Find general antiderivatives for the following functions:

1) \( f(x) = e^{3x} + x^2 \)
2) \( f(x) = e^{-x/2} - \frac{3}{\sqrt{x}} \)
3) \( s(t) = \cos t + \sin t - \sec^2 t \)
4) \( g(x) = \frac{1}{x^2} - \frac{1}{\sqrt{x}} \)
5) \( g(x) = \frac{(x+1)^2}{x} \)
6) \( f(x) = \frac{1}{\sqrt{1-x^2}} + \frac{1}{1+x^2} \)
7) \( f(x) = \sin(3x) + \cos(\pi x) \)
8) \( s(t) = 2t \cos(t^2) \)
9) \( g(x) = \frac{4x^3}{x^4+1} + xe^x \)

10) Suppose a robot is moving on a linear track so that its position (in meters) is given by:
\[
s(t) = 16t - \frac{t^4}{4}, \text{ for } t \in [0, 4],
\]
where \( t \) is time in seconds.

a) What is the robot’s velocity at \( t = 2 \) seconds?

b) When does the robot reverse direction?

c) What is the robot’s acceleration at \( t = 3 \) seconds?

11) Suppose a particle is moving on a linear track so that its acceleration (in m/s²) is given by:
\[
a(t) = 4 - \sin 4t, \text{ for } t \in [0, 4\pi],
\]
where \( t \) is time in seconds. If the particle’s initial position is \( s(0) = 2 \) m, and initial velocity is \( v(0) = -2 \) m/s, what is its position at any given time?

12) Suppose a particle is moving on a linear track so that its acceleration (in m/s²) is given by:
\[
a(t) = 4t + \sin 2t + e^{-3t}, \text{ for } t \in [0, 10],
\]
where \( t \) is time in seconds. If the particle’s initial position is \( s(0) = 2 \) m, and initial velocity is \( v(0) = -2 \) m/s, what is its position at any given time?
Solutions

1) \( F(x) = \frac{1}{3} e^{3x} + \frac{1}{3} x^3 + C \)

2) \( F(x) = -2e^{-x/2} - \frac{3}{5} \sqrt{x^5} + C \)

3) \( S(t) = \sin t - \cos t - \tan t + C \)

4) \( G(x) = -\frac{1}{x} - 2\sqrt{x} + C \)

5) \( g(x) = \frac{(x+1)^2}{x} = \frac{x^2 + 2x + 1}{x} = x + 2 + \frac{1}{x} \)
   
   so \( G(x) = \frac{x^2}{2} + 2x + \ln x + C \)

6) \( F(x) = \arcsin x + \arctan x + C \)

7) \( f(x) = -\frac{1}{3} \cos(3x) + \frac{1}{\pi} \sin(\pi x) + C \)

8) \( s(t) = \sin(t^2) + C \)

9) \( G(x) = \ln(x^4 + 1) + \frac{1}{2} e^{x^2} + C \)

10) a) \( s'(2) = v(2) = 16 - t^3 \bigg|_{t=2} = 8 \text{ m/s} \)

    b) When \( s'(t) = 16 - t^3 = 0 \), or \( t = 3\sqrt{16} \)

    c) \( s''(3) = \left(-3t^2 \bigg|_{t=3}\right) = -27 \text{ m/s}^2 \)

11) \( s(t) = 2t^2 + \frac{1}{16} \sin(4t) - \frac{9}{4} t + 2 \)

12) \( s(t) = \frac{2}{3} t^3 - \frac{1}{4} \sin 2t + \frac{1}{9} e^{-3t} - \frac{7}{6} t + \frac{17}{9} \)