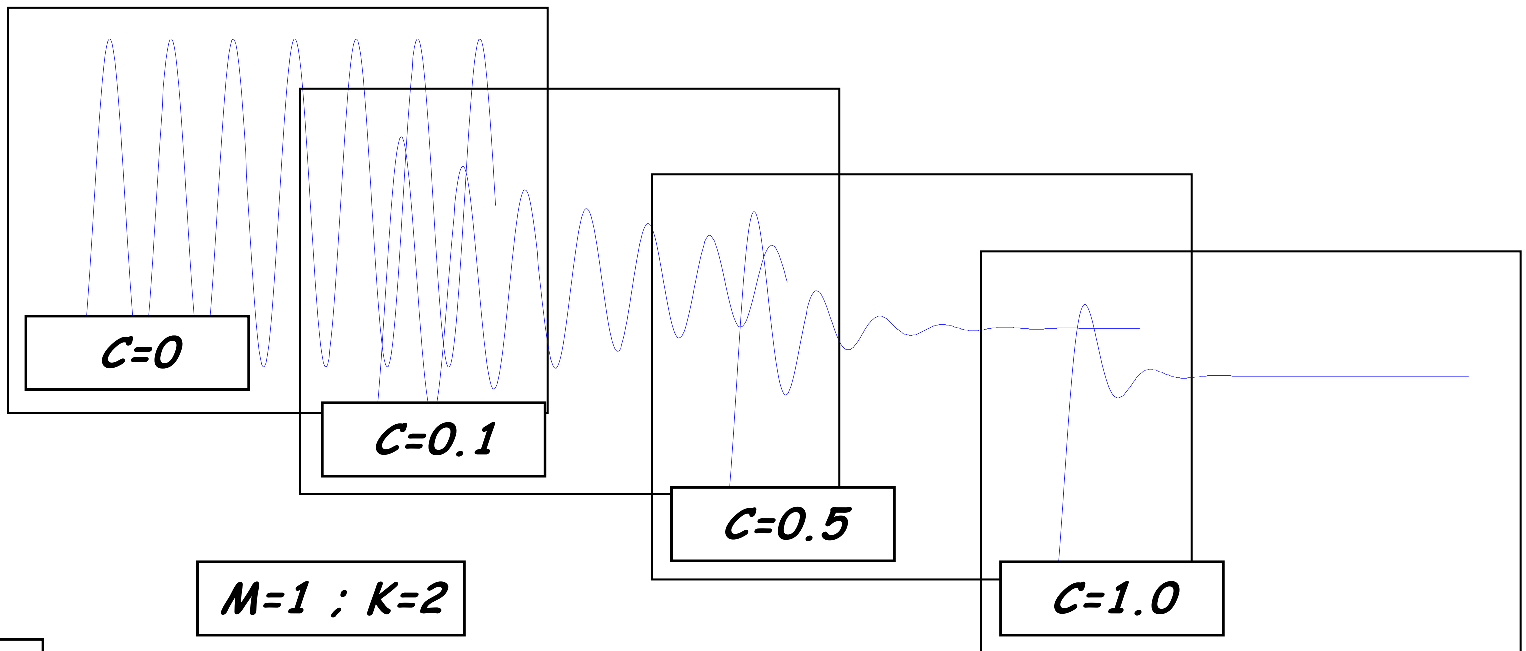
and

$$x(t) = \frac{F_0}{k}\left[1 - \frac{e^{-\zeta\omega_n t}}{m\omega_n\sqrt{1-\zeta^2}} \cos \omega_n \sqrt{1-\zeta^2}\, t - \psi\right] \tag{4.2.3}$$

Step Excitation

This can be simplified as

$$h(t) = \frac{e^{-\sigma t}}{m\omega_d} \sin \omega_d t \qquad\qquad x(t) = \frac{F_0}{k}\left[1 - \frac{e^{-\sigma t}}{m\omega_d} \cos \omega_d t - \psi\right]$$



Base Excitation

For base excitation,

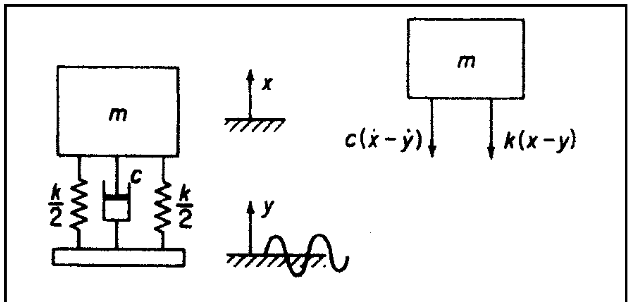


FIGURE 3.5.1. System excited by motion of support point.

$$m\ddot{x} = -k(x-y) - c(\dot{x}-\dot{y}) \qquad (3.5.1)$$

$$z = x - y \qquad (3.5.2)$$

the equation of motion is expressed as z=x-y and will result in

$$\ddot{z} + 2\zeta\omega_n\dot{z} + {\omega_n}^2 z = -\ddot{y} \qquad (4.2.4)$$

Notice that the F/m term is replaced by the negative of the base acceleration (ie, F=ma)

Base Excitation

For and undamped system initially at rest, the solution for the relative displacement is

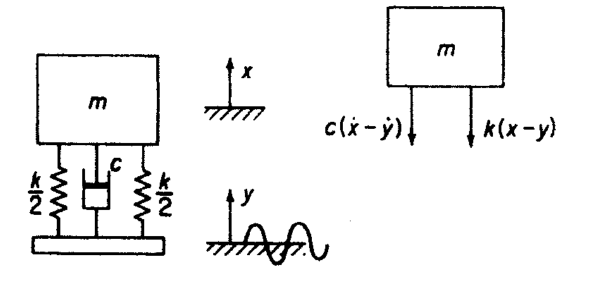
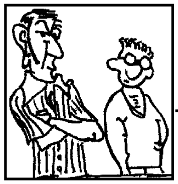


FIGURE 3.5.1. System excited by motion of support point.

$$z(t) = -\frac{1}{\omega_n}\int_0^t \ddot{y}(\xi)\sin\omega_n(t-\xi)\,d\xi \qquad (4.2.5)$$



Ramp Excitation and Rise Time

This solution must always be considered in two parts - the time less than and greater than t_1

The ramp of the force is

$$f(t) = F_0\left(\frac{t}{t_1}\right) \tag{4.4.1}$$

and h(t) for the convolution integral is

$$h(t) = \frac{1}{m\omega_n}\sin\omega_n t = \frac{\omega_n}{k}\sin\omega_n t \tag{4.4.1}$$

Ramp Excitation and Rise Time

The response for the first part of the ramp is

$$x(t) = \frac{\omega_n}{k}\int_0^t F_0 \frac{\xi}{t_1}\sin\omega_n(t-\xi)\,d\xi$$

$$= \frac{F_0}{k}\left(\frac{t}{t_1} - \frac{\sin\omega_n t}{\omega_n t_1}\right) \qquad t < t_1 \tag{4.4.2}$$

and the response of the step portion after t_1 is

$$x(t) = -\frac{F_0}{k}\left(\frac{t-t_1}{t_1} - \frac{\sin\omega_n(t-t_1)}{\omega_n t_1}\right) \qquad t > t_1$$

Ramp Excitation and Rise Time

The superposition of the two pieces of the solution gives the total response due to the force as

$$x(t) = \frac{F_0}{k}\left(1 - \frac{\sin\omega_n t}{\omega_n t_1} + \frac{\sin\omega_n (t - t_1)}{\omega_n t_1}\right) \quad t > t_1 \qquad (4.4.3)$$

Rectangular Pulse

The rectangular pulse is the sum of two different step functions - one positive and one negative shifted in time

Step Up

$$\frac{kx(t)}{F_0} = \left(1 - \cos\omega_n t\right) \quad t < t_1 \tag{4.4.4}$$

Step down

$$\frac{kx(t)}{F_0} = -\left(1 - \cos\omega_n\left(t - t_1\right)\right) \quad t < t_1 \tag{4.4.5}$$

Combined

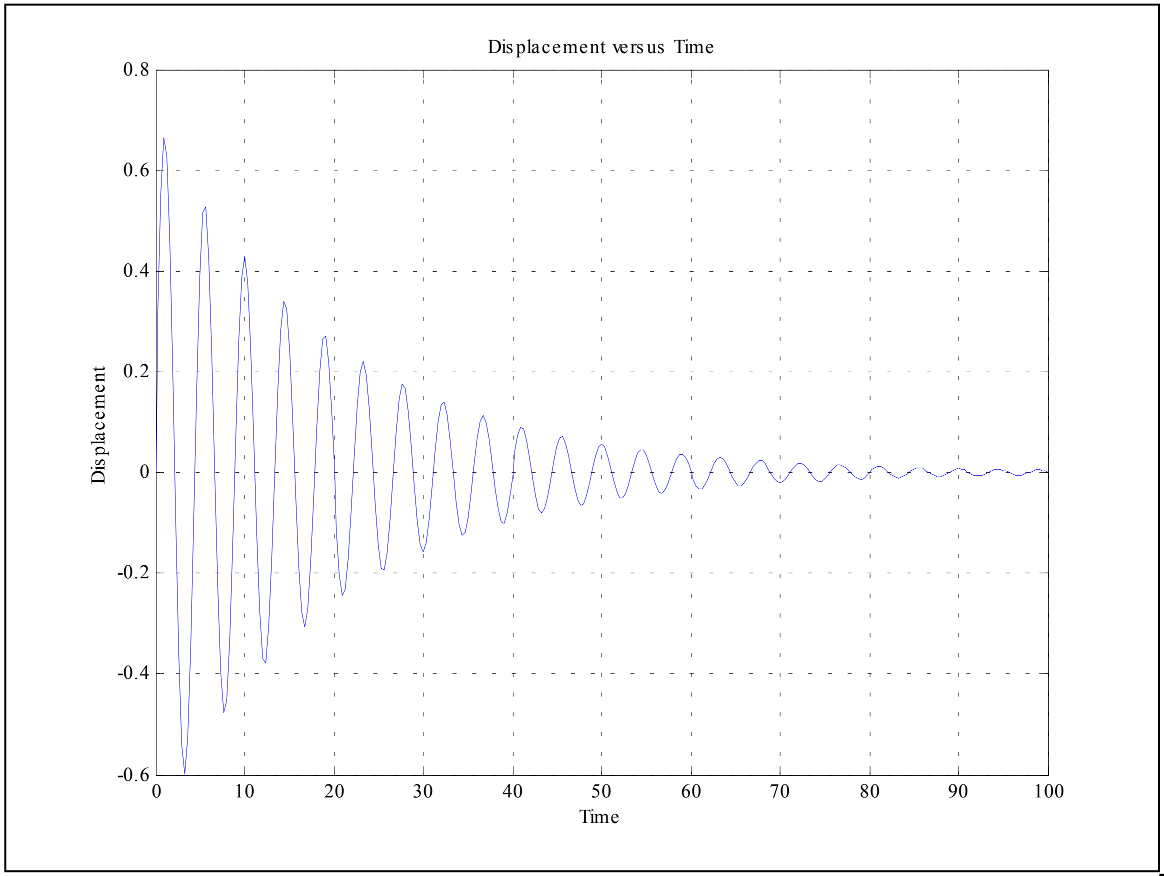
$$\frac{kx(t)}{F_0} = \left(1 - \cos\omega_n\left(t\right)\right) - \left(1 - \cos\omega_n\left(t - t_1\right)\right) \quad t < t_1 \tag{4.4.6}$$

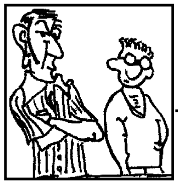
$$\frac{kx(t)}{F_0} = \left(-\cos\omega_n\left(t\right)\right) + \left(\cos\omega_n\left(t - t_1\right)\right) \quad t < t_1$$

MATLAB Examples - VTB3_1

VIBRATION TOOLBOX EXAMPLE 3_1

```
>> m=1;   c=.1;  k=2;   tf=100;   F0=1;
>> vtb3_1(m,c,k,F0,tf)
>>
```

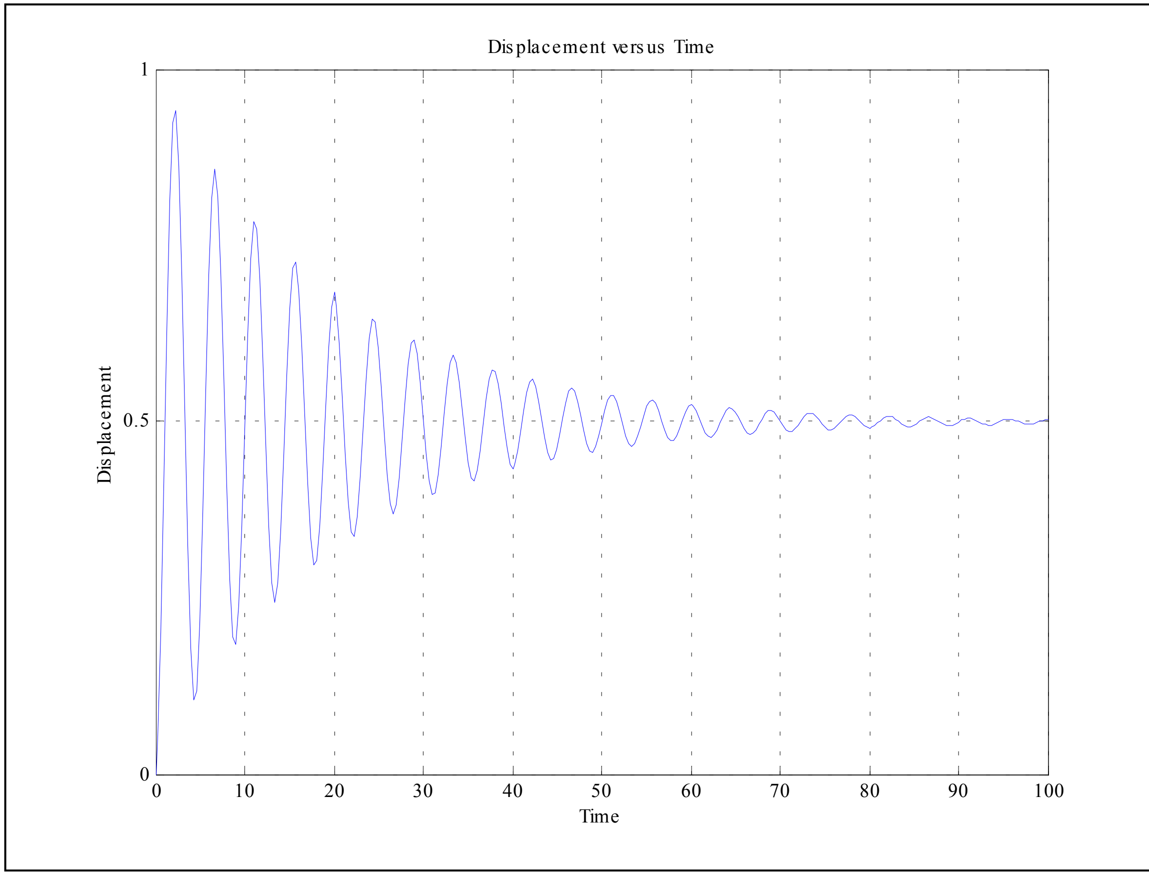




MATLAB Examples - VTB3_2

VIBRATION TOOLBOX EXAMPLE 3_2

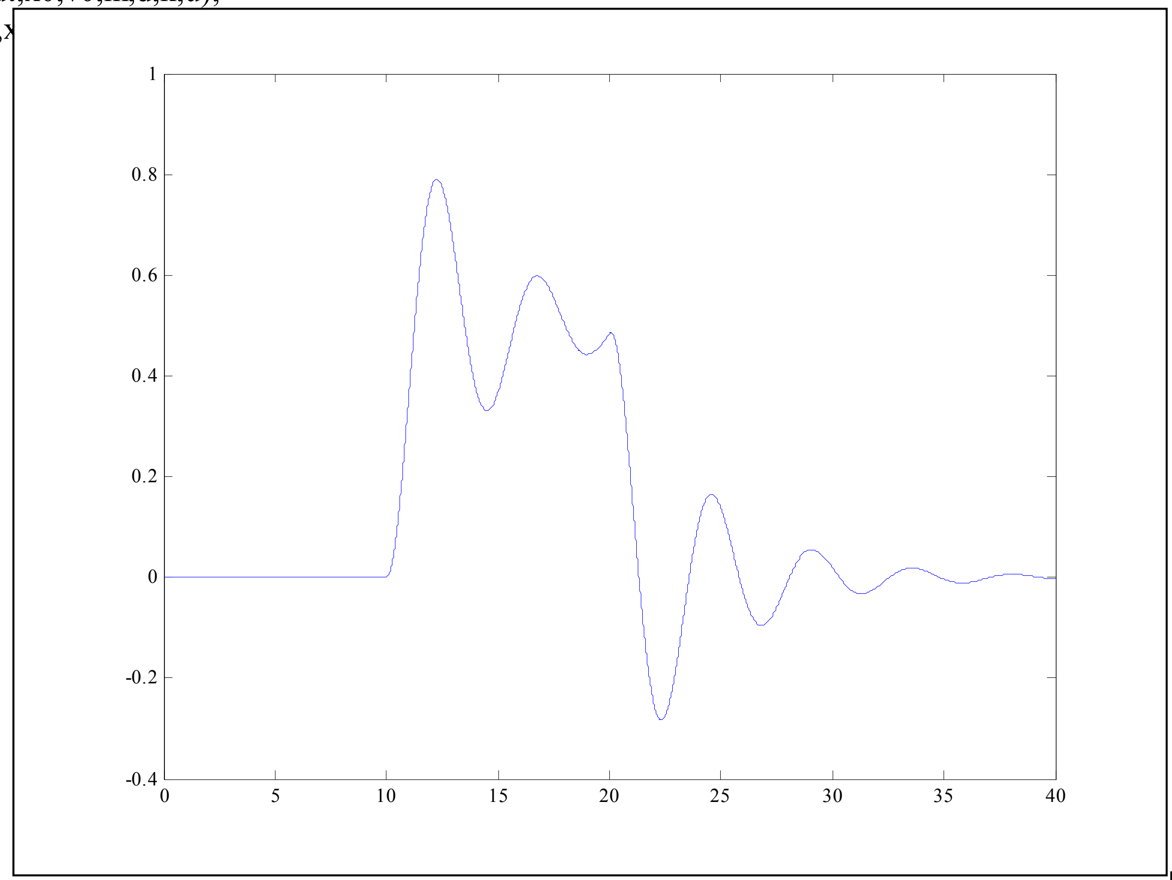
```
>> m=1;   c=.1;  k=2;   tf=100;   F0=1;
>> VTB3_2(m,c,k,F0,tf)
>>
>>
```



MATLAB Examples - VTB1_4

VIBRATION TOOLBOX EXAMPLE 1_4 – pulse

```
>> clear; p=1:1:1000; pp=p./p; ppp=[(p./p-p./p) pp (p./p-p./p) (p./p-p./p)];
>> x0=0;    v0=0;    m=1;    d=.5;    k=2;    dt=.01;    n=4000;
>> u=ppp; [x,xd]=VTB1_4(n,dt,x0,v0,m,d,k,u);
>> t=0:dt:n*dt; plot(t,x);plot(t,x
>>
```



MATLAB Examples - VTB1_4

VIBRATION TOOLBOX EXAMPLE 1_4 – ramp up (basically a static problem)

```
>> clear; x0=0;    v0=0;    m=1;    d=.5;    k=2;    dt=.01;    n=3000;
>> t=0:dt*100:n; u=t./3000; [x,xd]=VTB1_4(n,dt,x0,v0,m,d,k,u);
>> t=0:dt:n*dt; plot(t,x);
>>
```