The Roles of the S₁ and S₂ Scattering Matrix Terms on Half-Widths and Their Temperature Dependence for the Water Vapor-Nitrogen System

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Abstract. The effect of the first (S_1) and second (S_2) order successive approximations to the S matrix on the half-width and the temperature dependence of the half-width are studied for the water vapor-nitrogen system. The calculations are done via the Robert-Bonamy semi-classical formalism. It is found that the influence of the S_1 and S_2 terms depends on the rotational states of the transition. For the H_2O-N_2 system, half-widths for H_2O transitions with low J quantum numbers are dominated by the S_2 terms. Transitions with high J values the half-width is determined largely by the S_1 term. For transitions with intermediate values of J both the S_1 and S_2 terms contribute. The temperature dependence of the half-width. It is found that the standard power law temperature dependence of the half-width is only valid for transitions where $Re(S_2)$ dominates. When S_1 dominates the temperature dependence is negative and when both terms contribute the temperature dependence is not modeled by the power law.

Keywords: half-width, temperature dependence of the half-width, H_2O-N_2 . **PACS:** 30

INTRODUCTION

The complex the Robert-Bonamy (CRB) formalism [1] to calculate the half-width and line shift for a ro-vibrational transition is a semi-classical method based on a second order successive expansion of the S matrix. This approach yields a first order term that is purely isotropic and imaginary, S_1 , and a complex second order term, S_2 . Depending on the collision interaction system each term can contribute more or less. In this work it is found that for the H₂O-N₂ collision system the contributions of the S_1 and S_2 terms changes from transition to transition. The mechanism for this is detailed below. The CRB calculations used a potential which consists of the leading electrostatic components for the H_2O-N_2 (the dipole and quadrupole moments of H_2O with the quadrupole moment of N_2), atom-atom interactions, and isotropic induction and London dispersion interactions in the S_1 term. Here the atom-atom part of the potential is expanded to 8th order to insure convergence of this part of the potential [2]. The isotropic component of the atom-atom potential are fit to an effective isotropic (or "hetero-molecular") Lennard-Jones 6-12 potential which is used to define the trajectory of the collisions within the semi-classical model of Robert and Bonamy [1]. All parameters used in the calculations are the best available from the literature and can be found in Ref. [3].

RESULTS AND DISCUSSION

The calculation of the half-width and line shift for a ro-vibrational transition $f \leftarrow i$ is made by averaging over all collisionally induced transitions. In the collision the absorbing molecule is driven from state *i* to state *i*' (or *f* to state *f*') with an energy gap $\Delta E_{i \rightarrow i'}$ in the same collision the perturbing molecule goes from state J_2 to J_2 with an energy gap $\Delta E_{J2 \rightarrow J2}$. If the energy gaps match each other the collision is said to be on resonance and the collisional contribution to the half-width will be large. These components are in the $Re(S_2)$ term. For conditions where the energy gaps do not match the collision is said to be off resonance and the collisional contribution is small. In such cases it is the S₁ term and the $Im(S_2)$ term that contribute to the half-width What is observed for the H₂O-N₂ system is that for low J states of water the collisions are on resonance and the half-width is determined mostly from collisional contributions, the $Re(S_2)$ term. For high J states most of the collisions are off resonance. In such cases the half-width is determined from vibrational contributions (S₁ and $Im(S_2)$).. These tend to be narrow lines.

The temperature dependence of the half-width was worked out by Birnbaum [4] assuming on resonance collisions and a single term in the intermolecular potential. The result is the standard power law seen in the literature, for example see Wagner et al. [5]. We have observed that for low J lines, which are determined mostly by the real part of S₂, the power law model works well. (Note, all figures are for the v₂ band of water.) Figure 1a shows the fit for the 2 $_0 _2 \leftarrow 3 _{-1} _3$ transition. The slope (temperature exponent) is 0.74 and the correlation coefficient is 1 to four significant figures indication the power law model is nearly perfect for this line. For the high J lines, where the S₁ and *Re*(S₂) terms determine the half-width, the temperature exponents can be negative and the power law is approximate, see Fig. 1b. What is interesting is the intermediate J region where both the S₁ and S₂ parts contribute to the half-width. Figure 1c is a plot for the 9 $_0 _9 \leftarrow 8 _{-1} _8$ transition. The temperature dependence of these lines is not modeled by the power law.

Caution should be used when modeling the temperature dependence of the halfwidth of N₂-broadened water vapor transitions. When the collisions are on resonance the usual power law model correctly describes the temperature dependence. For high J lines where the collisions are far off resonance the power law approximately models the temperature behavior. However for intermediate J transitions the power law fails to model the observed temperature dependence of the half-width. Similar results are found for O₂- and air-broadening of water vapor transitions. This could have large effects in modeling radiative transfer in mediums where the temperature is changing, i.e. Earth's atmosphere, combustion media, etc. For such cases a more sophisticated model is needed.



FIGURE 1. Plots of $\ln{\gamma(T)/\gamma(T_0)}$ versus $\ln{T_0/T}$ to determine the temperature exponent in the temperature range 150-400 K. (a) 2 $_0 _2 \leftarrow 3 _{-1 - 3} v_2$ band transition, (b) 15 $_0 _{-15} \leftarrow 16 _{-1 - 16} v_2$ band transition, (c) 9 $_0 _9 \leftarrow 8 _{-1 - 8} v_2$ band transition.

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REFERENCES

- ¹ D. Robert and J. Bonamy, J. Phys. Paris. **40**, 923 (1979).
- ² S.P. Neshyba and R.R. Gamache, J. Quant. Spectrosc. Radiat. Transfer **50**, 443 (1993).
- ³ G. Durry, V. Zeninari, B. Parvitte et al., J. Quant. Spectrosc. Radiat. Transfer **94**, 387 (2005).
- ⁴ George Birnbaum, Intermolecular Forces **12**, 487 (1967).
- ⁵ Georg Wagner, Manfred Birk, Robert. R. Gamache et al., J. Quant. Spectrosc. Radiat. Transfer **92**, 211 (2004).