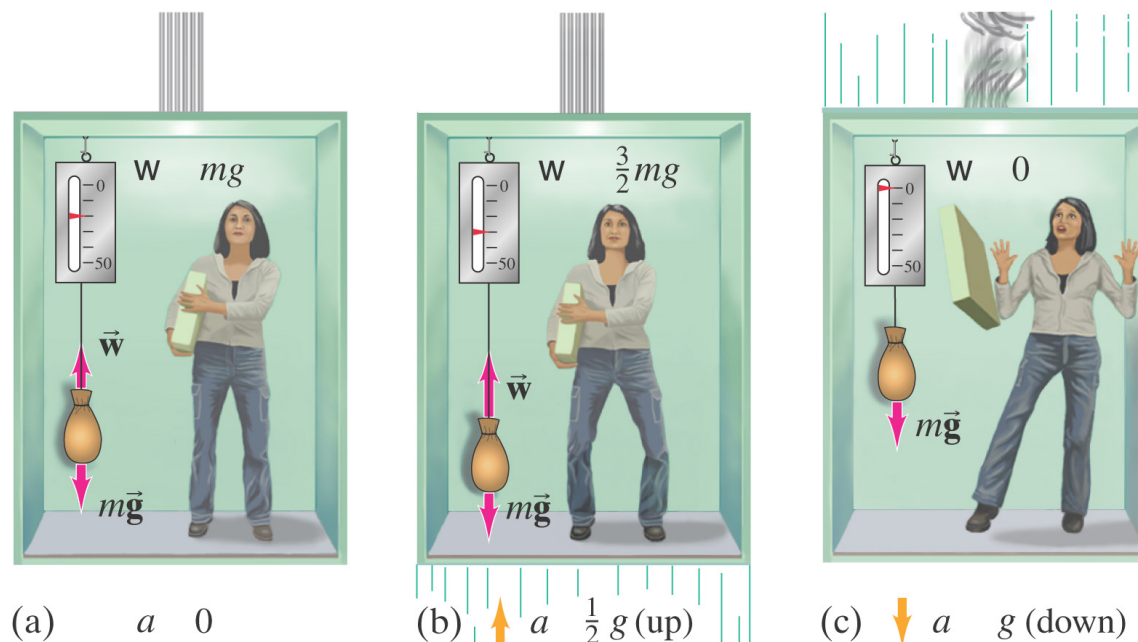


# 5-8 Satellites and “Weightlessness”

Objects in orbit are said to experience **weightlessness**. They do have a gravitational force acting on them, though!

The satellite and all its contents are in **free fall**, so there is no **normal force**. This is what leads to the experience of weightlessness.



## 5-8 Satellites and “Weightlessness”

More properly, this effect is called **apparent weightlessness**, because the gravitational force still exists. It can be experienced on Earth as well, but only briefly:



## Calculating the Local Force of Gravity

$$F = G \frac{m_1 m_2}{r^2}$$

**So the local gravitational acceleration depends upon your distance from the center of mass.**

**Usually this is your distance from the center of the Earth**

## **ConceptTest 5.9 Fly Me Away**

You weigh yourself on a scale inside an airplane that is flying with constant speed at an **altitude of 20,000 feet**.

How does your measured weight in the airplane compare with your weight as measured on the surface of the Earth?

- 1) greater than**
- 2) less than**
- 3) same**

## **ConceptTest 5.9 Fly Me Away**

You weigh yourself on a scale inside an airplane that is flying with constant speed at an **altitude of 20,000 feet**.

How does your measured weight in the airplane compare with your weight as measured on the surface of the Earth?

1) **greater than**

2) **less than**

3) **same**

At a high altitude, you are farther away from the center of Earth. Therefore, the gravitational force in the airplane will be less than the force that you would experience on the surface of the Earth.

## ConceptTest 5.10 Two Satellites

Two satellites A and B of the same mass are going around Earth in concentric orbits. The distance of satellite B from Earth's center is twice that of satellite A. What is the *ratio* of the centripetal force acting on B compared to that acting on A?

- 1)  $1/8$
- 2)  $1/4$
- 3)  $1/2$
- 4) it's the same
- 5) 2

## ConceptTest 5.10 Two Satellites

Two satellites A and B of the same mass are going around Earth in concentric orbits. The distance of satellite B from Earth's center is twice that of satellite A. What is the *ratio* of the centripetal force acting on B compared to that acting on A?

1) 1/8

2) 1/4

3) 1/2

4) it's the same

5) 2

Using the Law of Gravitation:

$$F = G \frac{Mm}{R^2}$$

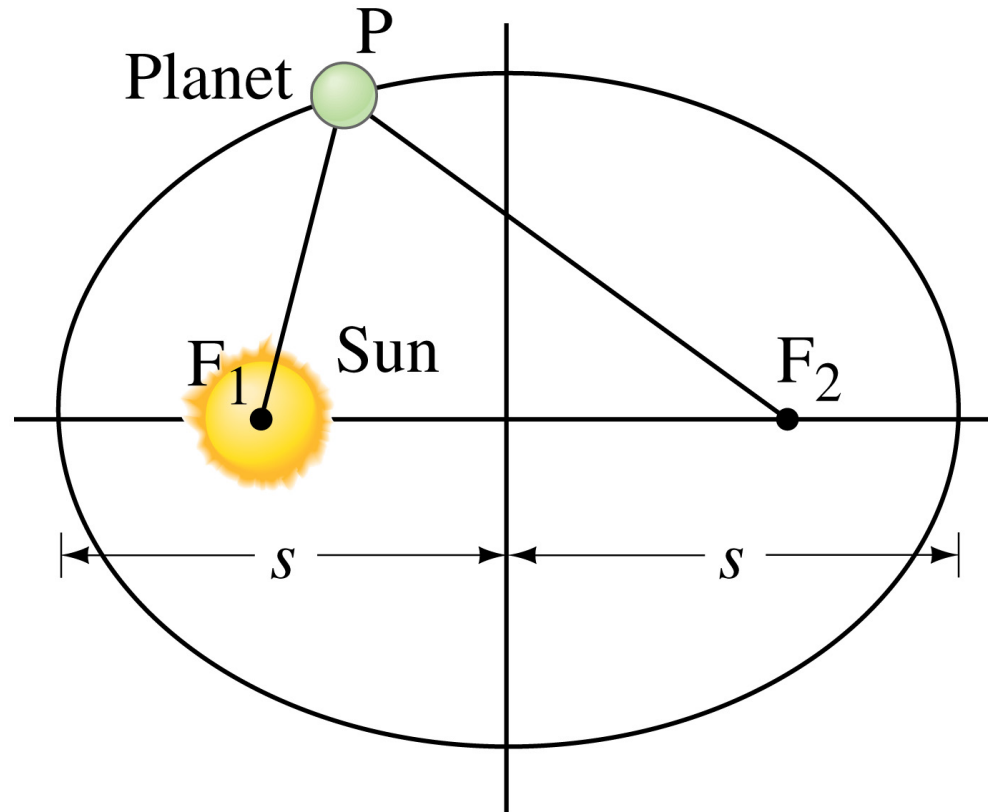
we find that the ratio is 1/4.

Note the  $1/r^2$  factor

## 5-9 Kepler's Laws and Newton's Synthesis

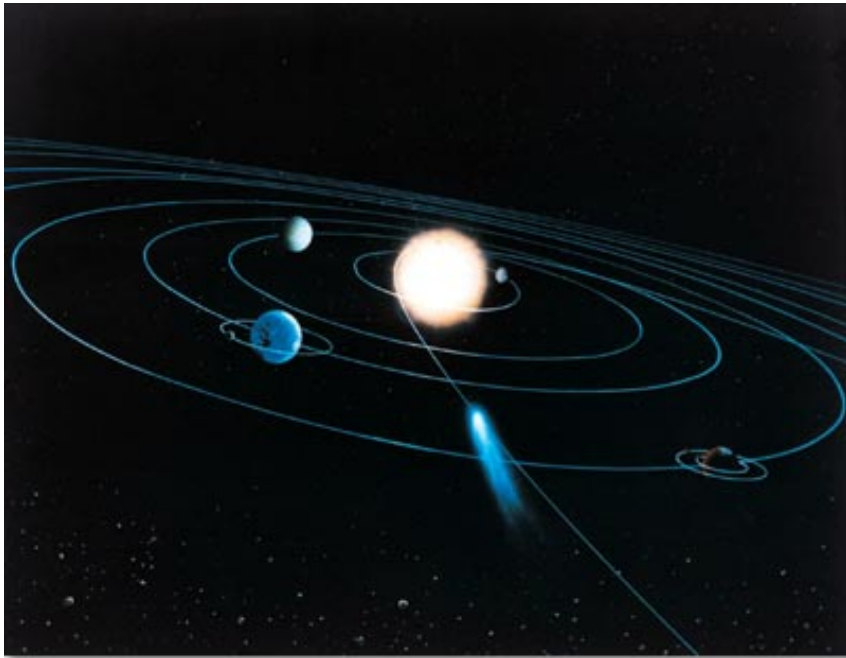
Kepler's laws describe planetary motion.

1. The orbit of each planet is an ellipse, with the Sun at one focus.





# Gravity and the Solar System



The direction of motion (both orbital velocity and rotation) of all the planets and moons follows a “right-hand” rule. (exceptions are

Just as you calculate satellite orbits by equating the centripetal force and Earth’s gravity:

$$mv^2/r = GMm/r^2$$

$$v = 2\pi r/P$$

The same principle applies to the planets, comets asteroids that make up the solar system

Newton’s Law of Universal Gravitation actually came from observations of the orbits of the planets

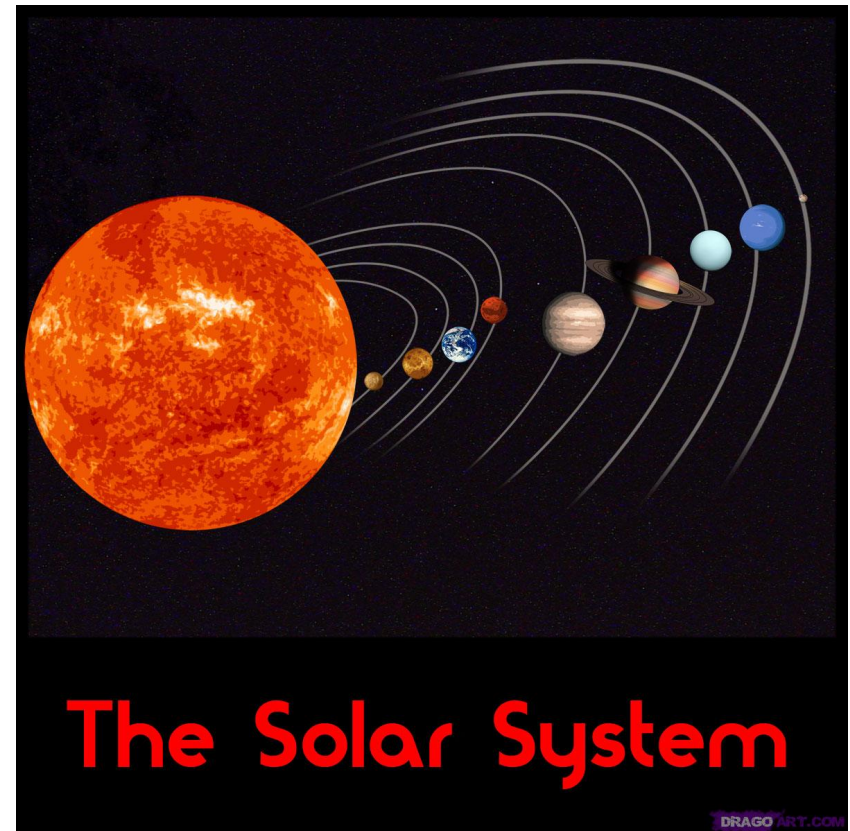
**In class we discussed what's "wrong" with this picture. Can you remember:**

**The actual scale of the solar system?**

**Why the planets could never string out in a line like this?**

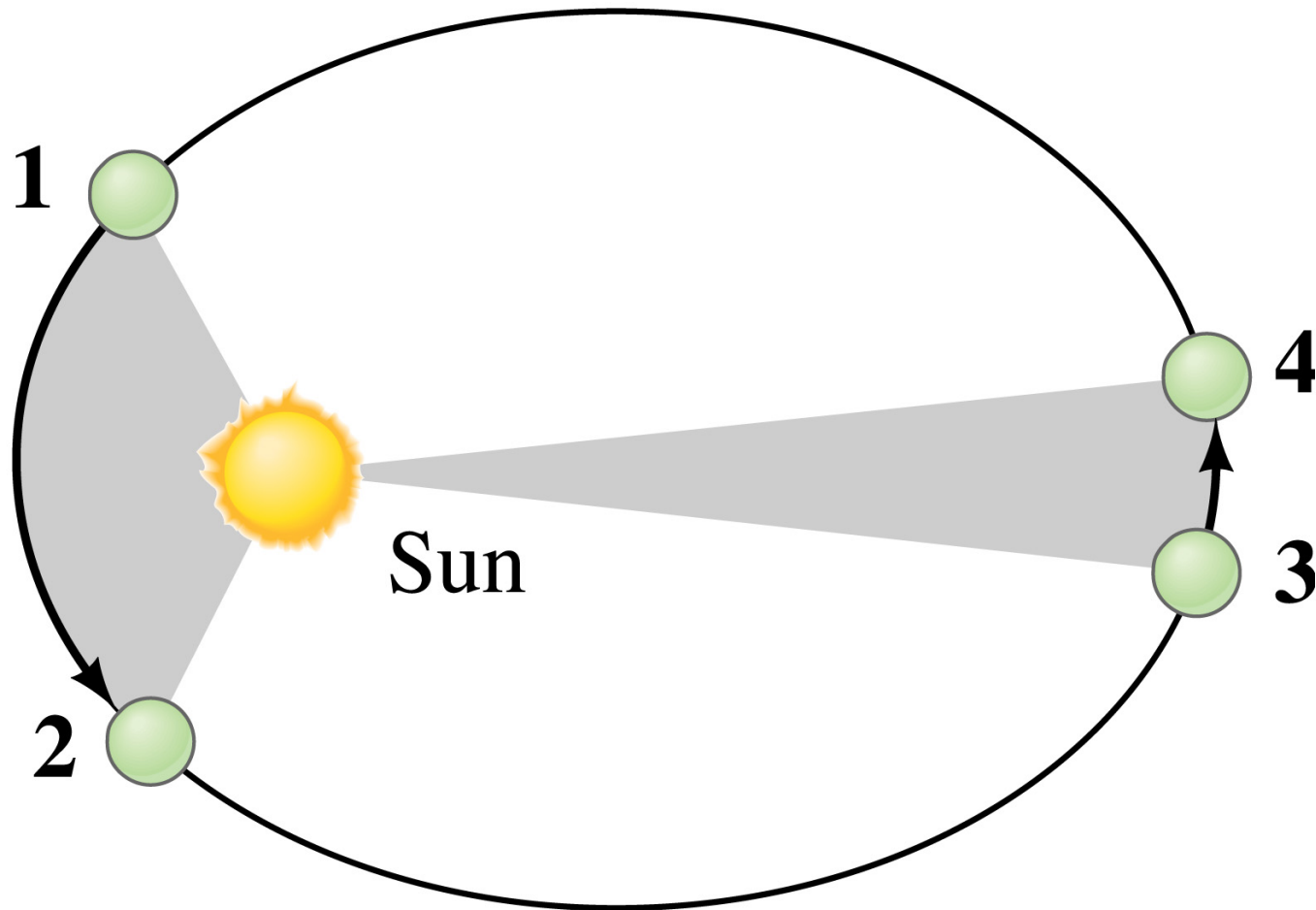
**Which planet moves the fastest,  
based on your understanding of  
Newton's law of gravity ?**

- 1. Earth**
- 2. Mars**
- 3. Venus**
- 4. Mercury**
- 5. Saturn**
- 6. Jupiter**
- 7. Uranus**
- 8. Neptune**



## 5-9 Kepler's Laws and Newton's Synthesis

2. An imaginary line drawn from each planet to the Sun sweeps out equal areas in equal times.



# Kepler's 3<sup>rd</sup> Law

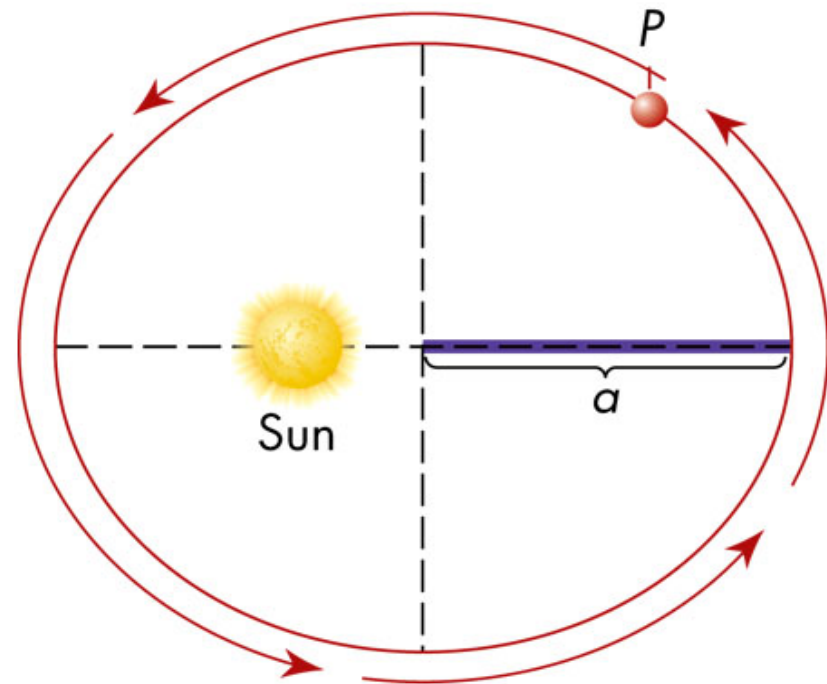
- The amount of time a planet takes to orbit the Sun is related to its orbit's size
- The square of the period,  $P$ , is proportional to the cube of the **semimajor axis**,  $a$

$$P^2 \propto a^3$$

$$P^2/a^3 = \text{Constant}$$

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$$P^2 \text{ years} = a^3 \text{ AU}$$



$P$  = time to complete orbit  
 $a$  = semimajor axis

## 5-9 Kepler's Laws and Newton's Synthesis

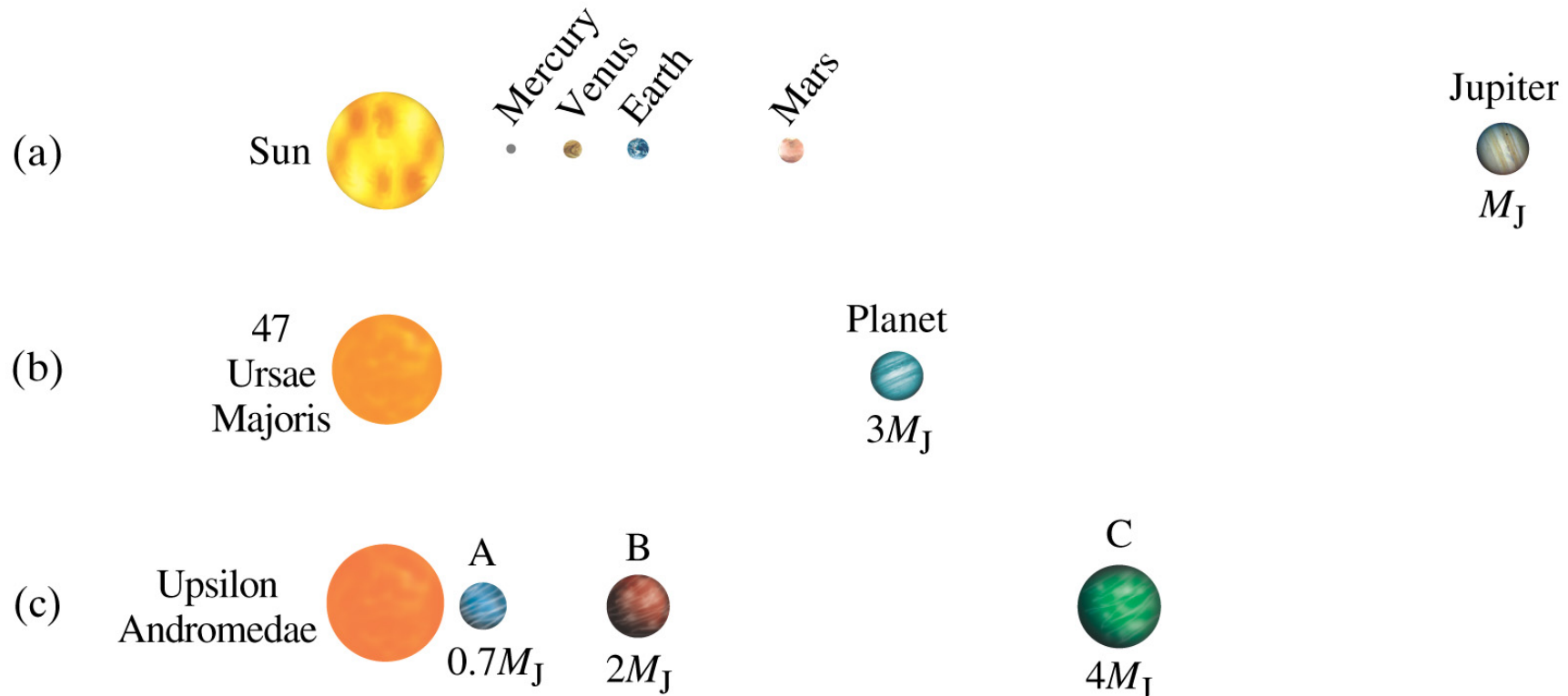
The ratio of the square of a planet's orbital period is proportional to the cube of its mean distance from the Sun.

**TABLE 5–2 Planetary Data Applied to Kepler's Third Law**

| <b>Planet</b> | <b>Mean Distance<br/>from Sun, <math>s</math><br/>(<math>10^6</math> km)</b> | <b>Period, <math>T</math><br/>(Earth years)</b> | <b><math>s^3/T^2</math><br/>(<math>10^{24}</math> km<sup>3</sup>/y<sup>2</sup>)</b> |
|---------------|--|---|---|
| Mercury       | 57.9   | 0.241   | 3.34  |
| Venus         | 108.2  | 0.615   | 3.35  |
| Earth         | 149.6  | 1.0   | 3.35  |
| Mars          | 227.9  | 1.88  | 3.35  |
| Jupiter       | 778.3  | 11.86   | 3.35  |
| Saturn        | 1427   | 29.5  | 3.34  |
| Uranus        | 2870   | 84.0  | 3.35  |
| Neptune       | 4497   | 165   | 3.34  |
| Pluto         | 5900   | 248   | 3.34  |

# 5-9 Kepler's Laws and Newton's Synthesis

**Kepler's laws can be derived from Newton's laws. Irregularities in planetary motion led to the discovery of Neptune, and irregularities in stellar motion have led to the discovery of many planets outside our Solar System.**



# Types of Forces in Nature

Modern physics now recognizes four fundamental forces:

1. **Gravity**
2. **Electromagnetism**
3. **Weak nuclear force (responsible for some types of radioactive decay)**
4. **Strong nuclear force (binds protons and neutrons together in the nucleus)**

## Summary of Chapter 5

- An object moving in a circle at constant speed is in uniform circular motion.

- It has a centripetal acceleration  $a_R = \frac{v^2}{r}$

- There is a centripetal force given by

$$\Sigma F_R = ma_R = m \frac{v^2}{r}$$

- The centripetal force may be provided by friction, gravity, tension, the normal force, or others.

- Newton's law of universal gravitation:

$$F = G \frac{m_1 m_2}{r^2}$$

- Satellites are able to stay in Earth orbit because of their large tangential speed.



## **ConceptTest 5.12 In the Space Shuttle**

**Astronauts in the  
space shuttle  
float because:**

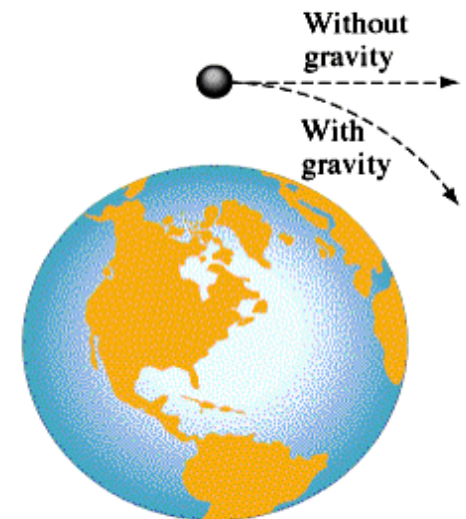
- 1) They are so far from Earth that Earth's gravity doesn't act any more.**
- 2) Gravity's force pulling them inward is cancelled by the centripetal force pushing them outward.**
- 3) While gravity is trying to pull them inward, they are trying to continue on a straight-line path.**
- 4) Their weight is reduced in space so the force of gravity is much weaker.**

## ConceptTest 5.12 In the Space Shuttle

Astronauts in the space shuttle float because:

- 1) They are so far from Earth that Earth's gravity doesn't act any more.
- 2) Gravity's force pulling them inward is cancelled by the centripetal force pushing them outward.
- 3) While gravity is trying to pull them inward, they are trying to continue on a straight-line path.
- 4) Their weight is reduced in space so the force of gravity is much weaker.

Astronauts in the space shuttle float because they are in “free fall” around Earth, just like a satellite or the Moon. Again, it is gravity that provides the centripetal force that keeps them in circular motion.



Follow-up: How weak is the value of  $g$  at an altitude of 300 km?

# Lecture 13

## Chapter 8

# Rotational Motion



## Summary of what we learned so far:

**Motion in a straight line (speed, distance, displacement, velocity, acceleration)**

**Vectors -components, magnitude, direction**

**Energy (Kinetic, Potential, Chemical, Heat, etc. Conservation of)**

**Momentum -Linear, conservation of. Impulse**

**Forces, Work done by., relation to Momentum**

**Newton's Laws: 1st, 2nd, 3rd, Gravity**

**Circular Motion -centripetal acceleration and force**

**Next: Extend our toolbox for rotational motion**

## Units of Chapter 8

- **Angular Quantities**
- **Constant Angular Acceleration**
- **Rolling Motion (Without Slipping)**
- **Torque**
- **Rotational Dynamics; Torque and Rotational Inertia**
- **Rotational Kinetic Energy**
- **Angular Momentum and Its Conservation**

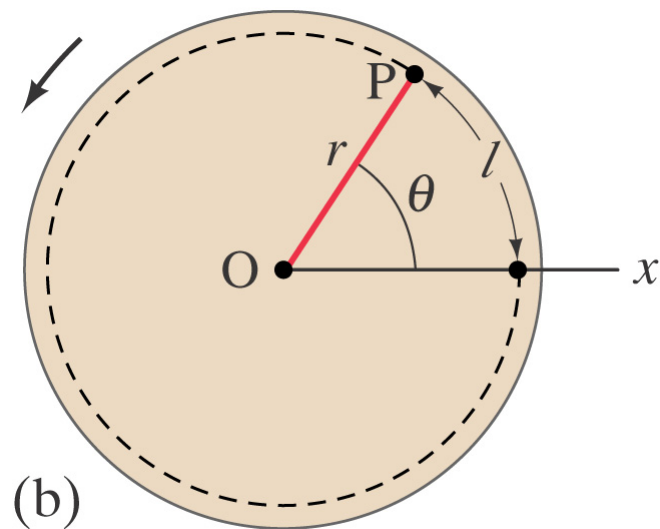
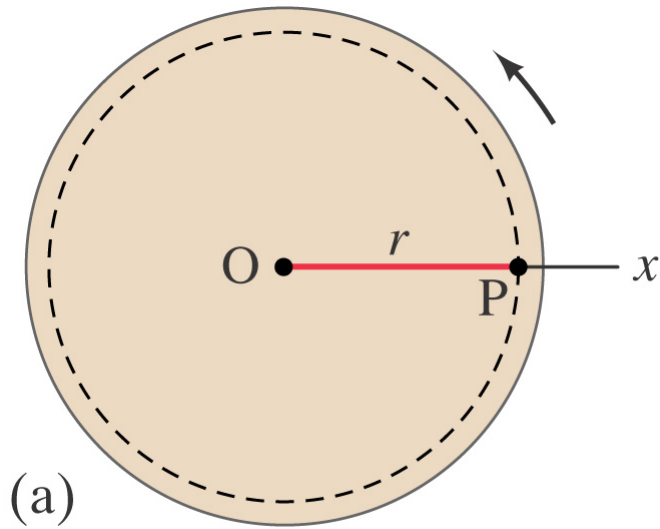
# Angular Quantities: Radians

In purely rotational motion, all points on an object move in circles around the axis of rotation (“O”). The radius of the circle is  $r$ . All points on a straight line drawn through the axis move through the same angle in the same time. The angle  $\theta$  in radians is defined:

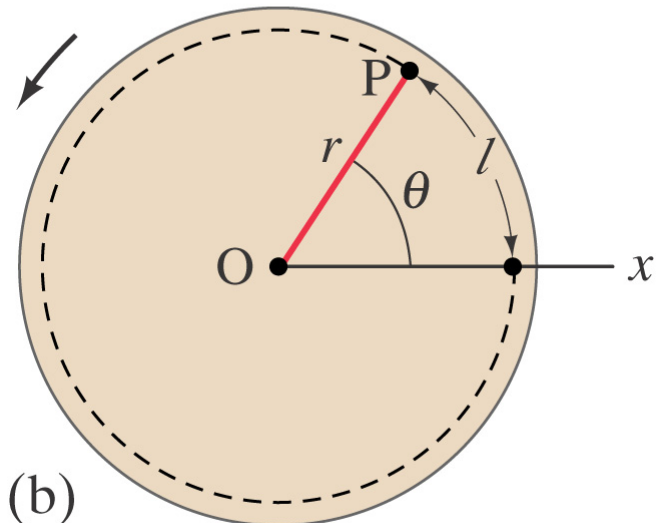
$$\theta = \frac{l}{r} \quad (8-1a)$$

where  $l$  is the arc length.

**Radians make the math EASIER!**



## $2\pi$ Radians in a Circle



For a circle of radius  $r$ , the total arc length is just the circumference  $2\pi r$

So by definition, the angle subtended is:

$$\theta = l/r = 2\pi r/r = 2\pi \text{ radians}$$

Another way to look at it: For what angle is the arc length equal to the radius?  
(think equilateral triangle, so 1 radian is a little bit smaller than 60 degrees)

**Express these angles in Radians:**

**A. 30°**

$$\pi/6 = 0.52 \text{ rad}$$

**B. 57°**

$$= 0.99 \text{ rad}$$

$$\times (2\pi/360)$$

**C. 90°**

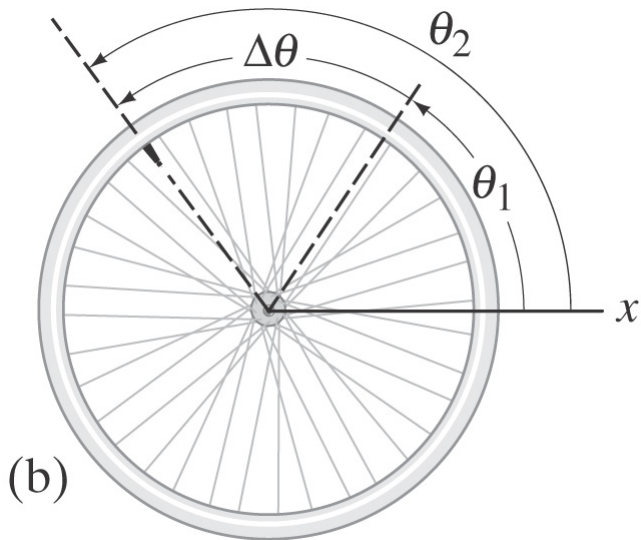
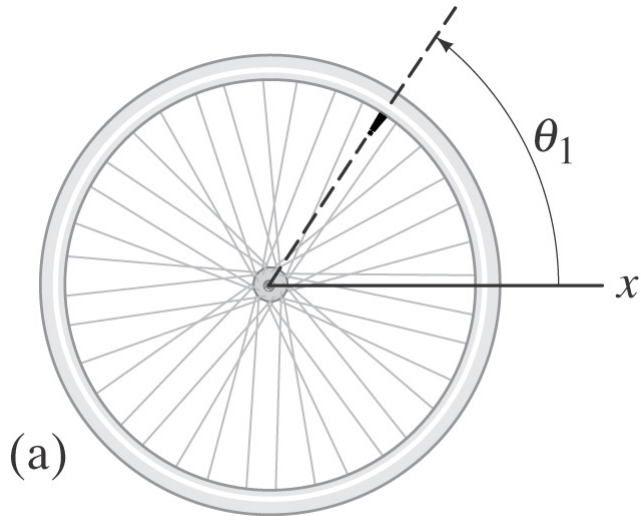
$$\pi/2 = 1.57 \text{ rad}$$

**D. 360°**

$$2\pi = 6.28 \text{ rad}$$



# Angular Displacement and Angular Speed



**Angular displacement:**

$$\Delta\theta = \theta_2 - \theta_1$$

**The average angular velocity is defined as the total angular displacement divided by time:**

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t} \quad (8-2a)$$

**The instantaneous angular velocity:**

$$\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} \quad (8-2b)$$

## **ConceptTest 8.3a**

## **Angular Displacement I**

An object at rest begins to rotate with a constant angular acceleration. If this object rotates through an angle  $\theta$  in the time  $t$ , through what angle did it rotate in the time  $1/2 t$ ?

- 1)  $1/2 \theta$
- 2)  $1/4 \theta$
- 3)  $3/4 \theta$
- 4)  $2 \theta$
- 5)  $4 \theta$

## ConceptTest 8.3a

## Angular Displacement I

An object at rest begins to rotate with a constant angular acceleration. If this object rotates through an angle  $\theta$  in the time  $t$ , through what angle did it rotate in the time  $1/2 t$ ?

1)  $1/2 \theta$

2)  $1/4 \theta$

3)  $3/4 \theta$

4)  $2 \theta$

5)  $4 \theta$

The angular displacement is  $\theta = 1/2 \alpha t^2$  (starting from rest), and there is a quadratic dependence on time. Therefore, in **half the time**, the object has rotated through **one-quarter the angle**.

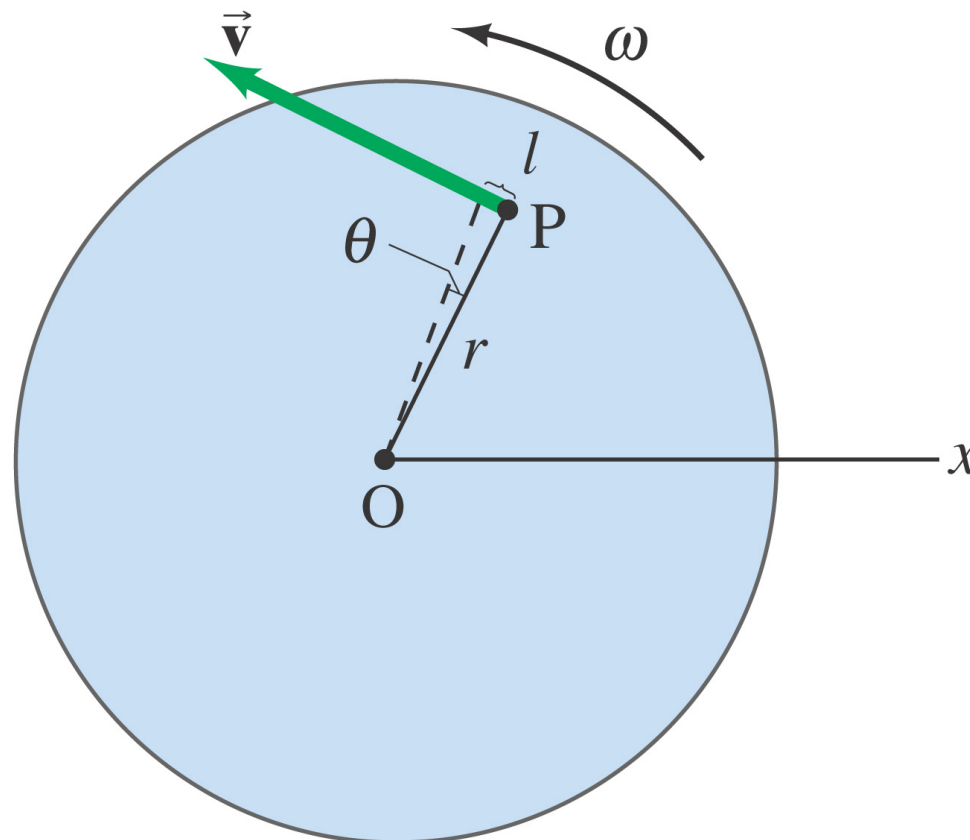
# Angular and Linear Velocity

Every point on a rotating body has an angular velocity  $\omega$  and a linear velocity  $v$ .

They are related:

$$v = r\omega$$

(8-4)



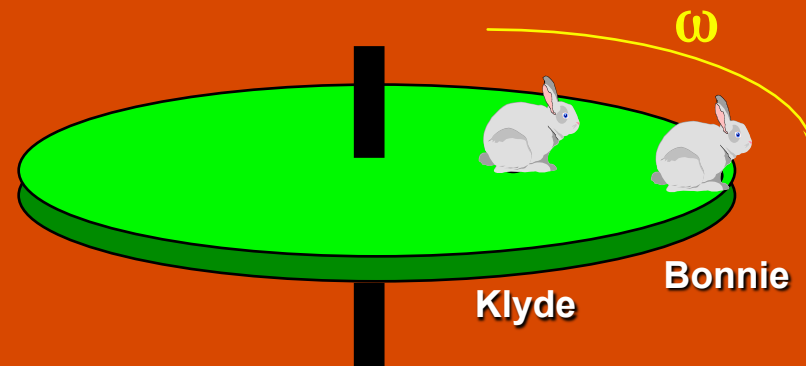
## ConceptTest 8.1a

## Bonnie and Klyde I

**Bonnie** sits on the outer rim of a merry-go-round, and **Klyde** sits midway between the center and the rim. The merry-go-round makes one complete revolution every two seconds.

**Klyde's angular velocity is:**

- 1) same as Bonnie's
- 2) twice Bonnie's
- 3) half of Bonnie's
- 4) 1/4 of Bonnie's
- 5) four times Bonnie's



## ConceptTest 8.1a

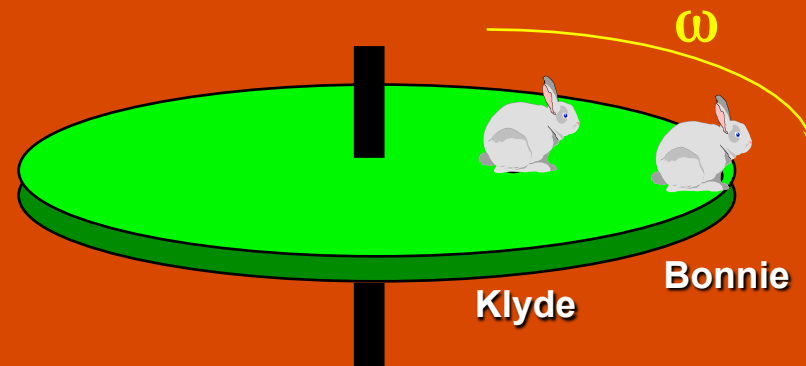
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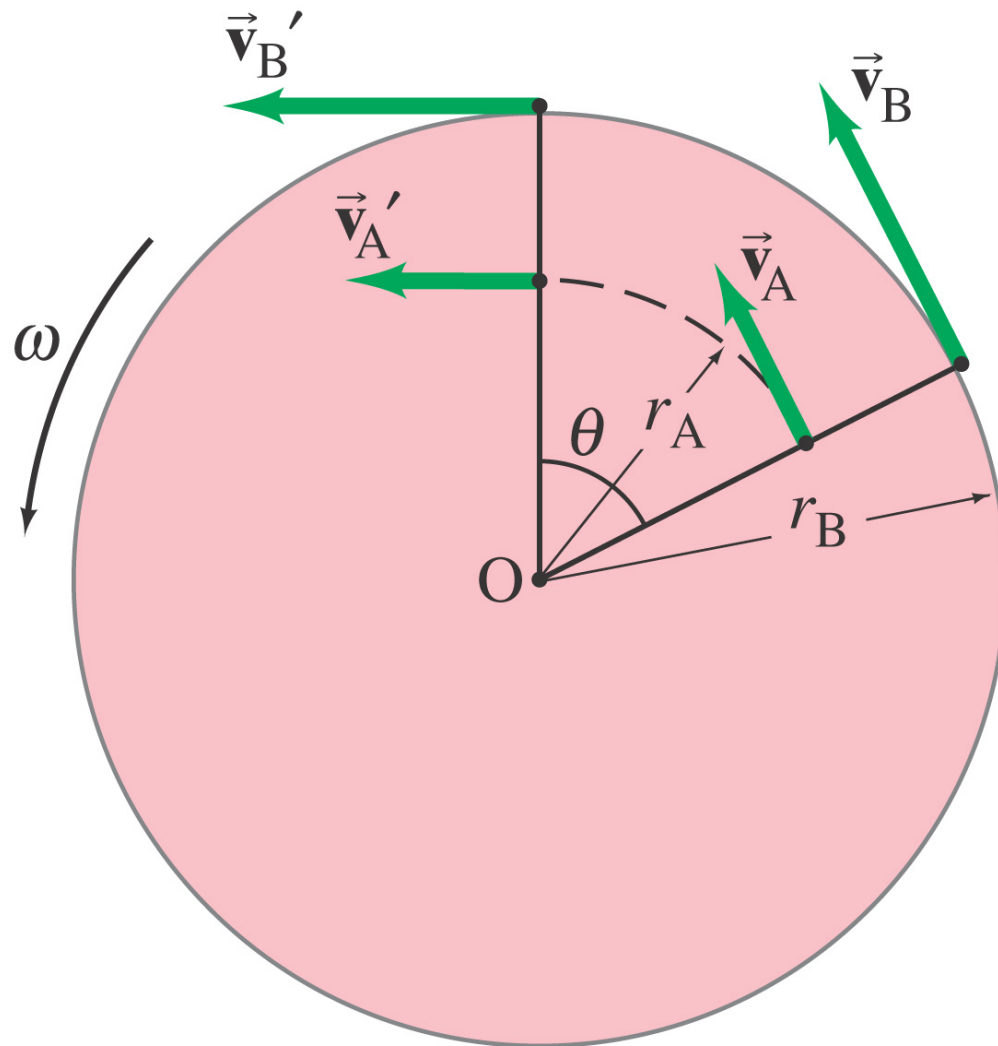
**Klyde's angular velocity is:**

- 1) same as Bonnie's
- 2) twice Bonnie's
- 3) half of Bonnie's
- 4) 1/4 of Bonnie's
- 5) four times Bonnie's

The **angular velocity**  $\omega$  of any point on a solid object rotating about a fixed axis *is the same*. Both Bonnie & Klyde go around one revolution ( $2\pi$  radians) every two seconds.



# Linear (or Tangential) Velocity depends on distance from axis.



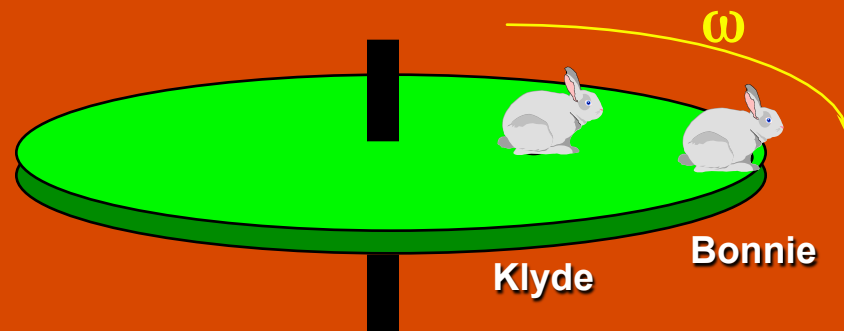
**Objects farther from the axis of rotation move faster, for a given constant angular velocity**

## ConceptTest 8.1b

## Bonnie and Klyde II

**Bonnie** sits on the outer rim of a merry-go-round, and **Klyde** sits midway between the center and the rim. The merry-go-round makes one revolution every two seconds. **Who has the larger linear (tangential) velocity?**

- 1) **Klyde**
- 2) **Bonnie**
- 3) **both the same**
- 4) **linear velocity is zero for both of them**





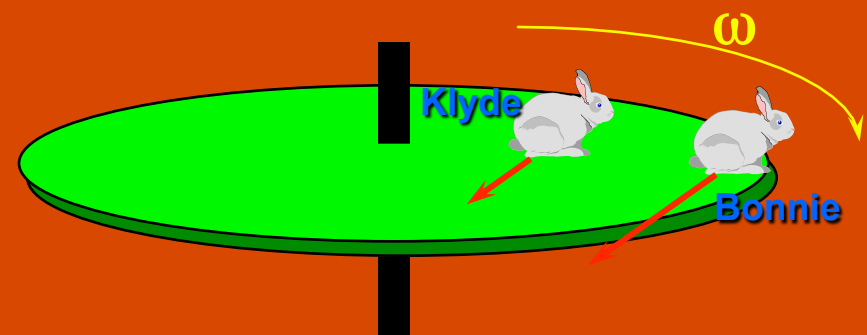
## ConceptTest 8.1b

## Bonnie and Klyde II

**Bonnie** sits on the outer rim of a merry-go-round, and **Klyde** sits midway between the center and the rim. The merry-go-round makes one revolution every two seconds. **Who has the larger linear (tangential) velocity?**

- 1) Klyde
- 2) **Bonnie**
- 3) both the same
- 4) linear velocity is zero for both of them

Their **linear speeds  $v$**  will be different since  $v = R\omega$  and **Bonnie is located further out** (larger radius  $R$ ) than Klyde.



**Follow-up:** Who has the larger centripetal acceleration?

## **ConceptTest 8.2**

Suppose that the speedometer of a truck is set to read the linear speed of the truck, but uses a device that actually measures the angular speed of the tires. If larger diameter tires are mounted on the truck instead, how will that affect the speedometer reading as compared to the true linear speed of the truck?

## **Truck Speedometer**

- 1) speedometer reads a higher speed than the true linear speed**
- 2) speedometer reads a lower speed than the true linear speed**
- 3) speedometer still reads the true linear speed**

## ConceptTest 8.2

Suppose that the speedometer of a truck is set to read the linear speed of the truck, but uses a device that actually measures the angular speed of the tires. If larger diameter tires are mounted on the truck instead, how will that affect the speedometer reading as compared to the true linear speed of the truck?

## Truck Speedometer

- 1) speedometer reads a higher speed than the true linear speed
- 2) speedometer reads a lower speed than the true linear speed
- 3) speedometer still reads the true linear speed

The linear speed is  $v = \omega R$ . So when the speedometer measures the same angular speed  $\omega$  as before, the linear speed  $v$  is actually higher, because the tire radius is larger than before.

**Can all the points on a rigid object  
(such as a wheel, or a centrifuge)  
be described by a single angular  
velocity value ?**

**1. Yes**

**2. No**

**3. Not Sure**



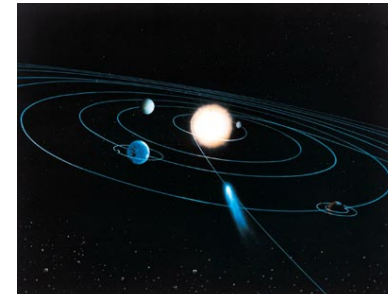
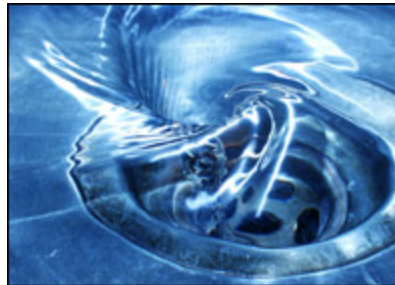
**What about a Non-Rigid object? (perhaps an uncooked egg, water going down a plug-hole, the solar system, etc)**

**Can any of these be described by a single angular velocity value ?**

**1. Yes**

**2. No**

**3. Not Sure**



# Angular Acceleration

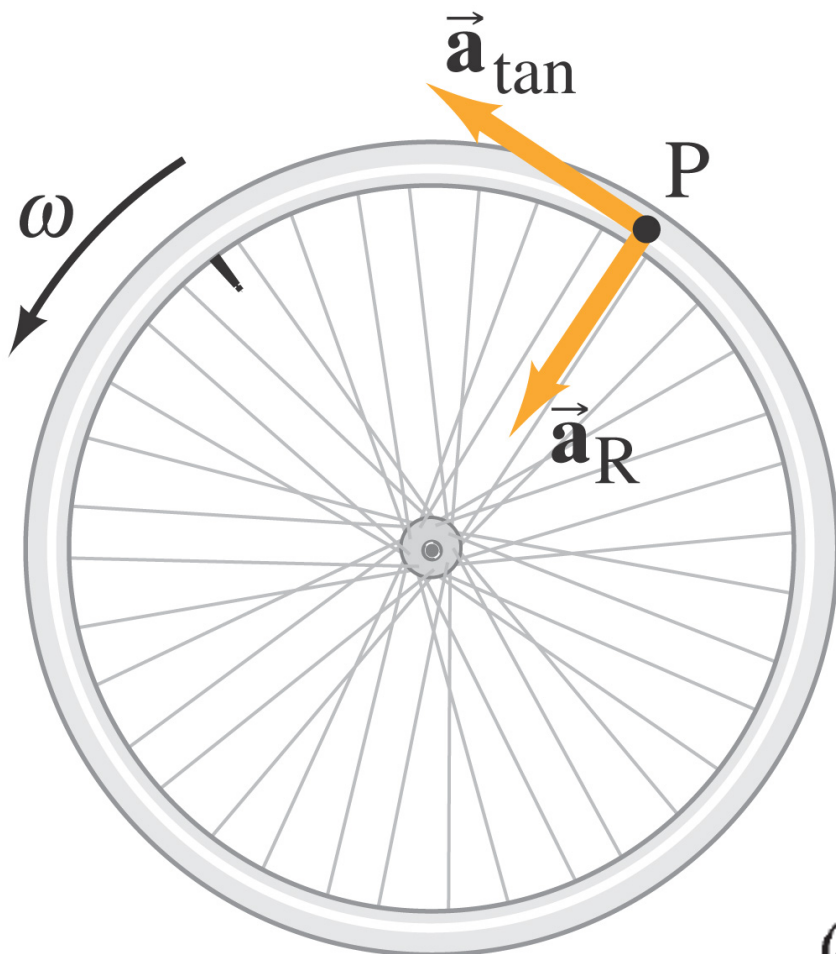
If the angular velocity of a rotating object changes, it has a **tangential (linear) acceleration**:

$$V_{\text{tan}} = r \omega$$

$$a_{\text{tan}} = r \alpha$$

**Remember this is not the same as centripetal acceleration:**

$$a_{\text{R}} = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = \omega^2 r$$



# Angular Acceleration

The angular acceleration is the rate at which the angular velocity changes with time:

$$\bar{\alpha} = \frac{\omega_2 - \omega_1}{\Delta t} = \frac{\Delta\omega}{\Delta t} \quad (8-3a)$$

The instantaneous acceleration:

$$\alpha = \lim_{\Delta t \rightarrow 0} \frac{\Delta\omega}{\Delta t} \quad (8-3b)$$

**Note:**

**Angular acceleration IS NOT centripetal Acceleration**

## **ConceptTest 8.3b**

## **Angular Displacement II**

An object at rest begins to rotate with a constant angular acceleration. If this object has angular velocity  $\omega$  at time  $t$ , what was its angular velocity at the time  $1/2 t$ ?

- 1)  $1/2 \omega$
- 2)  $1/4 \omega$
- 3)  $3/4 \omega$
- 4)  $2 \omega$
- 5)  $4 \omega$



## ConceptTest 8.3b

## Angular Displacement II

An object at rest begins to rotate with a constant angular acceleration. If this object has angular velocity  $\omega$  at time  $t$ , what was its angular velocity at the time  $1/2t$ ?

1)  $1/2 \omega$

2)  $1/4 \omega$

3)  $3/4 \omega$

4)  $2 \omega$

5)  $4 \omega$

The angular velocity is  $\omega = \alpha t$  (starting from rest), and there is a linear dependence on time. Therefore, in **half the time**, the object has accelerated up to only **half the speed**.

Here is the correspondence between linear and rotational quantities:

**TABLE 8–1 Linear and Rotational Quantities**

| <b>Linear</b>    | <b>Type</b>  | <b>Rotational</b> | <b>Relation</b>            |
|------------------|--------------|-------------------|----------------------------|
| $x$              | displacement | $\theta$          | $x = r\theta$              |
| $v$              | velocity     | $\omega$          | $v = r\omega$              |
| $a_{\text{tan}}$ | acceleration | $\alpha$          | $a_{\text{tan}} = r\alpha$ |

# Frequency

**The period is the time one revolution takes:**

$$T = \frac{1}{f}$$

**While the frequency is its reciprocal: the number of complete revolutions per second:**

**So from  $\omega = 2\pi/T$ , we get  $f = \omega/2\pi$**

**Frequencies are measured in hertz.**

$$1 \text{ Hz} = 1 \text{ s}^{-1}$$

**(8-8)**

# Constant Angular Acceleration Formulae

The equations of motion for **constant angular acceleration** are the same as those for **linear motion**, with the substitution of the **angular quantities** for the **linear ones**.

| <b>Angular</b>                                 | <b>Linear</b>                  |
|--|--------------------------------|
| $\omega = \omega_0 + \alpha t$                 | $v = v_0 + at$                 |
| $\theta = \omega_0 t + \frac{1}{2} \alpha t^2$ | $x = v_0 t + \frac{1}{2} at^2$ |
| $\omega^2 = \omega_0^2 + 2\alpha\theta$        | $v^2 = v_0^2 + 2ax$            |
| $\bar{\omega} = \frac{\omega + \omega_0}{2}$   | $\bar{v} = \frac{v + v_0}{2}$  |

**A centrifuge accelerates uniformly from rest to 15,000 rpm in 220s. Through how many revolutions did it turn in this time?**

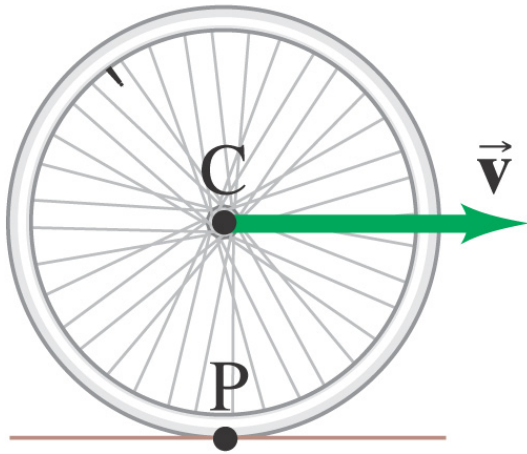
**A.  $2.75 \times 10^4$**

**B.  $5.5 \times 10^4$**

**C.  $1.65 \times 10^6$**

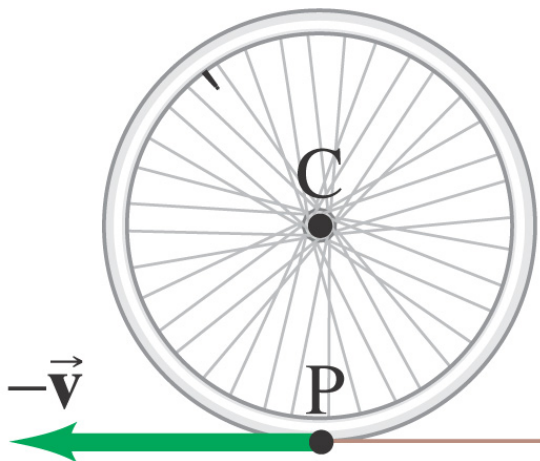


## Rolling Motion (Without Slipping)



(a)

In (a), a wheel is **rolling without slipping**. The point P, touching the ground, is **instantaneously at rest**, and the center moves with **velocity  $v$** .



(b)

In (b) the same wheel is seen from a **reference frame where C is at rest**. Now point P is **moving with velocity  $-v$** .

**The linear speed of the wheel is related to its angular speed:**

$$v = r\omega$$

# Summary of Chapter 8

- **Angles are measured in radians; a whole circle is  $2\pi$  radians.**
- **Angular velocity is the rate of change of angular position.**
- **Angular acceleration is the rate of change of angular velocity.**
- **The angular velocity and acceleration can be related to the linear velocity and acceleration.**
- **The frequency is the number of full revolutions per second; the period is the inverse of the frequency.**

## Summary of Chapter 8, cont.

- The equations for rotational motion with constant angular acceleration have the same form as those for linear motion with constant acceleration.
- Torque is the product of force and lever arm.
- The rotational inertia depends not only on the mass of an object but also on the way its mass is distributed around the axis of rotation.
- The angular acceleration is proportional to the torque and inversely proportional to the rotational inertia.

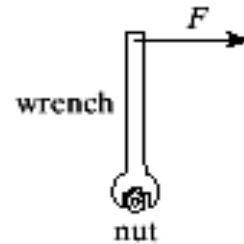


## ConceptTest 8.4

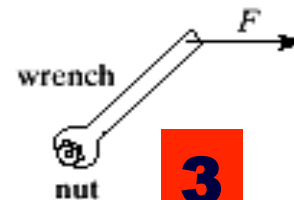
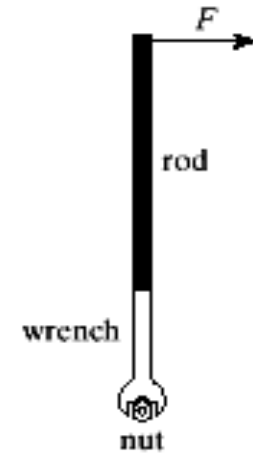
You are using a wrench to loosen a rusty nut. Which arrangement will be the most effective in loosening the nut?

## Using a Wrench

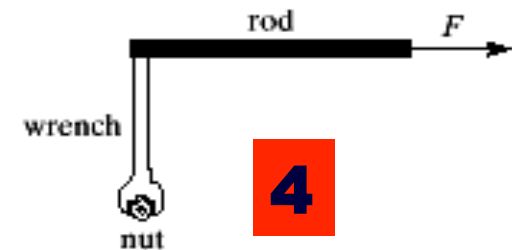
1



2



3



4

5) all are equally effective

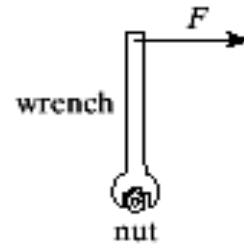
## ConceptTest 8.4

## Using a Wrench

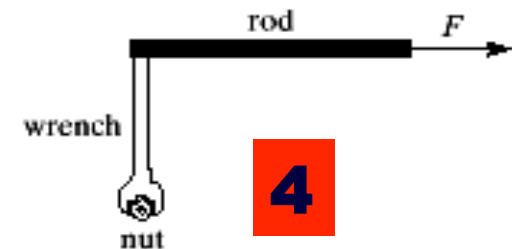
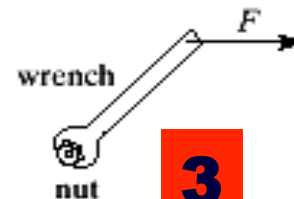
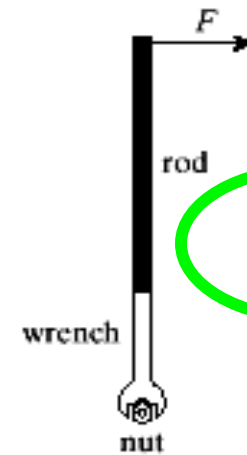
You are using a wrench to loosen a rusty nut. Which arrangement will be the most effective in loosening the nut?

Since the forces are all the same, the only difference is the lever arm. The arrangement with the **largest lever arm** (case #2) will provide the **largest torque**.

1



2



5) all are equally effective

Follow-up: What is the difference between arrangement 1 and 4?

## ***ConceptTest 8.5***

Two forces produce the same torque. Does it follow that they have the same magnitude?

## **Two Forces**

- 1) **yes**
- 2) **no**
- 3) **depends**

## **ConceptTest 8.5**

Two forces produce the same torque. Does it follow that they have the same magnitude?

## **Two Forces**

1) yes

2) no

3) depends

Because torque is the product of force times distance, two different forces that act at different distances could still give the same torque.

**Follow-up:** If two torques are identical, does that mean their forces are identical as well?

## ConceptTest 8.6

In which of the cases shown below is the torque provided by the applied force about the rotation axis biggest? For all cases the magnitude of the applied force is the same.

## Closing a Door

- 1)  $F_1$
- 2)  $F_3$
- 3)  $F_4$
- 4) all of them
- 5) none of them



## ConceptTest 8.6

In which of the cases shown below is the torque provided by the applied force about the rotation axis biggest? For all cases the magnitude of the applied force is the same.

## Closing a Door

- 1)  $F_1$
- 2)  $F_3$
- 3)  $F_4$
- 4) all of them
- 5) none of them

The torque is:  $\tau = F d \sin \theta$  and so the force that is at  $90^\circ$  to the lever arm is the one that will have the **largest torque**. Clearly, to close the door, you want to push **perpendicular!!**



Follow-up: How large would the force have to be for  $F_4$ ?

## **ConceptTest 8.7**

When a tape is played on a cassette deck, there is a tension in the tape that applies a torque to the supply reel. Assuming the tension remains constant during playback, how does this applied torque vary as the supply reel becomes empty?

## **Cassette Player**

- 1) torque increases**
- 2) torque decreases**
- 3) torque remains constant**

## **ConceptTest 8.7**

When a tape is played on a cassette deck, there is a tension in the tape that applies a torque to the supply reel. Assuming the tension remains constant during playback, how does this applied torque vary as the supply reel becomes empty?

## **Cassette Player**

- 1) torque increases
- 2) torque decreases
- 3) torque remains constant

As the supply reel empties, the lever arm decreases because the radius of the reel (with tape on it) is decreasing. Thus, as the playback continues, the applied torque diminishes.

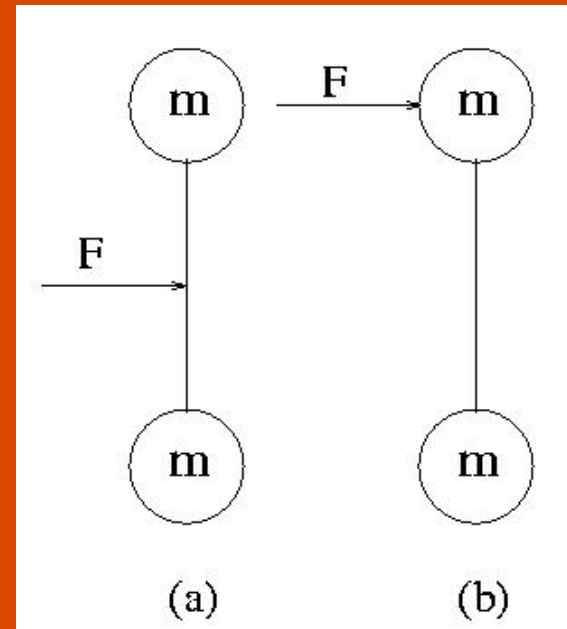


## ConceptTest 8.8a

## Dumbbell I

A force is applied to a dumbbell for a certain period of time, first as in (a) and then as in (b). In which case does the dumbbell acquire the greater center-of-mass speed?

- 1) case (a)
- 2) case (b)
- 3) no difference
- 4) It depends on the rotational inertia of the dumbbell.



## ConceptTest 8.8a

## Dumbbell I

A force is applied to a dumbbell for a certain period of time, first as in (a) and then as in (b). In which case does the dumbbell acquire the greater center-of-mass speed?

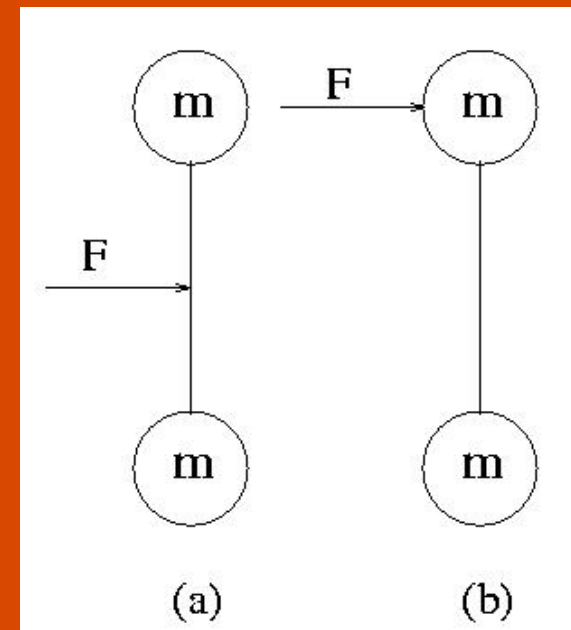
1) case (a)

2) case (b)

3) no difference

4) It depends on the rotational inertia of the dumbbell.

Because the same force acts for the same time interval in both cases, the change in momentum must be the same, thus the CM velocity must be the same.

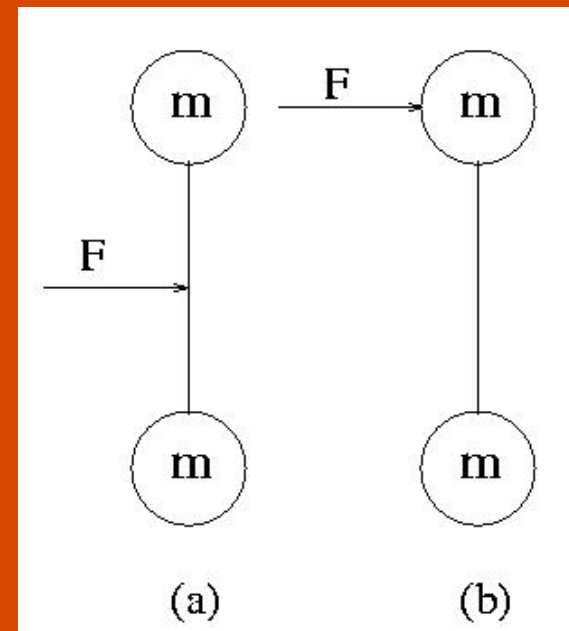


## ConceptTest 8.8b

## Dumbbell II

A force is applied to a dumbbell for a certain period of time, first as in (a) and then as in (b). In which case does the dumbbell acquire the greater energy?

- 1) case (a)
- 2) case (b)
- 3) no difference
- 4) It depends on the rotational inertia of the dumbbell.



## ConceptTest 8.8b

## Dumbbell II

A force is applied to a dumbbell for a certain period of time, first as in (a) and then as in (b). In which case does the dumbbell acquire the greater energy?

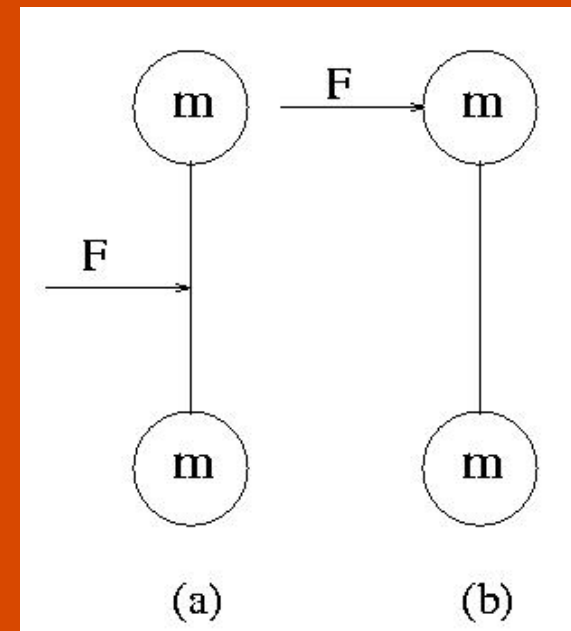
1) case (a)

2) case (b)

3) no difference

4) It depends on the rotational inertia of the dumbbell.

If the CM velocities are the same, the translational kinetic energies must be the same. Because dumbbell (b) is also rotating, it has rotational kinetic energy in addition.



## ConceptTest 8.9

## Moment of Inertia

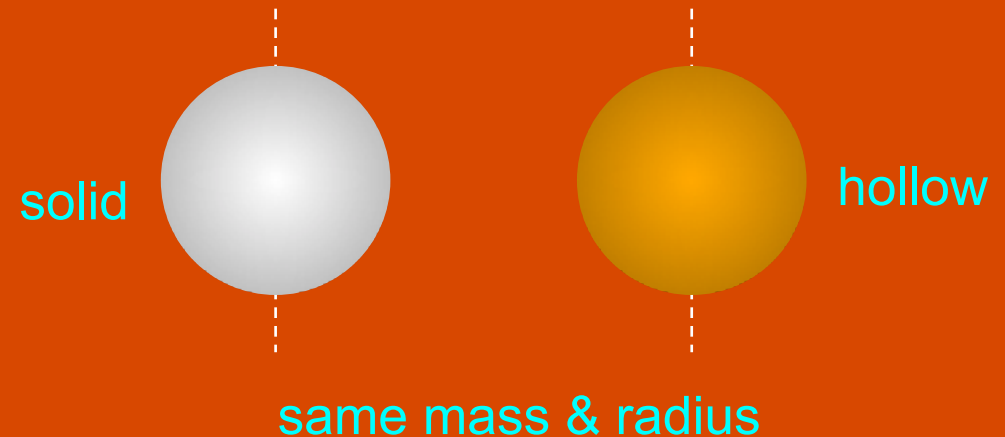
Two spheres have the same radius and equal masses. One is made of solid aluminum, and the other is made from a hollow shell of gold.

Which one has the bigger moment of inertia about an axis through its center?

a) solid aluminum

b) hollow gold

c) same



## ConceptTest 8.9

## Moment of Inertia

Two spheres have the same radius and equal masses. One is made of solid aluminum, and the other is made from a hollow shell of gold.

Which one has the bigger moment of inertia about an axis through its center?

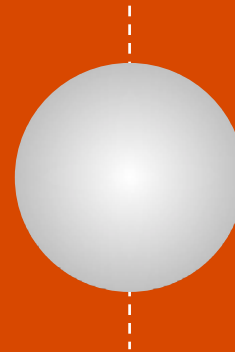
a) solid aluminum

b) hollow gold

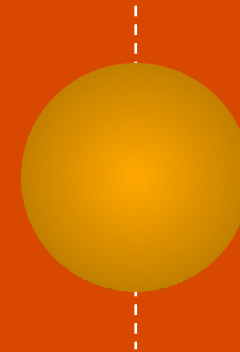
c) same

Moment of inertia depends on mass and distance from axis squared. It is bigger for the shell since its mass is located farther from the center.

solid



hollow



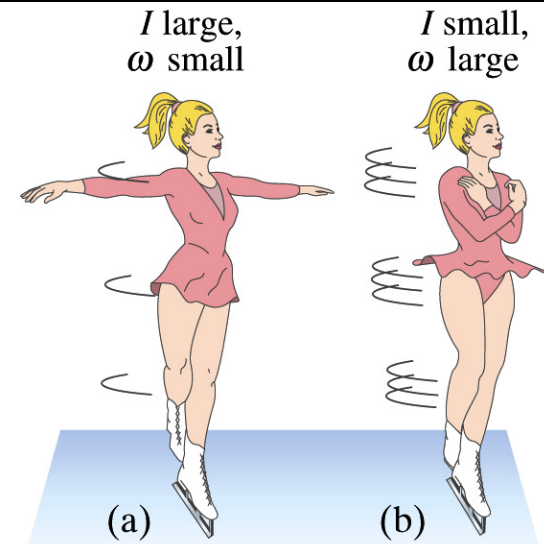
same mass & radius

## ConceptTest 8.10

## Figure Skater

A figure skater spins with her arms extended. When she pulls in her arms, she reduces her rotational inertia and spins faster so that her angular momentum is conserved. Compared to her initial rotational kinetic energy, her rotational kinetic energy after she pulls in her arms must be

- 1) the same
- 2) larger because she's rotating faster
- 3) smaller because her rotational inertia is smaller



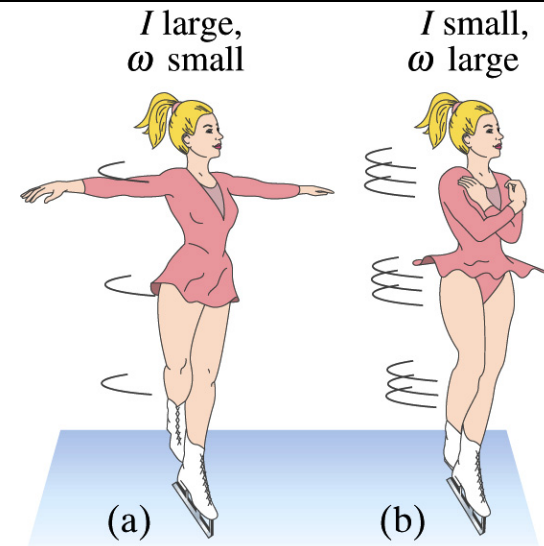
## ConceptTest 8.10

## Figure Skater

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- 1) the same
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- 3) smaller because her rotational inertia is smaller

$KE_{\text{rot}} = \frac{1}{2} I \omega^2 = \frac{1}{2} L \omega$  (used  $L = I\omega$  ).  
Since  $L$  is conserved, larger  $\omega$  means larger  $KE_{\text{rot}}$ . The "extra" energy comes from the work she does on her arms.



**Follow-up:** Where does the extra energy come from?



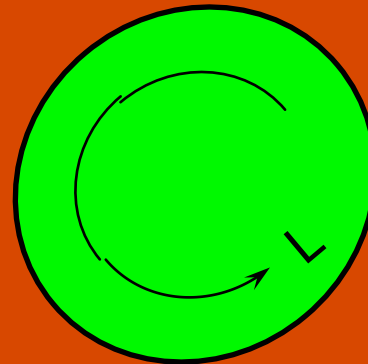
## ConcepTest 8.11

## Two Disks

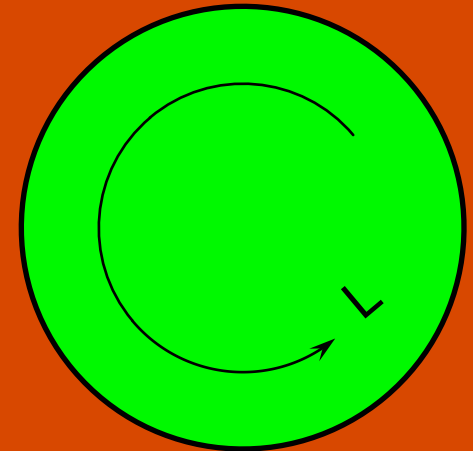
Two different spinning disks have the **same angular momentum**, but **disk 1** has more kinetic energy than **disk 2**.

- 1) disk 1
- 2) disk 2
- 3) not enough info

Which one has the bigger moment of inertia?



Disk 1



Disk 2

## ConceptTest 8.11

## Two Disks

Two different spinning disks have the **same angular momentum**, but **disk 1** has more kinetic energy than **disk 2**.

1) disk 1

2) disk 2

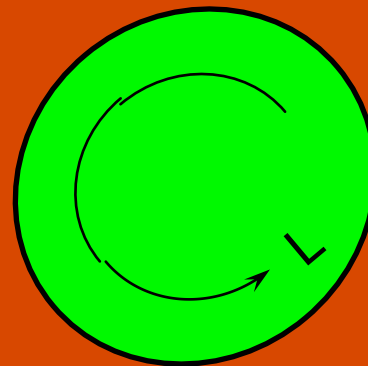
3) not enough info

Which one has the bigger moment of inertia?

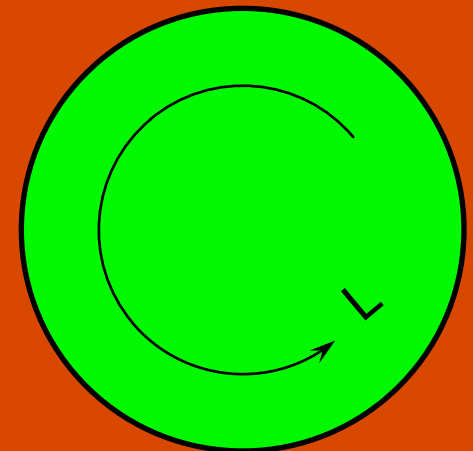
$$KE = \frac{1}{2} I \omega^2 = \frac{L^2}{2I}$$

(used  $L = I\omega$ ).

Since  $L$  is the same, bigger  $I$  means smaller KE.



Disk 1



Disk 2

## ConceptTest 8.12

## Spinning Bicycle Wheel

You are holding a spinning bicycle wheel while standing on a stationary turntable. If you suddenly flip the wheel over so that it is spinning in the opposite direction, the turntable will

- 1) remain stationary
- 2) start to spin in the same direction as before flipping
- 3) start to spin in the same direction as after flipping



## ConceptTest 8.12

## Spinning Bicycle Wheel

You are holding a spinning bicycle wheel while standing on a stationary turntable. If you suddenly flip the wheel over so that it is spinning in the opposite direction, the turntable will

- 1) remain stationary
- 2) start to spin in the same direction as before flipping
- 3) start to spin in the same direction as after flipping

The total angular momentum of the system is  $L$  upward, and it is conserved. So if the wheel has  $-L$  downward, you and the table must have  $+2L$  upward.

