

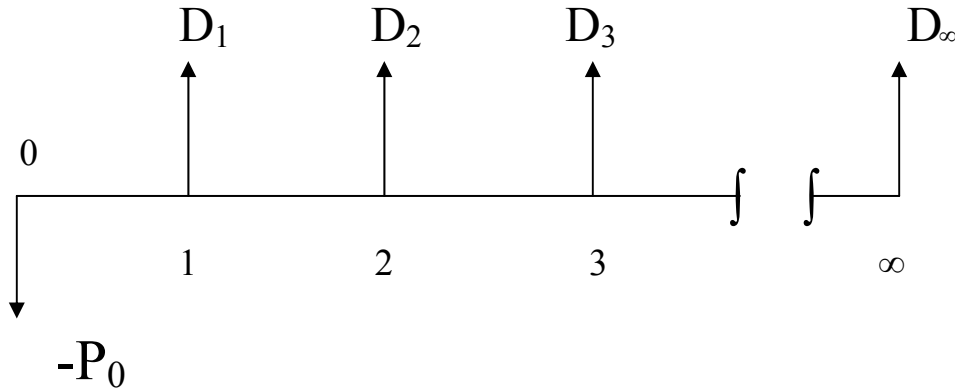
### Stock Valuation

This web document shows how we can use the techniques developed to handle time value problems to value stock. Two documents that we will refer to occasionally are “Compounding and Discounting” and “Annuity Derivations”.

We will assume that the stock may be purchased and sold at discrete points  $t = 1, t = 2, t = 3, \text{ etc.}$ , separated by a constant interval through time, and that the firm may distribute dividends of various amounts to shareholders at these discrete points.

We will further assume that if you purchase the stock, say at  $t = 0$ , you will be entitled to  $D_1$ , the dividend at  $t = 1$  but not to  $D_0$ , the dividend at  $t = 0$ . Of course, if you sell your stock at any future point, say at  $t = 3$ , you would then be entitled to the dividend  $D_3$  rather than the person purchasing the stock at that time. This distribution scheme mirrors reality if we assume that the discrete points are all ex-dividend days, or days when purchase of the stock no longer entitles you to the recently announced dividend.

If we assume that we purchase the stock at  $t = 0$  and hold the stock forever, a time diagram of all future cash flows for our cash flow will look as follows:



The purchase price of the stock will equal  $P_0$ . We will shortly show that we need not hold the stock until infinity, and that the value of the stock will be the same even if we plan to sell it in the very near future. In general, the dividend amounts may vary over time.

One model for the stock price would be the present value of all the cash flows from  $t = 1$  through  $t = \infty$ , discounted at an appropriate rate. This rate should be greater than the risk-

free rate because future dividends are uncertain. Using the present value discount formula for a stream of cash flows, the value of the stock should be:

$$V_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+r)^t} \quad (1)$$

Don't forget that in the above formula,  $r$  represents a rate of return that you think is appropriate for the level of risk associated with this stock. It will be much higher than the risk-free rate of return, and could be specific to the stock being valued. In fact, this discount rate is meant to capture the risk of the stock. The higher the risk, the higher the discount rate  $r$ , and the lower the resulting stock value. If the dividends are realized as expected, and if you can purchase the stock for a price equal to  $V_0$ , your return on the investment will equal  $r$ . This should be clear if you recall the concept of internal rate of return or IRR.

Notice that we use symbol  $V$  for the value at  $t = 0$  to differentiate from the price  $P$ . We allow for the possibility that the "market price" could differ from the theoretical value  $V$ .

Equation 1 expanded would look as follows:

$$V_0 = \frac{D_1}{(1+r)^1} + \frac{D_2}{(1+r)^2} + \frac{D_3}{(1+r)^3} \dots \frac{D_{\infty}}{(1+r)^{\infty}} \quad (2)$$

You might wonder how difficult it would be to use this equation for a stream of infinite terms. In a practical sense, dividend payments very far into the future may not be worth much in present value terms. A five dollar dividend paid in year 25 using a 10% discount rate is worth \$0.46, and after 50 years is only worth \$0.04.

If the patter of expected dividends reduces to two special cases, equation 2 reduces to a very compact and simple form. If dividends are constant, the dividend discount model reduces to:

$$V_0 = \frac{D}{r} \quad (3)$$

If dividends are growing at a constant rate  $g < r$ , the dividend discount model becomes:

$$V_0 = \frac{D_1}{r - g} \quad (4)$$

Both of these results are derived in the document “Annuity Derivations”. The constant dividend model, equation 3, is based on the present value of a perpetuity, and is sometimes used to model preferred stock since the dividends associated with preferred stock is more constant than common stock dividends.

The constant growth model, equation 4, is sometimes called the Gordon Growth Model. It is only appropriate if the growth rate  $g$  is less than the discount rate  $r$ . Otherwise the value would be infinite!

Before we continue, let us demonstrate with several examples. Consider a constant dividend of \$2, and a discount rate of 20%. Because dividends are constant, we can use equation 3:

$$V_0 = \frac{D}{r} = \frac{2}{.20} = \$10$$

What if dividends grow at a 10%, the discount rate is 20%, and  $D_1 = \$2$ ?

This means that  $D_2 = 2(1.1) = 2.20$   
 $D_3 = 2(1.1)^2 = 2.42$  etc.

In general,  $D_t = D_{t-1}(1 + g)$  or  $D_t = D_1(1 + g)^{t-1}$

But we need not do any of this for the valuation if we know the dividend at  $t = 1$ . We use equation 4:

$$V_0 = \frac{D_1}{r - g} = \frac{2}{.20 - .10} = \frac{2}{.10} = \$20$$

It should be expected that this value should be more than for the constant dividend example above, since both problems have identical dividends at  $t = 1$ , but in the second example it is growing at a 10% rate after  $t = 1$ .

Now we shall tackle the question of what happens if you don't want to hold the stock forever. What if we would buy the stock at  $t = 0$ , and sell at  $t = 1$ ? At that point we should get  $D_1$  and  $P_1$ , and the value at  $t = 0$  should be as follows:

$$V_0 = \frac{D_1 + P_1}{1 + r} = \frac{D_1}{1 + r} + \frac{P_1}{1 + r} \quad (5)$$

Further suppose that we sell the stock at  $t = 1$  for a “fair price”, that is,  $P_1 = V_1$ .

We could think of valuing the stock at  $t = 1$  in a similar manner as above, buying at  $t = 1$  and selling at  $t = 2$ :

$$V_1 = \frac{D_2 + P_2}{1 + r} \tag{6}$$

Now using  $V_1 = P_1$  and substituting (6) into (5):

$$V_0 = \frac{D_1}{1 + r} + \frac{\left[ \frac{D_2 + P_2}{1 + r} \right]}{1 + r} = \frac{D_1}{1 + r} + \frac{D_2 + P_2}{(1 + r)^2}$$

If we continue in this manner we can generalize:

$$V_0 = \frac{D_1}{(1 + r)^1} + \frac{D_2}{(1 + r)^2} + \frac{D_3}{(1 + r)^3} \dots \frac{D_n + P_n}{(1 + r)^n} \tag{7}$$

As long as you sell the stock for the discounted value of all future cash flows, you will get identical values  $V_0$ , if you use equation 4 (derived from equation 2), which represents discounting all future dividends, or equation 7, which represents holding the stock for  $n$  periods and then selling it at that point. Let us demonstrate this for the same example as above. We will buy the stock at  $t = 0$ , and sell at  $t = 1$ :



We need to discount both the dividend and the price of the stock that we get at  $t = 1$ . We know  $D_1 = \$2$ . To get  $P_1$ , we assume we will be able to sell it for value  $V_1$ , which can be obtained from the Constant Growth Model, equation 4, which in turn requires  $D_2$ .

As you can see, when dividends grow at a constant rate  $g$ , you can value the stock at any point in time, but you need the amount of the dividend one period forward from the point you want to value the stock. In this case, it is  $t = 2$ :

$$D_2 = D_1(1+g) = 2(1.1) = 2.2$$

Therefore,

$$V_1 = \frac{D_2}{r - g} = \frac{2.20}{.20 - .10} = \frac{2.20}{.10} = \$22$$

If we assume that  $P_1 = V_1 = \$22$ , we now use equation 7 and discount both the price and the dividend we receive at  $t = 1$ , using  $n=1$  (only the final term required here!):

$$V_0 = \frac{D_n + P_n}{(1+r)^n} = \frac{D_1 + P_1}{(1+r)^1} = \frac{2.0 + 22}{1.2} = \frac{24}{1.2} = \$20$$

Same result as using the dividend discount model directly! We are demonstrating with a numerical example that it makes no difference if we assume that we will hold the stock forever or hold for a finite period and sell it. This method also allows you to value stock with dividends that do not grow at a constant rate or have a rate of growth which exceeds the discount rate, provided that at some point in the future, your dividends will eventually start to grow at a constant rate which is below  $r$ .

You would need to find the dividends through the point where the constant growth starts, then find the value of the stock one period before that. Then discount all the dividends and the selling price you would receive up through that point.

Suppose that between  $t = 1$  and  $t = 3$  the growth is actually 20% but after that the growth is 10%. Just as above, let  $D_1 = 2$  and  $r = 20\%$ . You can not use the conventional growth equation 4 at  $t = 0$ , because your growth is not constant, and it is not less than  $r$  between the first and second time period.

$$D_1 = 2 \quad D_2 = 2(1.2) = 2.40 \quad D_3 = (2.40)(1.2) = 2.88 \quad D_4 = (2.88)(1.1) = 3.168$$

After this, dividends grow every year at 10%. You can use the equation 4 at  $t = 2$  but not before. At  $t = 2$ , your next dividend is 2.88 at  $t = 3$ , after which the growth is a constant 10%:

$$P_2 = V_2 = \frac{D_3}{r - g} = \frac{2.88}{.20 - .10} = \frac{2.88}{.10} = \$28.80$$

Now discount the first two dividends at  $t = 1$  and  $t = 2$  and the selling price at  $t = 2$ :

$$V_0 = \frac{D_1}{(1+r)^1} + \frac{D_2 + P_2}{(1+r)^2} = \frac{2}{1.2} + \frac{2.40 + 28.80}{(1.2)^2} = 1.66666 + 21.66666 = \$23.33$$

We would expect this stock to be worth more than the previous example because there is a greater growth rate for several periods before the constant growth rate of 10% kicks in.