

## Chapter 28 Time Value of Money

### Lump sum cash flows

1. For example, how much would I get if I deposit \$100 in a bank account for 5 years at an annual interest rate of 10%? Let's try using our calculator:

N	I/Y	PV	PMT	FV

2. How much would I get if I deposit \$100,000 in a retirement account that gives 8% p.a., assuming there are 25 years to my retirement?

N	I/Y	PV	PMT	FV

3. For example, let's calculate the present value of a \$100,000 cash flow to be received 10 years from today, assuming a 10% p.a. interest rate.

N	I/Y	PV	PMT	FV

4. A bank is promising me a lump sum payment of \$1,000,000 at the time of my retirement. If the interest rate is 8% p.a. and there are 25 years to my retirement, how much should I pay for this investment?

N	I/Y	PV	PMT	FV

5. A bank is promising a 6% p.a. interest rate on a deposit. How long will it take to double an investment at this rate?

N	I/Y	PV	PMT	FV

6. In how many years will my initial investment of \$100,000 grow to \$1,000,000 at the rate of 12% p.a.?

N	I/Y	PV	PMT	FV

7. If a bank is promising to double my investment in 12 years, what is the implied interest rate?

N	I/Y	PV	PMT	FV

8. I wish to invest \$100,000 for my retirement that is 25 years from today. At what rate should I invest this money if I wish to receive \$1,000,000 at the time of my retirement?

N	I/Y	PV	PMT	FV

### Annuity

9. If I borrow \$20,000 at the rate of 10% p.a. and agree to repay the loan in five equal annual installments, then what should be the installment amount?

N	I/Y	PV	PMT	FV

10. How much will I pay if I borrow \$100,000 at the rate of 8% p.a. and agree to repay the loan in ten equal annual installments?

N	I/Y	PV	PMT	FV

11. A bank is offering to give me an annual payment of \$25,000 for 20 years. Assuming the current interest rate is 12%, how much should I pay for this investment today?

N	I/Y	PV	PMT	FV

12. A bank is promising me an annual payment of \$50,000 for ten years. Assuming I require a return of 10%, how much should I pay for this investment today?

N	I/Y	PV	PMT	FV

13. If I borrow \$20,000 for a car loan to be repaid in monthly installments of equal value over three years and the interest rate is 12%, then what is the monthly installment amount?

You have to make the following changes:

1.  $N = \text{No. of years} * \text{Number of times in a year payment is made} = 3 * 12$
2.  $I/Y = \text{Annual Interest rate} / \text{Number of times in a year payment is made} = 12/12$
3.  $PV = \text{No change}$
4.  $PMT = \text{Amount of periodic payment}$
5.  $FV = \text{No change}$

N	I/Y	PV	PMT	FV

14. What will be the monthly installment if I borrow \$12,000 for a car to be repaid over five years? Assume an interest rate of 6% p.a.

N	I/Y	PV	PMT	FV

15. If I borrow \$10,000 to be repaid over 12 equal quarterly payments, find the quarterly installment assuming an interest rate of 6% p.a.?

N	I/Y	PV	PMT	FV

16. I wish to invest \$10,000 every six months. If the rate of return is 12% p.a. then what will be account value at the end of 25 years?

N	I/Y	PV	PMT	FV

17. Let's say I borrow \$20,000 for a car and agree to repay the loan in 36 monthly installments with **the first installment due today**. What will the monthly installment rate if the interest rate is 12% p.a.?

Again, remember to change your calculator setting to BEG for these problems.

N	I/Y	PV	PMT	FV

18. What will be the monthly installment if I borrow \$12,000 for a car to be repaid over five years at a rate of 6% p.a.? Assume that the first installment is due at the beginning (annuity due).

N	I/Y	PV	PMT	FV

### **Perpetuity**

19. How much should I pay for an investment product that pays \$50,000 every year forever if the interest rate is 10%?
20. How much should I pay for an investment product that pays \$25,000 in perpetuity if the interest rate is 5%?
21. If a bank is offering to pay \$25,000 in perpetuity for a one time investment of \$500,000 then what is the rate of return?

22. What is the rate of return if an investment returns \$50,000 every year forever for an initial investment of \$500,000?

23. If the current interest rate is 10% then what will be the perpetuity amount for an investment of \$2 million?

### **Uneven Cash Flows**

24. What is the PV of the following cash flow if the interest rate is 10% p.a.:

After 1 year: \$100

After 2 years: \$200

After 3 years: \$300

Let's start with TI 83 Plus calculator, enter values like following:

$NPV(\text{Interest rate}, 0, \{CF1, CF2, CF3\})$

$NPV(10, 0, \{100, 200, 300\}) = \$481.59$

If you are using TI BII calculator then you have to do the following steps:

Press CF

Enter 0 for C0, press ENTER and ↓ key

Enter 100 for CF1, press ENTER and ↓ key

Enter 1 for F1, press ENTER and ↓ key

Enter 200 for CF2, press ENTER and ↓ key

Enter 1 for F2, press ENTER and ↓ key

Enter 300 for CF3, press ENTER and ↓ key

Enter 1 for F3, press ENTER and ↓ key

Press NPV key

Enter 10 for I/Y, press ENTER and ↓ key

Finally, press CPT to calculate PV

24. How much should I pay for an investment product that pays \$25,000 in the first year, \$30,000 in the second year, and \$50,000 in the third year? Assume a discount rate of 5%.

### Nominal vs. Effective Rate of Interest

Bank A offers 10% p.a. interest rate on a one year deposit. Bank B offers 10% p.a. compounded semi-annually. What is the difference? Let's take a simple case where we deposit \$10,000 in each account on January 1

	Bank A	Bank B
January 1- Deposit	-10,000	-10,000
June 30 – Interest		500
	-----	-----
June 30 – Balance	10,000	10,500
December 31 – Interest	1,000	525
	-----	-----
December 31 Balance	11,000	11,025
	=====	=====

It is clear that Bank B pays \$25 extra interest. This is because it pays interest on interest earned in the first half of the year. Thus, the **Nominal Rate** (NOM) for Bank B is 10% p.a. and the **Effective Annual Rate** (EAR) is 10.25% p.a.

To convert Nominal rate of 10% compounded semi-annually into effective rate we can use our calculators. For TI83 Plus, we will say

$$\text{EFF}(\text{NOM}, \text{No. of times compounding in a year})$$

$$\text{EFF}(10,2) = 10.25$$

For TI BAI I we will use the following steps:

Press 2<sup>nd</sup> key and then press the number 2 key

Enter 10 for Nominal rate press ENTER and ↓ key

Enter 2 for C/Y- Compounding per year and press ENTER and ↑ key twice  
Press CPT to calculate the effective rate.

25. What is the Effective Annual Rate for the following nominal rates?
  - a. 12% annual compounded semi-annually
  - b. 12% annual compounded quarterly
  - c. 12% annual compounded monthly
  - d. 12% annual compounded daily
  
26. Gomez Electronics needs to arrange financing for its expansion program. Bank A offers to lend Gomez the required funds on a loan in which interest must be paid monthly, and the quoted rate is 8 percent. Bank B will charge 9 percent, with interest due at the end of the year. What is the difference in the effective annual rates charged by the two banks?
  
27. You want to borrow \$1,000 from a friend for one year, and you propose to pay her \$1,120 at the end of the year. She agrees to lend you the \$1,000, but she wants you to pay her \$10 of interest at the end of each of the first 11 months plus \$1,010 at the end of the 12<sup>th</sup> month. How much higher is the effective annual rate under your friend's proposal than under your proposal?