1) \[ f'(x) = e^x \sec^2(e^x) \]

2) \[ s'(x) = \frac{3x^2}{\sqrt{1-x^6}} \]

3) \[ y' = e^{-4x}(3\cos(3x) - 4\sin(3x)) \]

4) \[ g'(t) = \frac{t^4 + 3t^2 + 2t}{(t^2 + 1)^2} \]

5) \[ y' = -9x^{-4}\ln x = -\frac{9\ln x}{x^4} \]

6) \[ f'(x) = \frac{\ln x}{2x} \]

7) Using the limit definition of the derivative, calculate \( \frac{df}{dx} \) for \( f(x) = x^3 + 1 \).

\[
\begin{align*}
f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{[(x+h)^3 + 1] - [x^3 + 1]}{h} \\
&= \lim_{h \to 0} \frac{x^3 + 3hx^2 + 3h^2x + h^3 + 1 - x^3 - 1}{h} \\
&= \lim_{h \to 0} \frac{3hx^2 + 3h^2x + h^3}{h} \\
&= \lim_{h \to 0} \left(3x^2 + 3hx + h^2\right) = 3x^2
\end{align*}
\]

8) Suppose that \( e^{x^{2}-1} - y = 0 \). Find \( \frac{dy}{dx} \), then find the equation of the line tangent to the graph of this equation at the point \((1,1)\). Write your answer in slope-intercept form.

\[
\begin{align*}
\frac{d}{dx} \left(e^{x^{2}-1} - y\right) &= \frac{d}{dx}(0) = 0 \quad \text{or} \quad \frac{d}{dx} e^{x^{2}-1} - \frac{d}{dx} y = 0 \quad \text{and so} \quad \left(\frac{d}{dx} (x^2y^2 - 1)\right) e^{x^{2}-1} - \frac{dy}{dx} = 0 \\
\left(2xy^2 + 2x^2y\frac{dy}{dx}\right) e^{x^{2}-1} - \frac{dy}{dx} = 0 \quad \text{or} \quad \frac{dy}{dx} = \frac{2xy^2 e^{x^{2}-1}}{1 - 2x^2 ye^{x^{2}-1}}
\end{align*}
\]

At \((1,1)\), \( \frac{dy}{dx} = \frac{2xy^2 e^{x^{2}-1}}{1 - 2x^2 ye^{x^{2}-1}} = \frac{2}{1 - 2} = -2 \), and so the tangent line is \( y - 1 = -2(x - 1) \) or \( y = -2x + 3 \).

9) Suppose a ball is moving on a linear track so that its acceleration (in \( \text{m/s}^2 \)) is given by:

\[ a(t) = 16 - 3t^2 \], for \( t \geq 0 \),

where \( t \) is time in seconds. The ball has initial position \( s = 2 \) m, and initial velocity \( v = 0 \) m/s. a) What is the ball’s velocity after 3 seconds? b) When does it reverse direction? c) What is the ball’s position \( s(t) \)?

**Solution:** First find the velocity and position functions \( v(t) = 16t - t^3 + C \) Using \( v(0) = 0 \), \( C = 0 \), and so \( v(t) = 16t - t^3 \). Anti-differentiating again \( S(t) = 8t^2 - \frac{1}{4}t^4 + C \), With \( S(0) = 2 \), \( C = 2 \),

\[
S(t) = 8t^2 - \frac{1}{4}t^4 + 2 \]. Now (a) \( v(3) = 21 \). (b) Setting \( v(t) = 16t - t^3 = t(16 - t^2) = 0 \), \( t = 0, \pm 4 \), the only physically reasonable answer is \( t = 4 \). (c) \( S(t) = 8t^2 - \frac{1}{4}t^4 + 2 \)
10) Let \( f(x) = e^{-x^2/2} \).

a) Find the interval(s) on which \( f \) is increasing or decreasing.

Since \( f'(x) = -xe^{-x^2/2} \), \( f'(x) < 0 \) for \( x > 0 \), and \( f'(x) > 0 \) for \( x < 0 \). Hence \( f \) increases for \( x < 0 \), and decreases for \( x > 0 \).

b) Find local (relative) maximum and minimum values of \( f \). From part (a) there is only one critical point at \( x = 0 \). Since \( f \) increases for \( x < 0 \), and decreases for \( x > 0 \), \( f(0) = 1 \) is a relative maximum value.

c) Determine intervals where the function is concave up and concave down.

\[ f''(x) = (x^2 - 1)e^{-x^2/2} = (x+1)(x-1)e^{-x^2/2} \quad f''(x) < 0 \text{ for } -1 < x < 1, \text{ and hence } f \text{ is concave down there.} \]
For \( x < -1 \cup x > 1 \), \( f''(x) > 0 \) and so \( f \) is concave up there.

d) Determine any points of inflection.
Since concavity changes at \( x = \pm 1 \), there are inflection points at \((\pm 1, e^{-1/2})\).

e) Are there any asymptotes?
Since \( \lim_{x \to \pm \infty} f(x) = \lim_{x \to \pm \infty} e^{-x^2/2} = 0 \), there is a horizontal asymptote \( y = 0 \). There are no vertical asymptotes.

11) A storage bin with a ceiling and floor is to be constructed in the shape of a cylinder. The cost of the material used for the two circular surfaces is $10 per square foot. The material used for the lateral surface costs $20 per square foot. What are the dimensions of the cheapest bin that can be built with a volume of \( 1000\pi \) ft\(^3\)?

\[ V = \pi r^2 h. \]

The circular ends have total area \( 2\pi r^2 \).

The lateral surface area is \( 2\pi rh \).

Cost is \( C = (20)(2\pi rh) + (10)(2\pi r^2) \).

So the problem at hand is minimize

\[ C = 40\pi rh + 20\pi r^2 \]

subject to the constraint \( \pi r^2 h = 1000\pi \).

\[ \pi r^2 h = 1000\pi \text{ means } h = \frac{1000}{r^2}. \]

Using this here gives

\[ C = \frac{40,000\pi}{r} + 20\pi r^2 \]

and so

\[ C' = \frac{-40,000\pi}{r^2} + 40\pi r, \]

which gives critical points as \( r = 0 \), and \( r = 10 \).

The physically reasonable solution is \( r = 10 \text{ m} \) which gives \( r = 10, h = 10 \text{ m}. \)

Note: If you wish to solve the problem using implicit differentiation. The steps follow.
The volume is given by \( V = \pi r^2 h \).
The circular ends have total area \( 2\pi r^2 \).
The lateral surface area is \( 2\pi rh \).
Cost is \( C = (\$20)(2\pi rh) + (\$10)(2\pi r^2) \).

So the problem at hand is minimize \( C = 40\pi rh + 20\pi r^2 \) subject to the constraint \( \pi r^2 h = 1000\pi \). or \( r^2 h = 1000 \)

Assuming \( h = f(r) \):

\[
\frac{d}{dr}(r^2h) = \frac{d}{dr}(1000) \quad \text{and} \quad \frac{d}{dr} C = \frac{d}{dr}(40\pi rh + 20\pi r^2)
\]

\[
2rh + r^2 \frac{dh}{dr} = 0 \quad \text{and} \quad \frac{dC}{dr} = 0 = 40\pi h + 40\pi r \frac{dh}{dr} + 40\pi r
\]

\[
\frac{dh}{dr} = -\frac{2h}{r}
\]

which gives

\[
40\pi h + 40\pi r \left(-\frac{2h}{r}\right) + 40\pi r = 0 \quad \text{or} \quad r = h.
\]

Using \( r = h \) in \( r^2 h = 1000 \) gives \( r = h = 10 \) m

12) There is one positive value of \( x \) that solves the equation \( x^3 - 3x = 0 \).

a) Give a recursion equation for solving this problem using Newton’s method.

b) Starting with \( x_0 = \frac{3}{2} \), approximate the solution by \( x_2 \).

To solve \( x^3 - 3x = 0 \), let \( f(x) = x^3 - 3x \), and so the recursion formula is

\[
x_{n+1} = x_n - \frac{x_n^3 - 3x_n}{3x_n^2 - 3}
\]

Which can be simplified to \( x_{n+1} = \frac{2x_n^3}{3x_n^2 - 3} \)

Starting with \( x_0 = \frac{3}{2} \), approximate the positive solution by \( x_2 \). (Round to \( 10^{-6} \))

\[
x_1 = \frac{2x_0^3}{3x_0^2 - 3} = \frac{2(3/2)^3}{3(3/2)^2 - 3} = \frac{9}{5} = 1.8
\]

\[
x_2 = \frac{2x_1^3}{3x_1^2 - 3} = \frac{2(9/5)^3}{3(9/5)^2 - 3} = \frac{729}{420} \approx 1.735714
\]