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### **ARTICLE IN PRESS**

Journal of Quantitative Spectroscopy & Radiative Transfer III (IIII) III-III

Iournal of Ouantitative Spectroscopy & Radiative Transfer

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# Half-widths of $H_2^{16}O$ , $H_2^{18}O$ , $H_2^{17}O$ , $HD^{16}O$ , and $D_2^{16}O$ : II. Comparison with measurement

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Received 29 May 2002; accepted 3 September 2002

#### Abstract

- 9 Pressure-broadened half-widths of  $v_2$  transitions are determined using the complex Robert–Bonamy formalism for four isotopomers of water vapor: H<sub>2</sub><sup>18</sup>O, H<sub>2</sub><sup>17</sup>O, HD<sup>16</sup>O, and D<sub>2</sub><sup>16</sup>O. The calculations are made with 11 nitrogen and oxygen as the perturbing gases. A survey of the literature revealed N<sub>2</sub>-broadened measurements
- of 365, 61, 990, and 545  $v_2$  transitions of the H<sub>2</sub><sup>18</sup>O, H<sub>2</sub><sup>17</sup>O, HD<sup>16</sup>O, and D<sub>2</sub><sup>16</sup>O isotopomers, respectively, and O<sub>2</sub>-broadened measurements of 266 and 134  $v_2$  transitions of the HD<sup>16</sup>O, and D<sub>2</sub><sup>16</sup>O isotopomers, respectively. 13
- The calculated and measured half-widths are compared. In general, good agreement is observed when the 15 most recent atom-atom constants are employed.
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- Keywords: Half-width; H<sub>2</sub><sup>18</sup>O, H<sub>2</sub><sup>17</sup>O, HD<sup>16</sup>Oand D<sub>2</sub><sup>16</sup>O; ; Water vapor isotopomers 17

#### 1. Introduction

19 In a previous study [1], hereafter called part I, the collision-broadened half-widths of five isotopomers of water vapor, H<sup>16</sup><sub>2</sub>O, H<sup>18</sup><sub>2</sub>O, H<sup>17</sup><sub>2</sub>O, HD<sup>16</sup>O, and D<sup>16</sup><sub>2</sub>O, broadened by N<sub>2</sub>, and O<sub>2</sub> were 21 calculated via the complex Robert-Bonamy formalism. The half-widths of the lesser abundant isotopomers were compared with the values for the principal species. It was observed that for  $H_2^{18}O$ and H<sub>2</sub><sup>17</sup>O the differences are small, a few percent maximum, whereas for HD<sup>16</sup>O and D<sub>2</sub><sup>16</sup>O the 23 differences can be quite large, up to 43%. These differences will have large effects if one is relying 25 on the current databases [2,3] to interpret spectra since these databases use the half-widths of the principal species for all isotopomers. Half-widths determined from algorithms based on an average ratio between different isotopomers will have large errors because the ratios change greatly as a 27 function of the transition considered with little dependence on the rotational quantum numbers.



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1 Many questions must be asked of the results of part I before algorithms are developed to predict half-widths of one isotopomer of water vapor from another isotopomer. It was observed that a 3 relatively small change in the atom-atom parameters of D resulted in noticeable changes in the calculated half-widths. The change in the parameters came from a study [4] where the atom-atom 5 parameters for  $H_2$  and  $D_2$  were determined via ab initio calculation. The new values were then used to determine the atom-atom parameters for the interactions N-D and O-D using conventional 7 combination rules [5]. There are a number of different methods that have been proposed to determine heteronuclear potential parameters from homonuclear parameters [6-8] and references therein. Thus, 9 the resulting parameters have an uncertainty which depends on the method chosen to go from the homonuclear to the heteronuclear parameters. For some methods the uncertainty is quite large. Hence adjustment of the atom-atom parameters within  $\sim 15\%$  around the values given by the combination 11 formula of Hirchfelder et al. is not unreasonable provided there are reliable experimental data. For 13 water vapor the confidence in the measured database in 1994 was not high [9]. However, the situation is improving with multiple newer measurements available for the principle isotopic species. Toth [10] has studied the region from 604 to 2271 cm<sup>-1</sup>, the measurements of Zou and Varanasi [11] consider 15 the regions 950-2100 and 3000-4050 cm<sup>-1</sup>. Schermaul et al. [12,13] have investigated the region  $8600-15000 \text{ cm}^{-1}$ , Coheur et al. [14] have looked at the  $13000-25000 \text{ cm}^{-1}$  region, and Stevert 17 et al. [15] measured rotational transitions in the  $380-600 \text{ cm}^{-1}$  range. There are now a number of measurements of the half-widths for the lesser isotopomers of water 19 vapor [10,15-20] to which the calculations can be compared. Devi et al. [16,17] made measurements of air- and N<sub>2</sub>-broadening of  $v_2$  transitions for D<sub>2</sub>O, and for HDO, H<sup>16</sup><sub>2</sub>O and H<sup>18</sup><sub>2</sub>O. Rinsland et al. 21 [18] later studied pressure-broadening and pressure-induced line shifts for more than 100 transitions in the  $v_2$  band of D<sub>2</sub>O in air, nitrogen, and oxygen. This work was later extended [19] to consider 23 air-, nitrogen-, and oxygen-pressure broadening and pressure-induced line shifts for more than 200 transitions in the  $v_2$  band of HD<sup>16</sup>O. More recently, Toth has measured a large number of air- and 25 N<sub>2</sub>-broadened half-widths and pressure-induced frequency shifts for transitions of HDO and D<sub>2</sub>O from 709 to 1936 cm<sup>-1</sup> [20] and  $H_2^{16}O$ ,  $H_2^{18}O$ , and  $H_2^{17}O$ , from 604 to 2271 cm<sup>-1</sup> [10]. 27 In this work, the intermolecular potentials of the isotopomers of H<sub>2</sub>O are refined by fitting to 29 several transitions that have been measured for  $H_2^{16}O$  by a number of investigators. This is done for

nitrogen and oxygen broadening of water vapor. Next, using the ratios between H and D and the results of part I, values are predicted for the other isotopomers. Calculations are then made for N<sub>2</sub>and O<sub>2</sub>-broadened transitions of H<sub>2</sub>O that have been measured and the calculations compared with the measurements.

#### 2. Complex Robert-Bonamy formalism

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All calculations made employ the complex Robert–Bonamy formalism [21], which was described in part I; here only salient features are described. The CRB formalism is a complex valued semi-classical method which yields the half-width and line shift from a single calculation. The collision dynamics are correct to second order in time, which has important consequences in the description of close intermolecular collisions (small impact parameters). Also important for close collision systems is the incorporation in the CRB theory of a short range (Lennard-Jones 6–12 [22]) atom–atom com-

41 ponent to the intermolecular potential. This component has been shown to be essential for a proper

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- 1 description of pressure-broadening, especially in systems where electrostatic interactions are weak [23]. (Here, the notion of strong and weak collisions adopts the definition of Oka [24].)
- 3 The half-width, γ of a ro-vibrational transition f ← i is given in the complex Robert-Bonamy (CRB) formalism by minus the imaginary part of the diagonal elements of the complex relaxation 5 matrix. In computational form, the half-width is usually expressed in terms of the Liouville scattering matrices, S<sub>1</sub> and S<sub>2</sub> [25,26] which depend on the intermolecular potential.
- 7 The potential employed in the calculations consists of the leading electrostatic components for the  $H_2O-X$  pair (the dipole and quadrupole moments of  $H_2O$  with the quadrupole moment of  $N_2$
- 9 or  $O_2$ ), and an atom-atom component [27,28] and isotropic induction and dispersion components. The isotropic component of the atom-atom potential is used to define the trajectory of the collision
- 11 within the semi-classical model of Robert and Bonamy [21].
- The atom-atom potential is defined as the sum of pair-wise Lennard-Jones 6-12 interactions [22] 13 between atoms of the radiating molecule and the perturbing molecule, N<sub>2</sub> or O<sub>2</sub> and is given in
- terms of the Lennard-Jones parameters for the atomic pairs, the  $\varepsilon_{ij}$  and  $\sigma_{ij}$ . The heteronuclear atom-15 atom parameters are usually constructed from homonuclear-atom-atom parameters ( $\varepsilon_i$  and  $\sigma_i$ ) by
- "combination rules" [29], however as described above the resulting parameters may have some error. The atom-atom potential must be expanded in terms of the center-of-mass internuclear separation,
- *R*. The order of the expansion has been discussed by Labani et al. [30] and by Gamache et al.
- 19 [27,28,31]. Here the formulation of Neshyba and Gamache [28] expanded to eighth order is used.

#### 2.1. Details of CRB theory

The expressions for the  $S_1$  and  $S_2$  terms in the CRB formalism are described in detail in Refs. [31,32]. Note that the first-order (imaginary) term,  $S_1$ , depends only on the difference in the isotropic part of the interaction potential between the initial and final vibrational states of the radiator and is accounted for by the vibrational dependence of the dipole moment and polarizability of H<sub>2</sub>O, the ionization potentials of the collision molecules, and the polarizability of the perturbing molecule. The second-order terms are comprised of two basic parts; one describing the internal states of the radiating and perturbing molecules and another describing the interaction and dynamics of the collision. These are calculable in terms of a number of molecular constants describing the colliding pair: electrostatic moments, atom-atom potential constants, isotropic potential constants.

- Many of the molecular parameters are well known and the calculations use the best available values from the literature. The dipole and quadrupole moments of water vapor are taken from Refs.
- [33,34], respectively. The quadrupole moment of nitrogen is from Mulder et al. [35] and that for oxygen is from Stogryn and Stogryn [36]. The numerical values are listed in Table 1.
- The ionization potential of water is taken to be a vibrationally independent 12.6 eV [37]. The polarizability of nitrogen and oxygen are taken from Ref. [38] and are  $17.4 \times 10^{-25}$  cm<sup>3</sup> and  $15.8 \times 10^{-25}$  cm<sup>3</sup>, respectively. The ionization potential of nitrogen is 15.576 eV [39] and the value for
- 37 oxygen is 12.063 eV [37].

The reduced matrix elements for the internal states of the radiator and perturber must be deter-39 mined. For water vapor, these are evaluated using wave functions determined by diagonalizing the Watson Hamiltonian [40] in a symmetric top basis for the vibrational states involved in the transi-

41 tion. For the ground state of  $H_2^{16}O$  the Watson constants derived by Flaud and Camy-Peyret [41] are used. For  $H_2^{18}O$  and  $H_2^{17}O$  the Watson constants of Toth [42] are used. For HDO and D<sub>2</sub>O the

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Table	1
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Values of electrostatic moments for the water vapor, N<sub>2</sub>, and O<sub>2</sub>

Molecule	Multipole moment (esu)	Reference	
H <sub>2</sub> O	$\mu = 1.8549 \times 10^{-18}$ $Q_{xx} = -0.13 \times 10^{-26}$ $Q_{yy} = -2.5 \times 10^{-26}$ $Q_{zz} = 2.63 \times 10^{-26}$	[33] [34] [34] [34]	
N <sub>2</sub> O <sub>2</sub>	$egin{aligned} Q_{zz} &= -1.4  imes 10^{-26} \ Q_{zz} &= -0.4  imes 10^{-26} \end{aligned}$	[35] [36]	

1 Watson constants are those of Toth [43]. The rotational constants for  $N_2$  and  $O_2$  are 2.0069 and 1.4377 cm<sup>-1</sup>, respectively [44].

3 In the parabolic approximation, the isotropic part of the interaction potential is taken into account in determining the distance, effective velocity, and force at closest approach [21]. To simplify the tra-

- 5 jectory calculations, the isotropic part of the atom-atom expansion is fit to an isotropic Lennard-Jones 6-12 potential.
- 7 Part I showed marked improvement in the comparison of theory and experiment for HDO and  $D_2O$  by utilizing homonuclear-atom-atom parameters for deuterium derived by Wang [4] in the standard
- 9 combination rules. The difference in the constants is only  $\sim 10\%$  for  $\varepsilon$  and 0.5% for  $\sigma$  yet this change leads to improved theoretical values. Good and Hope [8] showed that different combination
- rules lead to variations in  $\varepsilon$  of ~15% and  $\sigma$  of ~1%. It would not seem unreasonable to adjust the atom-atom potential coefficients if sufficient experimental data exist. This is the case for the

13 principal species of water vapor for which many measurements exist [45]. For nitrogen broadening of  $H_2^{16}O$  six transitions were chosen for which there are multiple mea-

- 15 surements [10,14,46–52] and for which the calculations were too high for two lines, too low for two lines, and in agreement for the last two transitions. The potential parameters were adjusted to
- 17 give a good fit of all the lines. The parameters are  $\varepsilon_{\rm HN}$ ,  $\sigma_{\rm HN}$ ,  $\varepsilon_{\rm ON}$ , and  $\sigma_{\rm ON}$ . For the starting values the heteronuclear atom-atom parameters used are derived from homonuclear-atom-atom parameters
- 19 obtained by Bouanich [53] using the combination rules [29]. The final values correspond to a 10% lowering of  $\varepsilon_{ON}$ , a 10% increase in  $\varepsilon_{HN}$ , a 3.9% decrease in  $\sigma_{HN}$ , and no change in  $\sigma_{ON}$ . Note, a
- 21 full least-squares minimization was not thought worthwhile until the work of Ref. [45] is completed. For oxygen broadening of  $H_2^{16}O$ , there are not multiple measurements to allow fitting to an average.
- 23 A similar procedure to that used for H<sub>2</sub>O-N<sub>2</sub> with some of the data from Ref. [54] was done. The best fit was obtained by lowering  $\sigma_{HO}$  by 10% of the combination rule value. The other atom-atom
- 25 parameters ( $\varepsilon_{HO}$ ,  $\varepsilon_{OO}$ ,  $\sigma_{OO}$ ) are the combination rule values.

The atom-atom parameters for the deuterated species of water were derived by taking the combination rule values for are  $\varepsilon_{\text{HN}}$ ,  $\sigma_{\text{HN}}$ ,  $\varepsilon_{\text{ON}}$ ,  $\sigma_{\text{ON}}$  and  $\varepsilon_{\text{HO}}$ ,  $\varepsilon_{\text{OO}}$ ,  $\sigma_{\text{OO}}$  derived by taking the

- homonuclear-atom-atom parameters obtained by Bouanich [53] and the  $D_2$  parameters given by Wang [4], forming the D to H ratio of the parameters, and then scaling the best fit parameters
- by the same ratios to get  $\varepsilon_{\text{DN}}$ ,  $\sigma_{\text{DN}}$ ,  $\varepsilon_{\text{DO}}$ ,  $\sigma_{\text{DO}}$ . Since homonuclear-atom-atom parameters are not 31 available for the lesser isotopomers of H<sub>2</sub>O, the H<sub>2</sub><sup>18</sup>O and H<sub>2</sub><sup>17</sup>O atom-atom constants are those of the principal species. The values used in the calculations are given in Table 2.

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Table 2

Values of the heteronuclear atom-atom Lennard-Jones (6-12) parameters for the collision pairs considered in this work

Atomic pair	$\sigma/{ m \AA}$	$\varepsilon/k_{\rm B}$ (°K)
H-N	2.7807	22.506
H-O	2.565	24.13
D-N	2.7704	20.339
D-0	2.5524	21.81
O-N	3.148	39.51
0-0	3.010	51.73

Table 3

Statistics of the comparison of measured half-widths for isotopomers of water vapor. APD  $\equiv$  average percent difference, AAPD  $\equiv$  average absolute percent difference, SD  $\equiv$  standard deviation

Comparison	# Points	PD/APD	Max difference	SD
H <sub>2</sub> <sup>18</sup> O-N <sub>2</sub>				
Refs. [17–10]	5	13/36	55	34
HDO-N <sub>2</sub>				
Refs. [17–19]	21	-2.6/4.4	11	5.0
Refs. [17–20]	29	-2.4/5.0	16	6.1
Refs. [17–10]	2	29/29	43	20
Refs. [19–20]	261	0.32/1.8	16	2.7
Refs. [19–10]	32	20/20	62	16
Refs. [20–10]	53	19/20	68	18
$D_2O-N_2$				
Refs. [18–20]	125	-1.0/2.5	12	3.2

#### 1 3. Calculations

The measured data for H<sup>18</sup><sub>2</sub>O [10,17], H<sup>17</sup><sub>2</sub>O [10], HD<sup>16</sup>O [10,17,19,20], and D<sup>16</sup><sub>2</sub>O [16,18,20]
were taken and the N<sub>2</sub>- and O<sub>2</sub>-broadened values extracted. This yielded 365, 61, 990, and 545 N<sub>2</sub>-broadened measurements of v<sub>2</sub> transitions of the H<sup>18</sup><sub>2</sub>O, H<sup>17</sup><sub>2</sub>O, HD<sup>16</sup>O, and D<sup>16</sup><sub>2</sub>O isotopomers, respectively, and 266 and 134 O<sub>2</sub>-broadened measurements of v<sub>2</sub> transitions of the HD<sup>16</sup>O, and D<sup>16</sup><sub>2</sub>O, and D<sup>16</sup><sub>2</sub>O, and D<sup>16</sup><sub>2</sub>O, and D<sup>16</sup><sub>2</sub>O, and D<sup>16</sup><sub>2</sub>O, and D<sup>16</sup><sub>2</sub>O

respectively, and 200 and 154 O<sub>2</sub>-bloadened measurements of V<sub>2</sub> transmission of the HD O, and D<sub>2</sub> O isotopomers, respectively. The rotational quantum numbers were taken and used as input to the CRB codes. Calculations were made at a temperature of 296 K with nitrogen and oxygen as the perturbing

gases.

#### 9 4. Results

Tables of the results for each isotopomer and perturber have been prepared. Electronic copies of the complete tables can be obtained from one of the authors (RRG).

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Table 4

Statistics of the comparison of multiple measurements compared with measurement average for half-widths for isotopomers of water vapor. AAPD  $\equiv$  average absolute percent difference

Common measurements	# Compared	H <sub>2</sub> <sup>18</sup> O AAPD	# Compared	HDO AAPD	# Compared	D <sub>2</sub> O AAPD
2	5	20	239	2.1	125	1.2
3	0		49	6.6	0	
4	0		2	13	0	



Fig. 1. Measured [10,17] and CRB calculated half-widths for the  $H_2^{18}O-N_2$  system versus an energy-ordered index,  $J^*(J+1)+Ka-Kc+1$ . The range of the x-axis was chosen to demonstrate the comparison of the data.

The comparison of the calculated half-widths with the measurements is complicated by the fact that multiple measurements have been made for a number of transitions. Before comparing the calculations with the measurements, it is useful to first ask how well the measurements compare with each other. This is possible for nitrogen broadening of H<sup>18</sup><sub>2</sub>O, HDO and D<sub>2</sub>O for which there are 2, 4, and 2 independent measurements. The data were taken and the transitions for which there were multiple measurements were found and the average percent difference (APD), average absolute

7 percent difference (AAPD), maximum percent difference between the measurements and the standard deviation (SD) of the percent differenced determined. These data are presented in Table 3 along with

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Fig. 2. Measured [10] and CRB calculated half-widths for the  $H_2^{17}$ O-N<sub>2</sub>system versus an energy-ordered index,  $J^*(J+1)$ +Ka-Kc+1. The range of the *x*-axis was chosen to demonstrate the comparison of the data.

the number of comparisons made. The first column of the table gives the reference numbers for the data being compared. For H<sub>2</sub><sup>18</sup>O-N<sub>2</sub> there are only five lines that can be compared. The AAPD is
 about 36% between the measurements with a maximum difference of ~55%.

For HDO-N<sub>2</sub> there are four independent measurements allowing six intercomparisons to be made. 5 The number of transitions compared in the table are 21, 29, 2, 261, 32, and 53. Of particular interest

is the comparison of Refs. [19,20]. These are measurements by two different groups that compare 261 transitions. The AAPD is 2%, the SD is 3%, and the maximum difference is 16%. Several of

the other comparisons show an AAPD  $\sim 4\%$  and SD  $\sim 5\%$ . Three of the intercomparisons show 9 AAPD of 20%, 20%, and 29%, however the 29% is based on a comparison of two transitions.

The maximum deviations range from  $\sim 11\%$  to 68%. The comparison of Refs. [10,20] based on 53 transitions shows a slightly larger maximum difference of 68%. However it should be noted that

for high J transitions the values of the half-width can be rather small which can exaggerate small differences in the reported values.

For  $D_2O-N_2$  only two measurements were made allowing the single comparison. One hundred and twenty five transitions were measured in common and the AAPD is 2.5% with an SD of 3.2 and a maximum difference of 12%.

17 Unfortunately there are only single measurements for  $H_2^{17}O-N_2$ , HDO-O<sub>2</sub>, and D<sub>2</sub>O-O<sub>2</sub> and no measurements for  $H_2^{18}O-O_2$  or  $H_2^{17}O-O_2$  making comparisons between measurements impossible.

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Fig. 3. Measured [10,17,19,20] and CRB calculated half-widths for the HDO-N<sub>2</sub> system versus an energy-ordered index,  $J^{*}(J+1)+Ka-Kc+1$ . The range of the *x*-axis was chosen to demonstrate the comparison of the data.

In some cases there are two, three, or four measurements for a given transition. In such cases the calculations will be compared with the measurement average. One would expect to have more confidence for such cases if the measurements agree. In Table 4, the measurements are compared to the measurement average when multiple data are available for a transition. Listed are the number of common measurements, the number of comparisons, and the average absolute percent differences

(AAPD) for the systems where multiple measurements were made: H<sup>18</sup><sub>2</sub>O-N<sub>2</sub>, HDO-N<sub>2</sub>, and D<sub>2</sub>O-N<sub>2</sub>.
 For H<sup>18</sup><sub>2</sub>O-N<sub>2</sub> and D<sub>2</sub>O-N<sub>2</sub> only pairs of data are possible. For H<sup>18</sup><sub>2</sub>O-N<sub>2</sub> there are five pairs and the

APD is 20%, for  $D_2O-N_2$  there are 125 pairs with an AAPD of 1.24. HDO-N<sub>2</sub> has 239 pairs, 49 9 3-data comparisons, and two 4-data comparisons with AAPDs of 2.1%, 6.6%, and 12.8%, respec-

tively. These results indicate that having multiple data for a transition does not necessarily improve the data to compare with the calculations. For example, for HDO-N<sub>2</sub> comparing 2-data points for 239 transitions gives an AAPD of 2.1%, comparing 3-data points for 49 transitions yields an AAPD

13 of 6.6%, and when 4-data points are compared for two transitions the AAPD is 13%. Figs. 1–6 present portions of the comparisons of the measurements with the calculated values

- 15 for H<sub>2</sub><sup>18</sup>O-N<sub>2</sub>, H<sub>2</sub><sup>17</sup>O-N<sub>2</sub>, HDO-N<sub>2</sub>, D<sub>2</sub>O-N<sub>2</sub>, HDO-O<sub>2</sub>, and D<sub>2</sub>O-O<sub>2</sub>, respectively. Because of the number of data, plots of the entire data sets are too confusing and not as useful as regions where the
- 17 comparisons can be better seen. The regions were chosen to give a good number of comparisons and multiple measurements for a single transition when possible. The plots are the measured half-widths

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Fig. 4. Measured [18,20] and CRB calculated half-widths for the  $D_2O-N_2$  system versus an energy-ordered index,  $J^*(J+1)+Ka-Kc+1$ . The range of the *x*-axis was chosen to demonstrate the comparison of the data.

- 1 with error bars and the calculated half-widths (solid triangle symbols) versus a unique lower state rotational quantum number index,  $J^*(J+1)+Ka-Kc+1$ . Because there can be multiple transitions from
- 3 a lower rotational state to a number of upper rotational states, the points are shifted for each new transition with a given lower state index. This was done to allow better interpretation of the figures.
- In Table 5, the calculations are compared with the measurement average for the systems studied.
   Presented are the number of measurements per transition, the number of transitions compared, the
   APD and AAPD for H<sup>18</sup><sub>1</sub>O-N<sub>2</sub>, HDO-N<sub>2</sub>, and D<sub>2</sub>O-N<sub>2</sub>. This is of a similar format as Table 4 except
- 7 APD and AAPD for  $H_2^{18}O-N_2$ , HDO- $N_2$ , and  $D_2O-N_2$ . This is of a similar format as Table 4 except that in Table 4 the APDs were necessarily zero. For the comparison with the average of pairs for
- 9  $H_2^{18}$ O-N<sub>2</sub> there are five transitions and the result shows an AAPD of 24%, roughly the same as the comparison of experiment with experiment. This is clear from Fig. 1 which shows three points
- 11 from Ref. [17] which do not agree with the measurements of Ref. [10] or the calculations. The comparison with the average of pairs of measurements for 125 transitions for  $D_2O-N_2$  gives an APD
- 13 of -4.9% and an AAPD of 5.2%. These numbers are higher than the corresponding values in Table 4 indicating that the calculations are at least a few percent high. This is evident in Fig. 4.
- For HDO-N<sub>2</sub> comparisons can be made with averages of 2, 3, and 4 measured points. There are 239 pair transitions for which the measurements were averaged. The comparison with the calculations
- 17 shows -3.1 for the APD and 4.7 for the AAPD. There are 49 transitions for which three measured values are averaged to compare with the calculations. The APD and AAPD for this comparison

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Fig. 5. Measured [19] and CRB calculated half-widths for the HDO-O<sub>2</sub> system versus an energy-ordered index,  $J^{*}(J+1)+Ka-Kc+1$ . The range of the *x*-axis was chosen to demonstrate the comparison of the data.

are -7.5% and 7.9%, respectively. For the two 4-measurement averages the APD is -15 and the AAPD is 15. These values are comparable to those for the measurement-measurement comparisons,
 possibly indicating that the calculated half-widths are ~1% too large on average.

Finally, in Table 6 the comparisons for the single measurement with the calculation are made 5 for the systems considered here. For  $H_2^{18}O-N_2$  the results are quite good, 355 transitions compared,

APD equal to 0.74%, AAPD 6.4%. What is difficult to explain is that for  $H_2^{17}$ O-N<sub>2</sub> the results

7 are not as good. There are only 61 transitions that are compared. The APD between measurement and calculation is -7.0 and the AAPD is 9.3. The measurements are mostly those of Toth

9 [10] for which there are 21 transitions that he reports half-widths for both isotopomers,  $H_2^{18}O-N_2$  and  $H_2^{17}O-N_2$ . Comparing Toth's measurements for the two isotopomers gives an APD of 0.03% and

11 an AAPD of 4.6%. Thus, similar agreement is expected for the comparison of the  $H_2^{18}O-N_2$  and  $H_2^{17}O-N_2$  results. Perhaps the particular 61 transitions for  $H_2^{17}O-N_2$  system are not as well calcu-

13 lated compared with the 355 transitions for  $H_2^{18}$ O-N<sub>2</sub> system. For HDO the comparisons for N<sub>2</sub>- and O<sub>2</sub>-broadening are quite good, APD=-3.4% and 2.2%, re-

15 spectively. For D<sub>2</sub>O the comparisons give larger APDs, -6.8% and -7.8% for N<sub>2</sub>- and O<sub>2</sub>-broadening, respectively, indicating the calculations are too high. However, for nitrogen broadening the compar-

17 ison with the 125 transitions that were measured by two groups [16,20] the AAPD drops to  $\sim$ 5%. This fact is observed in Figs. 4 and 6.

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Fig. 6. Measured [18] and CRB calculated half-widths for the  $D_2O-O_2$  system versus an energy-ordered index,  $J^*(J+1)+Ka-Kc+1$ . The range of the x-axis was chosen to demonstrate the comparison of the data.

Table 5 Statistics for the comparison of CRB calculated values compared with multiple measurement average for half-widths for isotopomers of water vapor. AAPD  $\equiv$  average absolute percent difference, AAPD  $\equiv$  average absolute percent difference

# Measurements per transition	# Compared	H <sub>2</sub> <sup>18</sup> O-N <sub>2</sub> APD/AAPD	# Compared	HDO-N <sub>2</sub> APD/AAPD	# Compared	D <sub>2</sub> O-N <sub>2</sub> APD/AAPD
2	5	12/24	239	-3.1/4.7	125	-4.9/5.2
3	0		49	-7.5/7.9	0	_
4	0	_	2	-15/15	0	—

#### 1 5. Summary

CRB calculations of the pressure-broadened half-width were made for some two thousand v<sub>2</sub>
band transitions of the lesser isotopomers of water vapor for which measured values are available. Comparisons were made for 365 transitions for the H<sub>2</sub><sup>18</sup>O-N<sub>2</sub> system, 61 transitions for the H<sub>2</sub><sup>17</sup>O-N<sub>2</sub>
system, 990 and 266 transitions for the HD<sup>16</sup>O-N<sub>2</sub> and -O<sub>2</sub> systems, respectively, and 545 and 134 transitions for the D<sub>2</sub><sup>16</sup>O-N<sub>2</sub> and -O<sub>2</sub> systems, respectively. The atom–atom part of the intermolecular potential was adjusted for the principal isotopomers of water vapor from which the parameters for

the lesser isotopomers species were determined by scaling by the H/D ratio. The authors feel that for

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Table 6

Statistics for the comparison of CRB calculated values compared with a single measurement for half-widths for isotopomers of water vapor. AAPD  $\equiv$  average absolute percent difference, AAPD  $\equiv$  average absolute percent difference

System	# Compared	APD	AAPD
$H_2^{18}$ O-N <sub>2</sub>	355	0.74	6.4
$H_2^{\bar{1}7}$ O-N <sub>2</sub>	61	-7.0	9.3
HDO-N <sub>2</sub>	375	-3.4	6.9
$D_2O-N_2$	295	-6.8	7.5
HDO-O <sub>2</sub>	266	2.2	3.5
D <sub>2</sub> O-O <sub>2</sub>	134	-7.8	8.0

 the lesser isotopomers there are not enough measurements by different groups to make adjustments to the atom-atom parameters at this time. The H<sub>2</sub><sup>18</sup>O-N<sub>2</sub> and H<sub>2</sub><sup>17</sup>O-N<sub>2</sub> systems employed the parameters
 of the H<sub>2</sub><sup>16</sup>O-N<sub>2</sub> system.

The agreement of the calculations with the measurements was good, roughly on par with the 3 agreement between different measurements. The figures and statistics suggest that the calculations

for  $H_2^{17}O$  broadened by N<sub>2</sub> may be ~5% too high. The calculations for D<sub>2</sub>O broadened by N<sub>2</sub> and O<sub>2</sub> appear to be ~5% higher on average than measurement. The calculations for HD<sup>16</sup>O-N<sub>2</sub> may

be  $\sim 1\%$  too high compared with the measured values.

9 Before calculations of the half-width are made for many transitions of the isotopomers of water vapor to generate a database, a refinement of the atom-atom potential should be made. However, this

- 11 will require that more measurements are made. Currently, only the HD<sup>16</sup>O-N<sub>2</sub> system has had many measurements by several groups. The authors suggest that measurements of  $v_2$  band transitions be
- 13 made for N<sub>2</sub>- and O<sub>2</sub>-broadening of transitions of  $H_2^{18}O$ ,  $H_2^{17}O$ ,  $HD^{16}O$ , and  $D_2^{16}O$ . For atmospheric applications D<sub>2</sub>O can be left off the list; however, from the point of view of the theory it is interesting
- 15 to compare calculations for HDO and  $D_2O$ .

#### Acknowledgements

The authors would like to thank Ms. Caroline Claude for preparing many of the files of measured half-widths. The authors are pleased to acknowledge support of this research by the National Science Foundation through Grant No. ATM-9415454. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

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