VECTOR EXPANSION TECHNIQUES

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MODAL VECTOR EXPANSION

- Guyan condensation
- Dynamic Condensation
- Improved Reduced System
- System Equivalent Reduction Expansion Process
Many times, it may be necessary to expand experimental mode shapes for various studies. Unmeasured dofs may be needed for structural dynamic modification studies, correlation studies, model updating, to name a few. Expansion techniques used are:

- **Guyen/Irons condensation**
- **Dynamic condensation**
- **Improved Reduced System**
- **System Equivalent Reduction Expansion Process**
General Transformation

For all model reduction/expansion techniques, there is a relationship between the master dof (adof) and the deleted dof (ddof) which can be written in general terms as

\[
\{x_n\} = \begin{bmatrix} x_a \\ x_d \end{bmatrix} = [T]\{x_a\}
\]

- \(n\) denotes all FEM dof
- \(a\) denotes master or tested dof
- \(d\) denotes deleted or omitted dof
The general form of the expansion equation is:

\[
\mathbf{E}_n = \begin{bmatrix}
\mathbf{E}_a \\
\mathbf{E}_d
\end{bmatrix} = [T]\mathbf{E}_a
\]

- $X_F = \text{full set of dof's}$
- $X_A = \text{active set of dof's}$
Expansion Techniques

The different expansion forms are

**Guyan**

\[
[T_s] = \begin{bmatrix} [I] \\ [t_s] \end{bmatrix} = -[K_{dd}]^{-1}[K_{da}]
\]

**Dynamic**

\[
[T_f] = \begin{bmatrix} [I] \\ [t_f] \end{bmatrix} = -[B_{dd}]^{-1}[B_{da}]
\]

**IRS**

\[
[T_i] = \begin{bmatrix} [I] \\ -[K_{dd}]^{-1}[K_{da}] \end{bmatrix} + \begin{bmatrix} [0] \\ [0] \end{bmatrix} [M_n][T_s][M_a]^{-1}[K_a]
\]

**SEREP**

\[
[T_u] = [U_n][U_a]^T = \begin{bmatrix} [U_a][[U_a]^T][U_a]^T \end{bmatrix} = \begin{bmatrix} [U_d] & [U_d][[U_a]^T][U_a]^T \end{bmatrix}
\]

Notice the [I] in the upper partition for most techniques!
Modal Vector Expansion using SEREP

The SEREP transformation matrix can be used to expand (complete & smooth) the measured experimental vectors using

\[
\begin{bmatrix}
E_n \\
E_a \\
E_d
\end{bmatrix} =
\begin{bmatrix}
T_u \\
I
\end{bmatrix}
\begin{bmatrix}
E_a
\end{bmatrix}
\]

\[
= \begin{bmatrix}
U_a \\
U_d
\end{bmatrix}
\begin{bmatrix}
U_a
\end{bmatrix}^g
\begin{bmatrix}
E_a
\end{bmatrix}
\]
Modal Vector Expansion using SEREP

The expansion can be broken down into two parts:

**Completion**

\[ E_d = [U_d I U_a]^g E_a \]

**Smoothing**

\[ E'_a = [U_a I U_a]^g E_a \]