Lab # 1

Vertical Spring-Mass System

96.161 Honors Physics I Laboratory

Batman (and Robin)

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Objective: To study the kinematics and dynamics of a vertical spring-mass system

Theory and Procedure:

Measuring the force constant:

First, a 0.025 kg mass was attached to a spring, and the equilibrium extension of the spring was measured. Since the mass is stationary, the force exerted by the spring must be equal to the weight of the object, so the force was also recorded for the mass. This procedure was repeated with masses of 0.045 kg, 0.065 kg, 0.085 kg, and 0.105 kg. A graph of force vs. extension was made, and the slope of that graph is the spring constant.

Measuring oscillations:

Next, a 0.055 kg mass was attached to the spring, and it was set oscillating gently. The motion sensor recorded the position, velocity, acceleration, and force for several periods of simple harmonic motion. The period (T) was found by measuring the time the mass took to make 20 oscillations and dividing that time by 20. From T, the frequency ($f = 1/T$), and the angular frequency ($\omega = 2\pi f$) were derived. The displacement in simple harmonic motion is sinusoidal, i.e. $y = A\cos(\omega t)$. Thus the mass returns to the same position after each period, or at every time instant that " ωt" (the argument of the cosine function) equals "2π" (the period of a trigonometric function). Thus: $\omega T = 2\pi$, so $\omega = 2\pi/T = 2\pi f$. The amplitude of each graph (x_m, v_m, a_m, and F_m) was found by taking the difference between the maximum and minimum and dividing by two. This procedure was repeated with masses of 0.075 kg and 0.105 kg. Subsequently, the values of angular frequency (ω), period (T), and amplitudes (v_m, a_m, and F_m) were found by calculations involving the spring constant, the mass, and x_m.

e.g. $F = -kx = ma$. Thus, $-(k/m)x = a$

Also, in simple harmonic motion, $a = -\omega^2 x$.

Combining the two, we get $-\omega^2 x = -(k/m)x$ or $\omega = (k/m)^{1/2}$.

The value of ω found above was then used to find T with the equation $T = 2\pi/\omega$.

The amplitudes v_m, a_m, and F_m were found using the equations $v_m = x_m\omega$, $a_m = x_m\omega^2$, and $F_m = ma_m$, respectively. Finally, the percent difference was found between the "measured" and calculated values of all these variables.

Results and Analysis:

m (kg)	F = mg (N)	*l* (m)	L = *l*_max - *l* (m)
0.025	0.245	0.5848	0
0.045	0.441	0.5344	0.0504
0.065	0.637	0.4927	0.0921
0.085	0.833	0.4331	0.1517
0.105	1.029	0.3733	0.2115

Table 1: Different masses hanging at rest. The *l* column is the height of the mass according to the motion sensor. The F is calculated with mg, but it is equal to the spring force because the mass is at rest, and the spring force must be equal to weight.

As force (F) increases (with increasing mass), the spring stretch (L) also increases. F can be plotted against L to demonstrate the relationship.

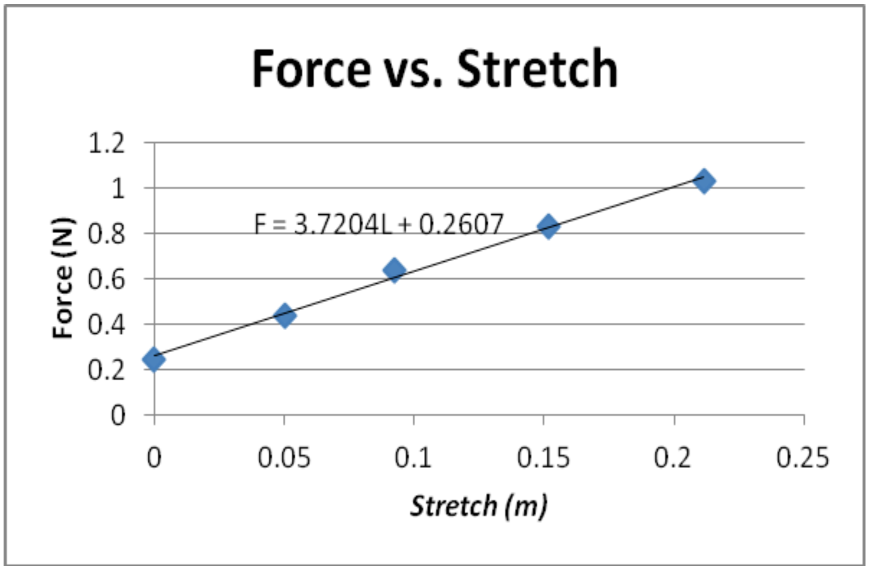

Fig 1: Spring force at different levels of stretch. They have a linear relationship.

Since F = kx, the slope of the graph in Fig 1 is the spring constant **k = 3.72 N/m**.

Next, a 0.055 kg mass was placed on the spring and set oscillating vertically. The force/motion sensors produced the following data:

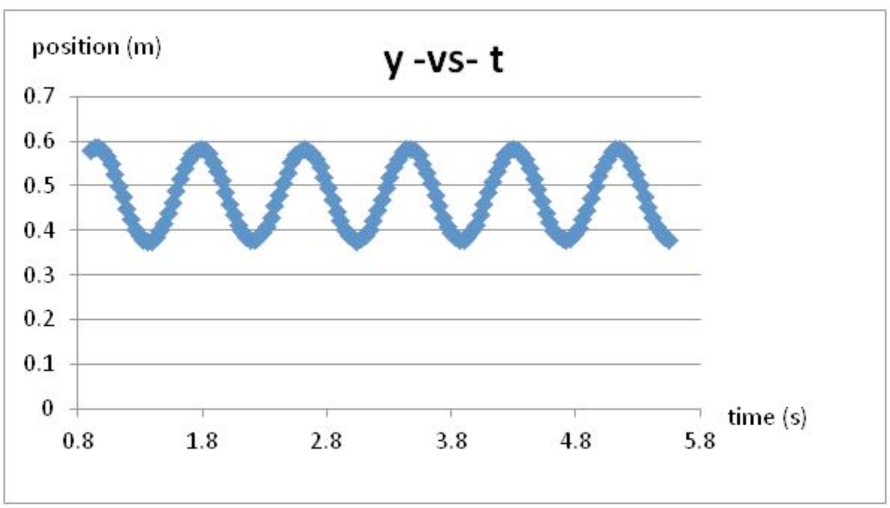

Fig 2: Position vs. time data for a 0.055 kg mass oscillating vertically.

The displacement amplitude of this data (x_m) was found by taking the difference between the maximum and minimum values and dividing by two. The max is 0.5869 m, and the min is 0.3839 m, so $\mathbf{x_m}$ = (0.5869 m -0.3839 m)/2 = **0.1015 m**.

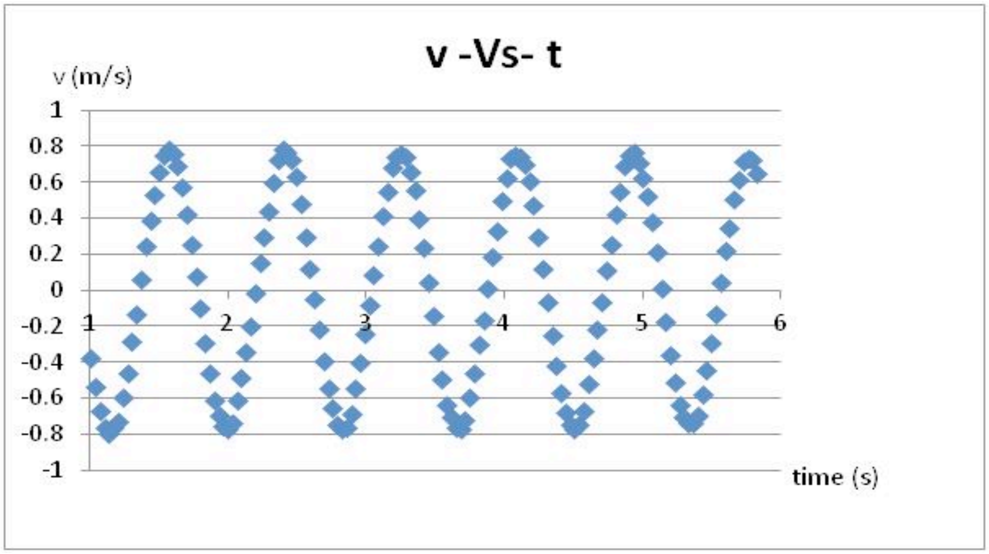

Fig 3: Velocity vs. time data for 0.055 kg pendulum.

As above, the velocity amplitude (v_m) was found by taking (max – min)/2.

So $\mathbf{v_m}$ = (0.7596 m/s – (-0.7424 m/s))/2 = **0.7510 m/s**.

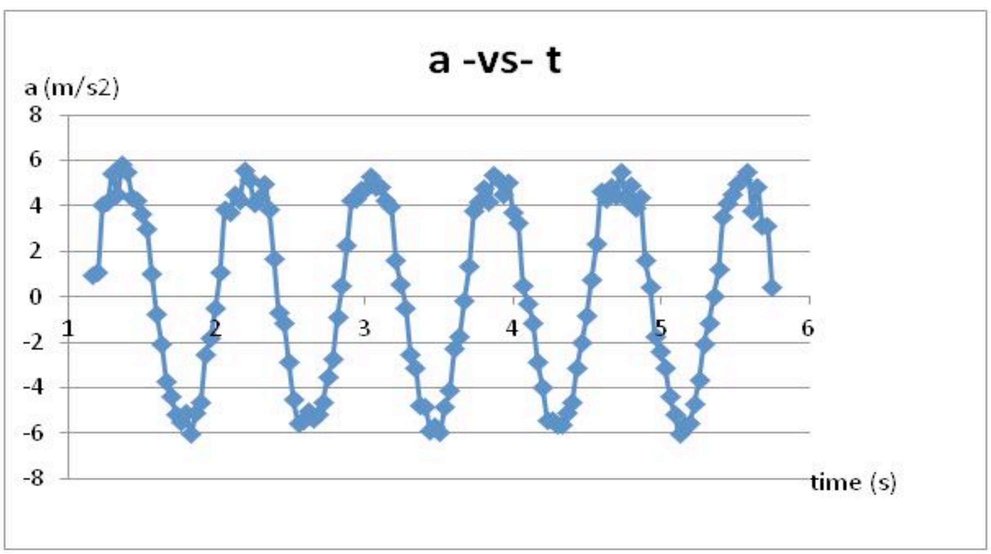

Fig 4: Acceleration vs. time for a 0.055 kg pendulum.

$\mathbf{a_m}$ = (5.9477 m/s^2 – (-5.7403 m/s^2))/2 = **5.8440 m/s^2**

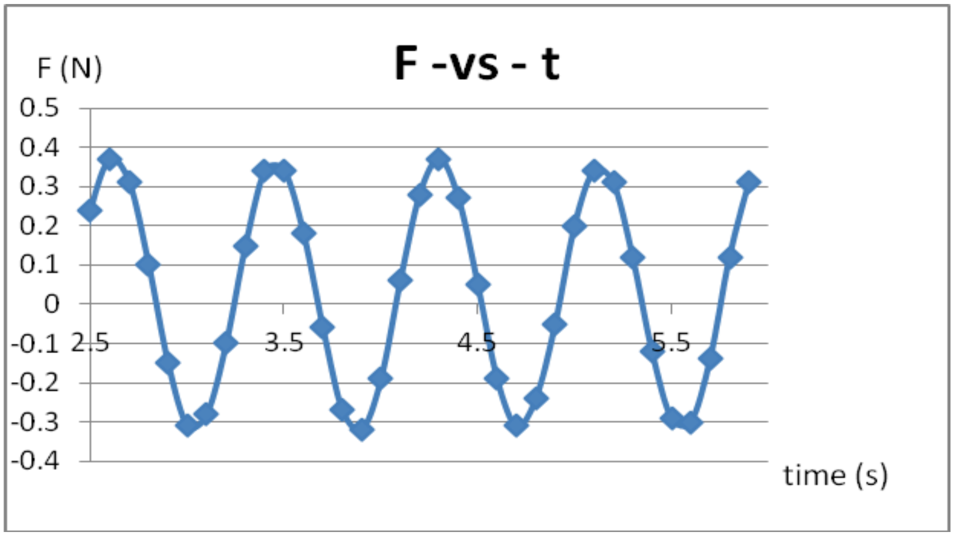

Fig 5: Force (measured by a force sensor) vs. time for a 0.055 kg pendulum. Notice it is directly proportional to fig 4 (acceleration vs. time).

$\mathbf{F_m}$ = (0.358 N – (-0.33 N))/2 = **0.344 N**

From Figs 2 – 5, some relationships are seen. The maximum points in the position graph (fig 2) correspond to the "zero" points on the velocity graph (fig 3) and vice-versa. The acceleration graph (fig 4) corresponds to the position graph flipped reflected over the x-axis, and the force graph (fig 5) corresponds to the acceleration graph. The same was done with masses of 0.075 and 0.105. The position graphs only for those two masses are attached below.

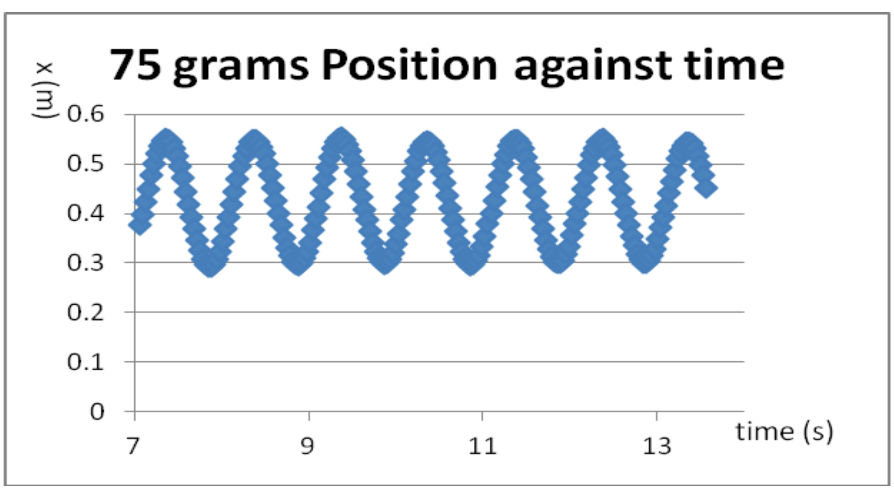

Fig 6: Position for 0.075 kg mass. Notice the frequency is lower than fig 2.

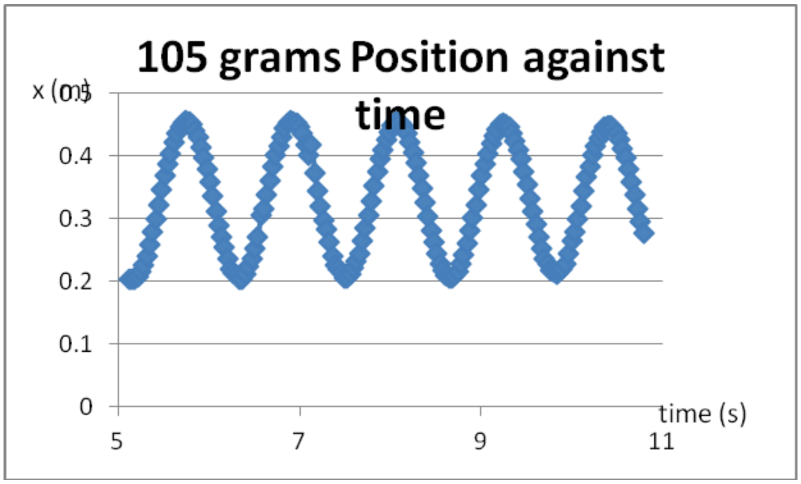

Fig 7: Position for 0.105 kg mass. Frequency is even lower.

All the data from the above graphs was compiled into the table below.

m (kg)	T (s)	f = 1/T (Hz)	ω = 2πf (rad/s)	x_m (m)	v_m (m/s)	a_m (m/s^2)	F_m (N)
0.055	0.802	1.247	7.834	0.1015	0.751	5.844	0.344
0.075	0.967	1.034	6.498	0.0632	0.422	3.213	0.231
0.105	1.142	0.876	5.502	0.0924	0.522	3.379	0.315

Table 2: Measured values.

As mass increases, period increases. This is because ω decreases when mass increases (according to $\omega = (k/m)^{1/2}$), and period increases as ω decreases (according to $T = 2\pi/\omega$). The amplitudes don't vary predictably with mass because they're all based on the height to which the mass is raised initially. The same values were then found by derivation based on x_m and k.

m (km)	ω = (k/m)^(1/2) (rad/s)	T = 2π/ω (s)	v_m = x_mω (m/s)	a_m = x_mω^2 (m/s^2)	F_m = ma_m (N)
0.055	8.2	0.76	0.83	6.9	0.38
0.075	7.0	0.89	0.45	3.1	0.24
0.105	6.0	1.06	0.55	3.3	0.34

Table 3: Derived values. The value of k comes from the slope of fig 1 (3.72 N/m). The values for x_m are from table 2.

The differences between the values in table 3 and table 2 are compared in Table 4. The percent differences are typically less than 10%. (The worst deviations are for the 0.055 kg weight.)

m (kg)	Δω %diff.	ΔT %diff.	Δv_m %diff.	Δa_m %diff	ΔF_m %diff.
0.055	4.9	4.9	10.6	16.1	9.3
0.075	8.1	8.1	5.3	2.5	1.8
0.105	7.9	7.9	5.2	3.2	8.7

Table 4: Comparison between measured and derived values.

Discussion: We can compare the spring potential energy at x_m with the kinetic energy at v_m to see whether energy is conserved.

At 0.055 kg: $U_{Spring} = 0.5kx^2 = 0.5 * 3.7204$ N/m $* (0.1015$ m$)^2 = 0.0192$ J

$K = 0.5mv^2 = 0.5 * 0.055$ kg $* (0.7510$ m/s$)^2 = 0.0155$ J %diff = 19.3%

At 0.075 kg: $U_{Spring} = 0.00743$ J $K = 0.00668$ J %diff = 10.1%

At 0.105 kg: $U_{Spring} = 0.0159$ J $K = 0.0143$ J %diff = 10.1%

Other than the 0.055-kg data, the numbers agree within 10%, which means that the potential energy in the spring when it is completely stretched becomes kinetic energy when the velocity is at a max. The maximum velocity occurs whenever the spring is at zero stretch because there is no spring potential there.

Conclusions: The kinematics, dynamics and energy conservation equations in an oscillating spring were investigated by measuring motion variables and analyzing their relationship to force and spring constant. The experiments validated theory within an uncertainty of about 10%.