CHAPTER 2
Time Value of Money

- The language of Finance
- The most important lesson

6-1

Time Value of Money (TVM)

- Basic tool for evaluating financial decisions
- A dollar received today is more valuable than a dollar received tomorrow. Why?

6-2

Try this

- Option A: Receive $100 today
- Option B: Receive $100 after a year
- Which one would you pick?

6-3

Option A is better

Why?

- Because you can invest $100 received today and get more than $100 at the end of year.

6-4

Try this

Assume an annual interest rate of 10%

- Option A: Receive $100 today
- Option B: Receive $110 after a year
- Which one would you pick?

6-5

Both options are the same

Why?

Because you can invest $100 at 10 percent and get $110 at the end of the year.

6-6

Terms

In Time-Value-of-Money (TVM) terms:

$100 is the present value (PV) of $100
10 percent is the rate (I/Y or Rate) of return
1 is the number of periods (N or NPER)
$100 is the future value (FV) of $100

6-7

Let's solve using calculator

N =1
I/Y = 10
PV = -100*
PMT = 0**
FV = ?

* Since you are investing, cash is going out and it is an outflow or has a negative sign
** We will talk about PMT later, for the time being we will keep it 0.

6-8

Let's solve using Excel

=FV(RATE, NPER, PV, PMT, TYPE*)

=FV(0.10, 1, -100, 0, 0) = 110

* We will always keep it 0

6-9

Types of cash flows

The previous example is what is called a SINGLE OR LUMP-SUM CASH FLOW

Other types we will study:
1) ANNUITY
2) PERPETUITY
3) UNEVEN CASH FLOWS

6-10

Example 1: FV of Lump-Sum CF

What is the future value of $100 after 2 years if the interest rate is 10 percent?

Calculator: N= 2, I/Y= 10, PV= -100, PMT = 0, FV = ?
FV= $121

Excel:=FV(RATE, NPER, PV, PMT, TYPE)
=FV(0.10, 2, -100, 0, 0) = ?
FV = $121

6-11

Practice 1: FV of Lump-sum CF

If I deposit $100,000 in a retirement account that gives 8 % p.a., what will be balance after 25 years?

6-12

Example 2: PV of Lump-sum CF

NOW WE WILL LEARN TO CALCULATE PV

What is the present value of $100,000 to be received after 10 years if the interest rate is 10 % p.a.?

Calculator: N= 10, I/Y= 10, PV= ?, PMT= 0, FV= $100,000

PV = -$38,554.33

Excel: =PV(RATE, NPER, PMT, FV, TYPE)

=PV(0.10, 10, 0, 100000, 0) = ?

PV = -$38,554.33

6-13

Practice 2: PV of Lump-sum CF

Bank is offering to pay me $1 million after 25 years. If the interest rate is 8% p.a., how much should I pay for this investment?

6-14

Example 3: N of Lump-sum CF

NOW WE WILL LEARN TO CALCULATE N

How long will it take to double my investment of $1,000 if the interest rate is 6% p.a.?

Calculator: N= ?, I/Y= 6, PV= -1000, PMT= 0, FV= 2,000

N = 11.9 years

Excel: =NPER(RATE, PMT, PV, FV, TYPE)

=NPER(0.06, 0, -1000, 2000,0) = ?

NPER = 11.9 years

6-15

Practice 3: N of Lump-sum CF

How long will it take for my investment of $10,000 to grow to $ 100,000 if the interest rate is 12 % p.a.?

6-16

Example 4: RATE (I/Y) of Lump-sum CF

NOW WE WILL LEARN TO CALCULATE RATE

Bank is offering to double my investment of $1,000 in 12 years. What is the interest rate?

Calculator: N= 12, I/Y= ?, PV= -1000, PMT= 0, FV= 2,000

I/Y = 5.95 %

Excel: =RATE(NPER, PMT, PV, FV, TYPE)

=RATE(12, 0, -1000, 2000,0) = ?

RATE = 5.95 %

6-17

Practice 4: RATE (I/Y) of Lump-sum CF

Bank is offering to pay $100,000 after 25 years if I invest $10,000 today. What is interest rate?

6-18

Multiple Cash Flows

- Annuities
- Perpetuities
- Uneven cash flows

6-19

Annuity

- Annuity is a series of cash flows of equal value occurring periodically for a specified period of time.
- Examples: Car/House loan installments
  Rent payments
- PMT key is used for Annuity CF value
- We will study Annual & Monthly annuity

6-20

Example 5: PMT of an annual Annuity

I borrow $20,000 to be repaid in FIVE EQUAL ANNUAL INSTALLMENTS. If the interest rate is 10 % p.a., what is the installment amount?

Calculator: N= 5, I/Y= 10, PV= 20000, PMT= ?, FV= 0
PMT = -$5275.95

Excel: =PMT(RATE, NPER, PV, FV, TYPE)
=PMT(0.10, 5, 20000, 0,0) = ?
PMT = -$5275.95

6-21

Practice 5: PMT of an annual Annuity

I borrow $100,000 to be repaid in TEN EQUAL ANNUAL INSTALLMENTS. If the interest rate is 8 % p.a., what is the installment amount?

6-22

Example 6: FV of an annual Annuity

If I deposit $10,000 every year in a stock account that pays 12 % p.a., how much money will I have after 25 years?

Calculator: N=25, I/Y=12, PV=?, PMT= 10000, FV= ?
FV = $1.33 million

Excel: =FV(RATE, NPER, PMT, PV, TYPE)
=FV(0.12, 25, 10000, 0,0) = ?
FV = $1.33 million

6-23

Practice 6: FV of an annual Annuity

Anna is 20 years old. She starts investing $1,000 every year in a stock account that pays 12 % p.a.. What will be the balance after 45 years?

6-24

Example 7: PMT of monthly Annuity

I take a car loan of $10,000 to be repaid in THIRTY SIX EQUAL MONTHLY INSTALLMENTS (3 YEARS). If the interest rate is 10 % p.a., what is the installment amount?

Calculator: N= 3*12, I/Y= 10/12, PV= 10000, PMT= ?, FV= 0

PMT = -$322.67

Excel: =PMT(RATE, NPER, PV, FV, TYPE)

=PMT((0.10/12), (3*12), 10000, 0,0) = ?

PMT = -$322.67

6-25

Practice 7: PMT of monthly Annuity

I take a house loan of $100,000 to be repaid in THIRTY YEARS OR (12*30 = 360) 360 EQUAL MONTHLY INSTALLMENTS. If the interest rate is 6% p.a., what is the installment amount?

6-26

Perpetuity

- Annuity is a series of cash flows of equal value occurring periodically for an infinite time period
- Examples: Pension payments
  Social security payments
- We cannot use calculator for this, rather the formula is

  PV of a Perpetuity = PMT/RATE

6-27

Example 8: PV of a Perpetuity

Bank offers to pay me an annual pension of $10,000 every year forever. If the interest rate is 10 % p.a., how much should I pay for this investment?

PV = PMT/RATE = 10000/0.10 = $100,000

6-28

Practice 8: PV of a Perpetuity

Bank offers to pay me an annual pension of $20,000 every year forever. If the interest rate is 5 % p.a., how much should I pay for this investment?

6-29

Example 9: RATE of a Perpetuity

PV = PMT/RATE can be modified to solve for

RATE = PMT/PV

Bank offers to pay me an annual pension of $10,000 every year forever if I invest $200,000 today. What is the interest rate?

RATE = PMT/PV = 10000/200000 = 0.05 or 5%

6-30

Practice 9: RATE of a Perpetuity

RATE = PMT/PV

Bank offers to pay me an annual pension of $25,000 every year forever if I invest $250,000 today. What is the interest rate?

Uneven Cash Flows

- Uneven CF's mean multiple cash flows with different values
- Examples: An investment that will pay:

  $100 after year 1 and $200 after year 2.
- We cannot use TVM keys in calculator, we have to use an application called NPV

Example 10: PV of Uneven Cash Flows

What is the present value of an investment that pays $100 after Year 1 and $200 after Year 2. The interest rate is 10% p.a.

Calculator:PV = NPV(I/Y, 0, {Cf1, Cf2,...Cfn})

= NPV(10,0,{100,200}) = $256.2

Excel: =NPV(RATE, Cf1, Cf2...Cfn)

= NPV(0.10,100,200) = $256.20

Example 10: PV of Uneven Cash Flows

BA II Plus Calculator

Press CF

Press 2nd KEY then CE|C KEY then ENTER then 2ND KEY then CPT KEY

CF0 = 0, press Enter and ↓ key

CF1 = 100, press Enter and ↓ key

F01 = 1, press Enter and ↓ key

CF2 = 200, press Enter and ↓ key

F02 = 1, press Enter and ↓ key

Press NPV KEY

I = 10, press Enter and ↓ key

Press CPT KEY

NPV = $256.20

Practice 10: PV of Uneven Cash Flows

What is the present value of an investment that pays $100 after Year 1, $200 after Year 2, and $300 after Year 3. The interest rate is 10% p.a.