Comparison of Some
System Modeling Approaches
OVERVIEW

System models can be generated using a variety of techniques including both analytical and experimental representations.

Physical models, modal models, reduced substructure components, and hybrid models are all considered.

Only system model approximations that are feasible for both analytical and experimental representations are considered (Models that can only be generated analytically such as those using residual flexibility, constraint modes, etc. are not included herein).

A comparison of the various techniques on the same system model is presented so that similarities and differences can be noted.
EQUATION OF MOTION

The equation of motion for a multiple dof system can be written as

\[
[M_n]\{\ddot{x}_n\} + [C_n]\{\dot{x}_n\} + [K_n]\{x_n\} = \{F_n(t)\}
\]

These equations can be recast in modal space using the modal transformation equation as

\[
[U_n]^{T}[M_n][U_n]\{\ddot{p}_n\}
+ [U_n]^{T}[K_n][U_n]\{p_n\} = [U_n]^{T}\{F_n(t)\}
\]
MODAL SPACE REPRESENTATION

This yields an uncoupled set of 'm' modal equations described by modes which are linearly independent and orthogonal w.r.t the system matrices

\[
\begin{bmatrix}
\cdots \\
M_n \\
\cdots \\
\end{bmatrix} \{\ddot{p}_n\} + \begin{bmatrix}
\cdots \\
K_n \\
\cdots \\
\end{bmatrix} \{p_n\} = [U_n]^T \{F_n(t)\}
\]

This is shown pictorially below
Now combining two physical components together (Component A and B) interconnected through a coupling matrix results in the following

\[
\begin{bmatrix}
  [M^A_n] \\
  [M^B_n]
\end{bmatrix}
+ [M^{TIE}] \begin{bmatrix}
  \{x^A\} \\
  \{x^B\}
\end{bmatrix}
+ \begin{bmatrix}
  [K^A_n] \\
  [K^B_n]
\end{bmatrix}
+ [K^{TIE}] \begin{bmatrix}
  \{x^A\} \\
  \{x^B\}
\end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\]
PHYSICAL TO PHYSICAL SYSTEM MODEL

A physical to physical system model is shown pictorially below

- total number of equations is sum of number of equations describing Component A and Component B
- accuracy is not affected by truncation
- accuracy only affected by the accuracy of the two component models
MODAL TO MODAL SYSTEM MODEL

Now combining two modal components together (Component A and B) involves the modal representation of each system and the connection matrix cast in modal space of the two components, results in the following

\[
\begin{align*}
\begin{bmatrix}
\ddots & & \\
& \tilde{M}^A & \\
& & \ddots \\
\end{bmatrix}
+ \begin{bmatrix}
\ddots & & \\
& \tilde{M}^B & \\
& & \ddots \\
\end{bmatrix}
+ [U]^T [\Delta M] [U]
\end{align*}
\left\{ \begin{array}{c}
\tilde{p}^A \\
\tilde{p}^B \\
\end{array} \right\}
\]

\[
\begin{align*}
\begin{bmatrix}
\ddots & & \\
& \tilde{K}^A & \\
& & \ddots \\
\end{bmatrix}
+ \begin{bmatrix}
\ddots & & \\
& \tilde{K}^B & \\
& & \ddots \\
\end{bmatrix}
+ [U]^T [\Delta K] [U]
\end{align*}
\left\{ \begin{array}{c}
\tilde{p}^A \\
\tilde{p}^B \\
\end{array} \right\} = \{0\}
\]

\[ [U] = \begin{bmatrix} U^A \\ U^B \end{bmatrix} \]
MODAL TO MODAL SYSTEM MODEL

A modal to modal system model is pictorially shown below

- total number of equations is sum of number of modes used to describe Component A and Component B
- accuracy can be strongly affected by truncation
MODAL TO PHYSICAL SYSTEM MODEL

Now the equation for a physical Component A and Modal Component B can be written as

\[
\begin{bmatrix}
M_{cc}^A & M_{cr}^A \\
M_{rc}^A & M_{rr}^A
\end{bmatrix}
\begin{bmatrix}
\ddot{x}_c^A \\
\ddot{x}_r^A
\end{bmatrix}
= \begin{bmatrix}
M^B \\
\dddot{p}^B
\end{bmatrix}
\]

\[
\begin{bmatrix}
K_{cc}^A & K_{cr}^A \\
K_{rc}^A & K_{rr}^A
\end{bmatrix}
\begin{bmatrix}
\ddot{x}_c^A \\
\ddot{x}_r^A
\end{bmatrix}
+ \begin{bmatrix}
\dddot{p}^A \\
\dddot{p}^A
\end{bmatrix}
= \{0\}
\]

Introducing a constraint between the two components gives

\[
\begin{bmatrix}
x_c^A \\
x_r^A \\
p^B
\end{bmatrix}
= \begin{bmatrix}
0 & [U_c^B] \\
[1] & 0
\end{bmatrix}
\begin{bmatrix}
x_r^A \\
p^B
\end{bmatrix}
= [T^{AB}]
\begin{bmatrix}
x_r^A \\
p^B
\end{bmatrix}
\]

MODAL TO PHYSICAL SYSTEM MODEL

Now the combined system equation for a physical Component A and Modal Component B constrained to each other can be written as

\[
\begin{align*}
\left[ T^{AB} \right]^T \begin{bmatrix}
M^{A}_{cc} & M^{A}_{cr} \\
M^{A}_{rc} & M^{A}_{rr}
\end{bmatrix} \begin{bmatrix}
\{x^A_r\} \\
\{p^B\}
\end{bmatrix} + \left[ T^{AB} \right]^T \begin{bmatrix}
K^{A}_{cc} & K^{A}_{cr} \\
K^{A}_{rc} & K^{A}_{rr}
\end{bmatrix} \begin{bmatrix}
\{x^A\} \\
\{p^B\}
\end{bmatrix} &= \{0\}
\end{align*}
\]
MODAL TO PHYSICAL SYSTEM MODEL

A modal to physical system model is pictorially shown below

- total number of equations is sum of number of physical dof used to describe Component A and number of modes used to describe Component B minus the number of constraints written
- accuracy can be strongly affected by truncation of the modal component
REDUCED TO REDUCED PHYSICAL SYSTEM MODEL

Now two different components can be represented as reduced physical components and can be interconnected through the use of a coupling matrix.

The reduced mass and stiffness matrices can be written as

\[
[M_a] = [T]^T [M_n][T]
\]

\[
[K_a] = [T]^T [K_n][T]
\]

With the relationship between the 'n' full set of dof and the 'a' reduced set of dof given as

\[
\{x_n\} = \begin{bmatrix} x_a \\ x_d \end{bmatrix} = [T]\{x_a\}
\]
REDUCTION SCHEMES

Several reduction schemes can be utilized. The transformation matrix for some techniques are listed below with a pictorial representation of the reduced model

GUYAN

\[
[T_s] = \begin{bmatrix} [I] \\ -[K_{dd}]^{-1} [K_{da}] \end{bmatrix}
\]

IMPROVED REDUCED SYSTEM

\[
[T_i] = \begin{bmatrix} [I] \\ -[K_{dd}] [K_{da}] \end{bmatrix} + [t_i]
\]

\[
[t_i] = \begin{bmatrix} 0 \\ 0 \end{bmatrix} [K_{dd}]^{-1} M_n [T_s] [M_a]^{-1} [K_a]
\]

SYSTEM EQUIVALENT REDUCTION EXPANSION PROCESS

\[
[T_U] = [U_n] [U_a]^g
\]
REDUCED TO REDUCED PHYSICAL SYSTEM MODEL

Now the two different reduced components can be interconnected through the use of a coupling matrix and can be written as

\[
\begin{bmatrix}
[M_a^A] & [M_a^B] \\
[M_a^B] & [M_a^B]
\end{bmatrix}
+ [M^{TIE}] \begin{bmatrix}
\{\ddot{x}_A\} \\
\{\ddot{x}_B\}
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
[K_a^A] & [K_a^B] \\
[K_a^B] & [K_a^B]
\end{bmatrix}
+ [K^{TIE}] \begin{bmatrix}
\{\ddot{x}_A\} \\
\{\ddot{x}_B\}
\end{bmatrix}
= \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]
REDUCED TO REDUCED PHYSICAL SYSTEM MODEL

A reduced to reduced physical system model is pictorially shown below

- total number of equations is sum of number of equations used to describe Component A and Component B
- accuracy can be strongly affected by the 'a' dof selected for the Guyan and IRS techniques
- accuracy is strongly affected by the number of modes (similar to that of Modal to Modal) used for the SEREP reduction technique
HYBRID SYSTEM MODEL

Now through the use of drive point and transfer impedances at the attachment locations and response locations, the system characteristics can be written in terms of the component FRF relations as follows.

The response of the coupled system at point 'i' on Component A due to a force at point 'j' on Component A is

\[
\tilde{h}_{ij}^{Z} = h_{ij}^{A} - h_{ij}^{A} \int_{ic} \left( \left[ h_{cc}^{A} \right] + \left[ h_{cc}^{B} \right] \right)^{-1} \{ h_{ij}^{A} \}
\]

The response of the coupled system at point 'i' on Component A due to a force at point 'j' on Component B is

\[
\tilde{h}_{ij}^{Z} = h_{ij}^{A} \int_{ic} \left( \left[ h_{cc}^{A} \right] + \left[ h_{cc}^{B} \right] \right)^{-1} \{ h_{ij}^{B} \}
\]
HYBRID SYSTEM MODEL

A hybrid system model is pictorially shown below

- total number of equations is only a function of the number of connection points and the number of input-output locations desired
- accuracy can be strongly affected by the number of modes used to synthesize FRFs for analytical applications
- accuracy can be strongly affected by the inclusion of rotational dofs in experimental data
EXAMPLE

A system model is formed using two planar beams to illustrate the different system modeling techniques

- tubular aluminum section 3" x 1.5" x 3/16" thick
- short beam was modeled free-free
  50" long, 14 beams, 15 nodes
- long beam was modeled with soft springs to ground
  140" long, 28 beams, 29 nodes

(Note: length not to scale)
EXAMPLE

The following models were studied

- Physical to Physical - reference solution
- Modal to Modal - 5 modes - component A and B
- Modal to Modal - 5 modes comp A - 10 modes comp B
- Modal to Modal - 10 modes - component A and B
- Modal to Physical - 5 modes comp A - physical comp B
- Physical to Reduced - Physical A to
  Guyan Reduced B (1,10,15,20,29 adof)
- Physical to Reduced - Physical A to
  SEREP Reduced B (1,10,15,20,29 adof)
- Physical to Reduced - Physical A to
  Guyan Reduced B (1,4,7,10,20 adof)
- Physical to Reduced - Physical A to
  SEREP Reduced B (1,4,7,10,20 adof)
- Physical to Reduced - Physical A to
  Guyan Reduced B (1,4,7,10,15,20,23,26,29) adof
- Physical to Reduced - Physical A to
  SEREP Reduced B (1,4,7,10,15,20,23,26,29 adof)
- Reduced to Reduced - Guyan Reduced A (1,3,8,13,15 adof)
  to Guyan Reduced B (1,10,15,20,29 adof)
- Reduced to Reduced - SEREP Reduced A (1,3,8,13,15 adof)
  to SEREP Reduced B (1,10,15,20,29 adof)
- Hybrid - Component A (5 modes) - Component B (5 modes)
- Hybrid - Component A (10 modes)-Component B (10 modes)
FULL PHYSICAL REFERENCE SOLUTION

A full solution was obtained for reference. The two beams were assembled with stiff springs to constrain the two beams to each other.

Mode | Reference
--- | ---
1 | 9.63
2 | 30.98
3 | 60.14
4 | 81.64
5 | 85.49
6 | 135.83
7 | 196.22
8 | 279.88
9 | 305.97
10 | 407.31
11 | 525.90

(Note: length not to scale)
MODAL TO MODAL SOLUTION

Using modal component representations for both components, the following system models were developed:

- 5 modes Component A - 5 modes Component B
- 10 modes Component A - 10 modes Component B
- 5 modes Component A - 10 modes Component B

FULL SPACE PHYSICAL MODEL

COMPONENT A

MODAL SPACE MODEL

MODAL SPACE MODEL

COMPONENT B

MODAL TIE MATRIX

FULL SPACE PHYSICAL MODEL
MODAL TO MODAL SOLUTION

The system model results were

<table>
<thead>
<tr>
<th>Mode</th>
<th>Reference</th>
<th>5A+5B</th>
<th>10A+10B</th>
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</table>

- using 5 modes yielded good results for lower modes but truncation affected higher modes
- using 10 modes improved results significantly
- using 5 modes for A and 10 modes for B also produced very good results - truncation of B is more critical than truncation of A
MODAL TO PHYSICAL SOLUTION

Using modal representation for Component A and a physical representation for Component B, the following system models were developed:

- 5 modes Component A - Physical Component B
- 10 modes Component A - Physical Component B
The system model results were

<table>
<thead>
<tr>
<th>Mode</th>
<th>Reference</th>
<th>5A + B</th>
<th>10A + B</th>
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<td>11</td>
<td>525.90</td>
<td>527.64</td>
<td>527.19</td>
</tr>
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</table>

- using 5 modes for Component A yielded very good results for the modes of the system since truncation of A is not as critical as truncation of B
- using 10 modes for Component A doesn't appreciably change the results
- since Component B is a physical representation, there is no concern of truncation for this component
PHYSICAL TO REDUCED SOLUTION

Using a physical representation for Component A and a reduced physical representation for Component B, the following system models were developed:

- 5 well distributed dof for Component B
- 5 poorly distributed dof for Component B
- 9 well distributed dof for Component B

Both Guyan and SEREP were used for the reduction of Component B.
PHYSICAL TO REDUCED SOLUTION - 5 GOOD DOF

The system model results were

<table>
<thead>
<tr>
<th>Mode</th>
<th>Reference</th>
<th>Guyan</th>
<th>SEREP</th>
</tr>
</thead>
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</table>
The following can be seen from these results

- Guyan reduced component looses accuracy after the second mode of the system even though this is a reasonably good selection of points

- SEREP reduced component has good results for the first 6 modes of the system
The system model results were

<table>
<thead>
<tr>
<th>Mode</th>
<th>Reference</th>
<th>Guyan</th>
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<td>279.88</td>
<td>660.78</td>
<td>661.83</td>
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</tbody>
</table>
The following can be seen from these results:

- Guyan reduced component loses accuracy after the second mode of the system.

- This is a poor selection of points and Guyan reduction is not expected to provide good results.

- SEREP reduced component has good results for the first 6 modes of the system even though the selection of points is poor for Guyan reduction.

- This indicated that SEREP can provide much better results with fewer arbitrarily selected dofs.
The system model results were

Good Distribution - 9 DOF

<table>
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<tr>
<th>Mode</th>
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<th>Guyan</th>
<th>SEREP</th>
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PHYSICAL TO REDUCED SOLUTION - 9 GOOD DOF

The following can be seen from these results

- Guyan reduced component produces better results as more well distributed dof are added to the component

- the results degrade as frequency increases for the Guyan reduced component as expected

- SEREP reduced component produces better results for the modes of the system
REDUCED TO REDUCED SOLUTION

Using a reduced physical representation for Component A and B (both Guyan and SEREP), a system model was developed using only 5 dof for each of the components.
### REDUCED TO REDUCED SOLUTION - 5 GOOD DOF

The system model results were

<table>
<thead>
<tr>
<th>Mode</th>
<th>Reference</th>
<th>Guyan</th>
<th>SEREP</th>
</tr>
</thead>
<tbody>
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<td>667.07</td>
</tr>
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</table>

- Guyan reduced component looses accuracy after the second mode of the system even though this is a reasonably good selection of points
- SEREP reduced component has good results for the first 6 modes of the system
- the SEREP reduced models produced better results when compared to the Guyan reduced approach
HYBRID SOLUTION

Using a hybrid model representation (FRFs) for Component A and B, the following system models were developed with FRFs synthesized as follows:

- 5 modes for Component A and B
- 10 modes for Component A and B
- All modes for Component A and B
HYBRID SOLUTION

Figure 17a - 5 Modes for Each Component

Figure 17b - 10 Modes for Each Component

Figure 17c - All Modes for Each Component
HYBRID SOLUTION

The system model results were

<table>
<thead>
<tr>
<th>Mode</th>
<th>Reference</th>
<th>5A+5B</th>
<th>10A+10B</th>
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The following can be seen from these results

- as more modes are added for the synthesis of the FRFs the results improve
- notice that the case of 5A+5B produces the same results as the SEREP to SEREP reduced modes with 5 modes; this is expected since both models are using essentially the same component representations
**COMPARISON**

**MODAL – SEREP REDUCED - HYBRID**

The system model results for each of the different approaches using 5 modes are essentially identical.

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OBSERVATIONS

The following observations can be made

- SEREP reduced models produce better system model approximations when compared to the equivalent Guyan reduced approach
- SEREP is also not affected by dof location selected

- while not specifically shown in this study, the IRS technique will generally produce better results when compared to Guyan but would not exceed the results obtained using SEREP

- SEREP models produced the same results as Hybrid when the same number of modes are used to describe the components
- SEREP models produced the same results as Modal to Modal when the same number of modes are used to describe the components
- truncation will affect the reduced component and synthesized FRF components in the same way