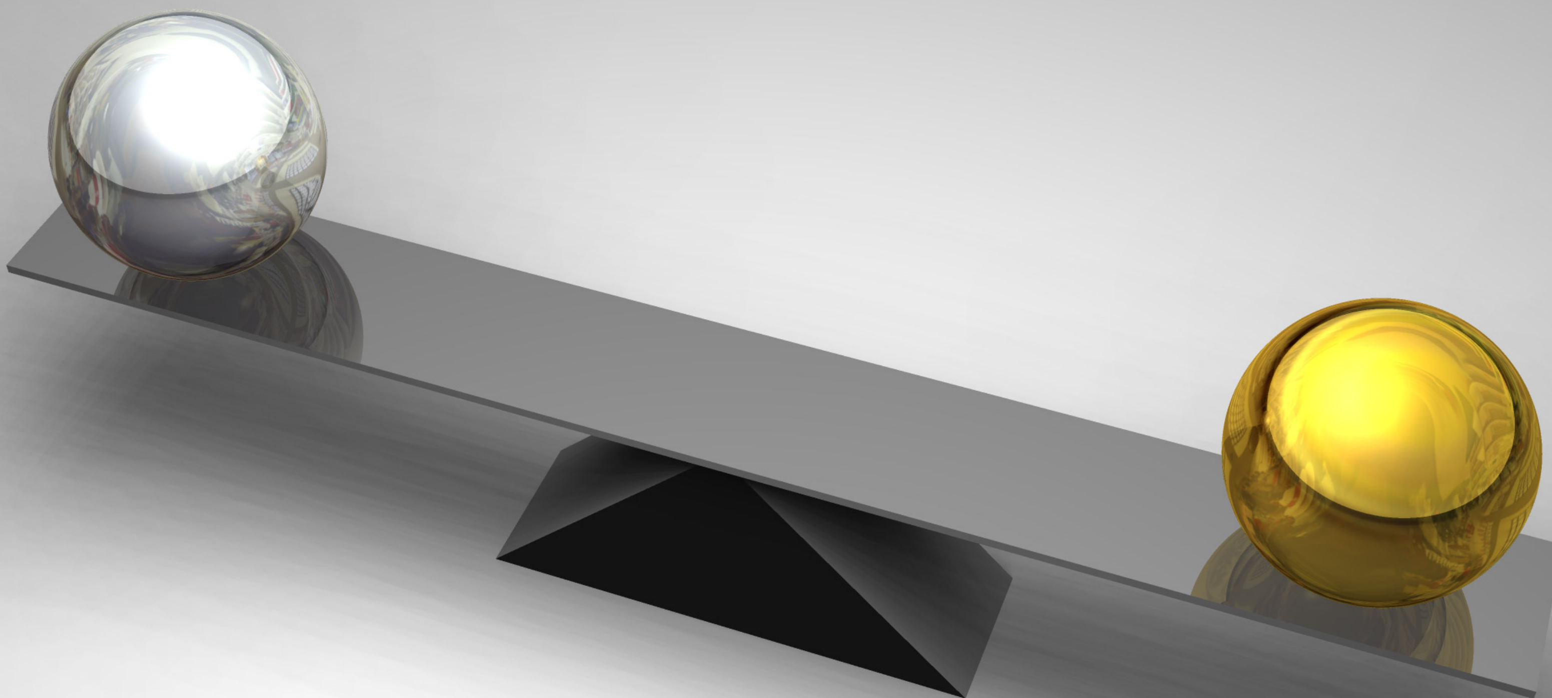
STATICS

Equilibrium and Strength of Materials

Equilibrium

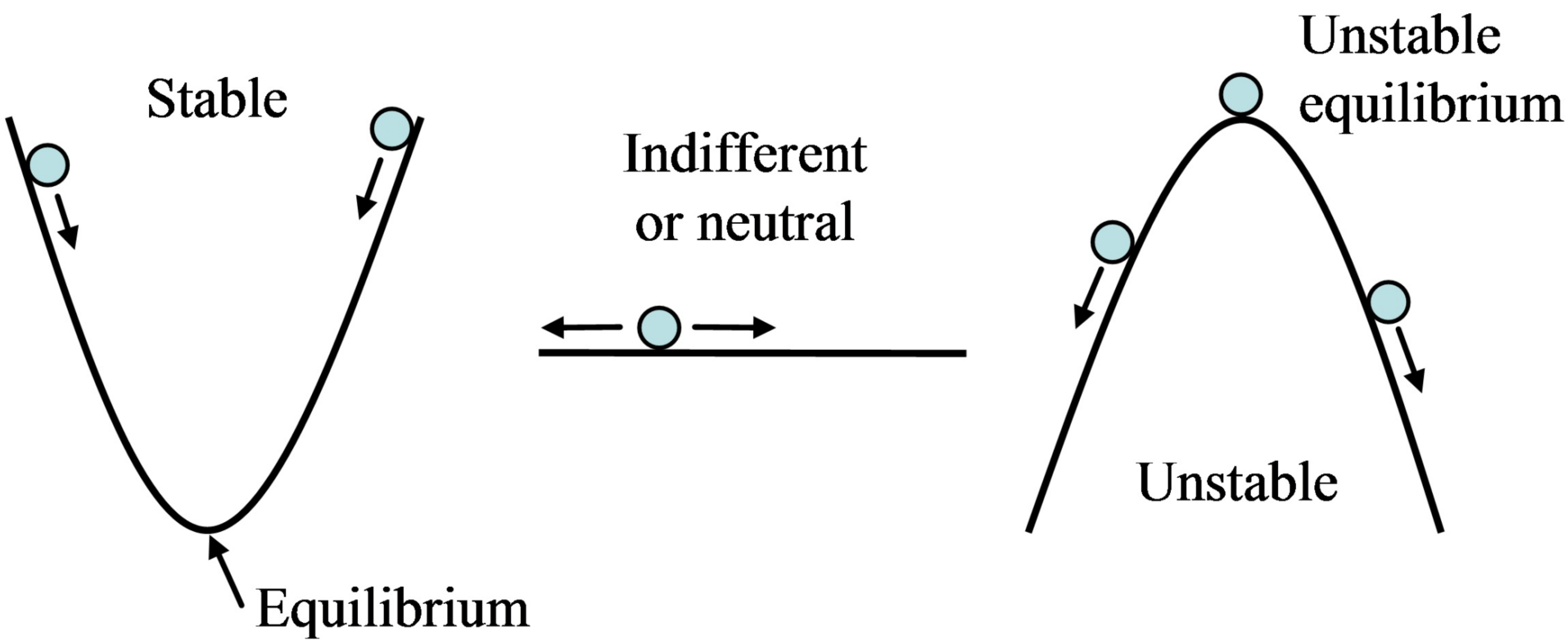


Latin for “*equal forces*”

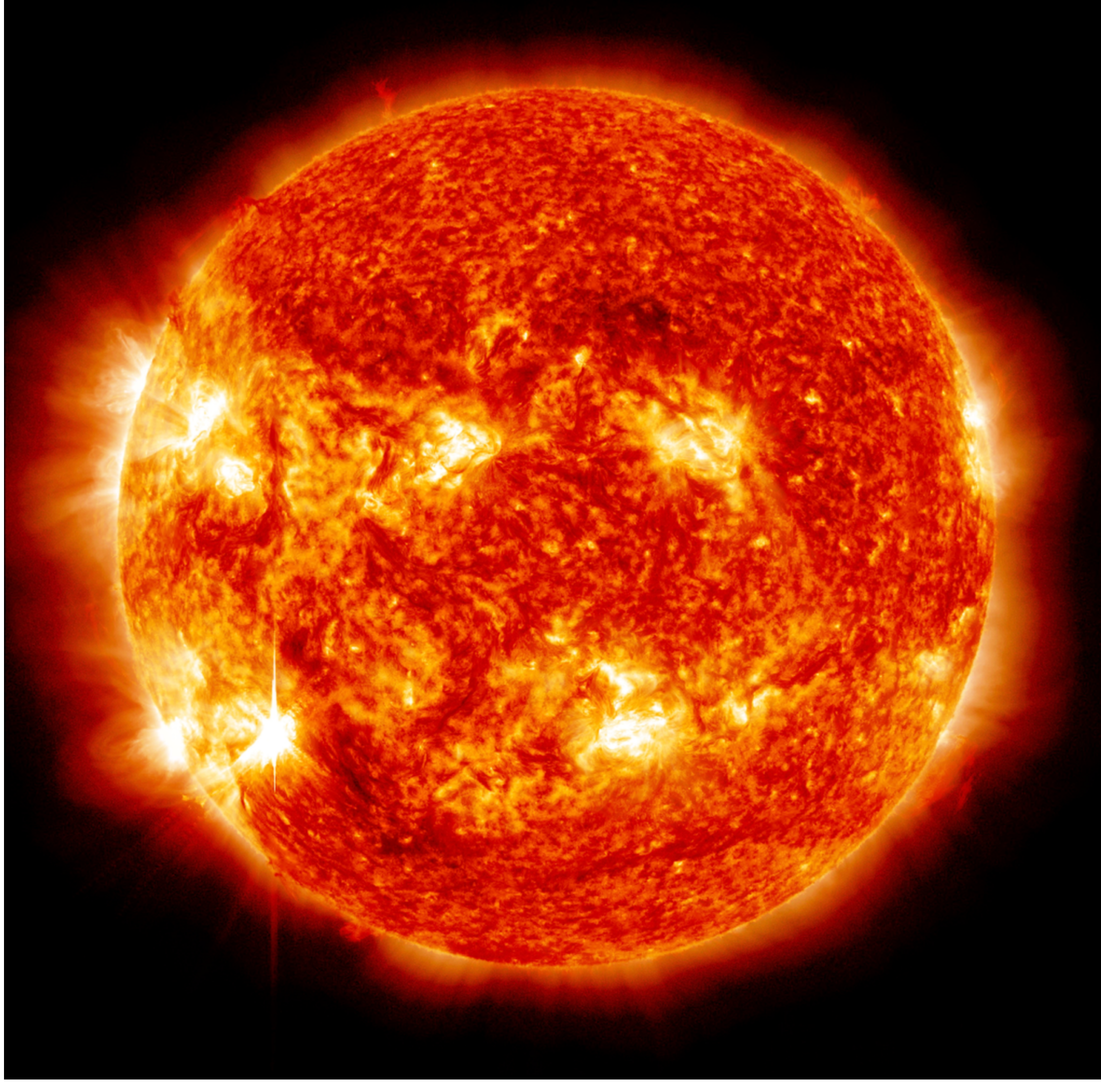
As we know, equal forces means no net force, and thus no acceleration.

Newton’s 2nd Law

Three types of Equilibrium



Sun



Our sun is in equilibrium (good thing for us).

The force of fusion pushing out from the center of the sun is equal to the force of gravity trying to collapse it.

Is this a stable, unstable, or neutral equilibrium?

Equilibrium

Two Conditions for Equilibrium:

1. Net Force acting on an object is 0.

$$\Sigma F = 0$$

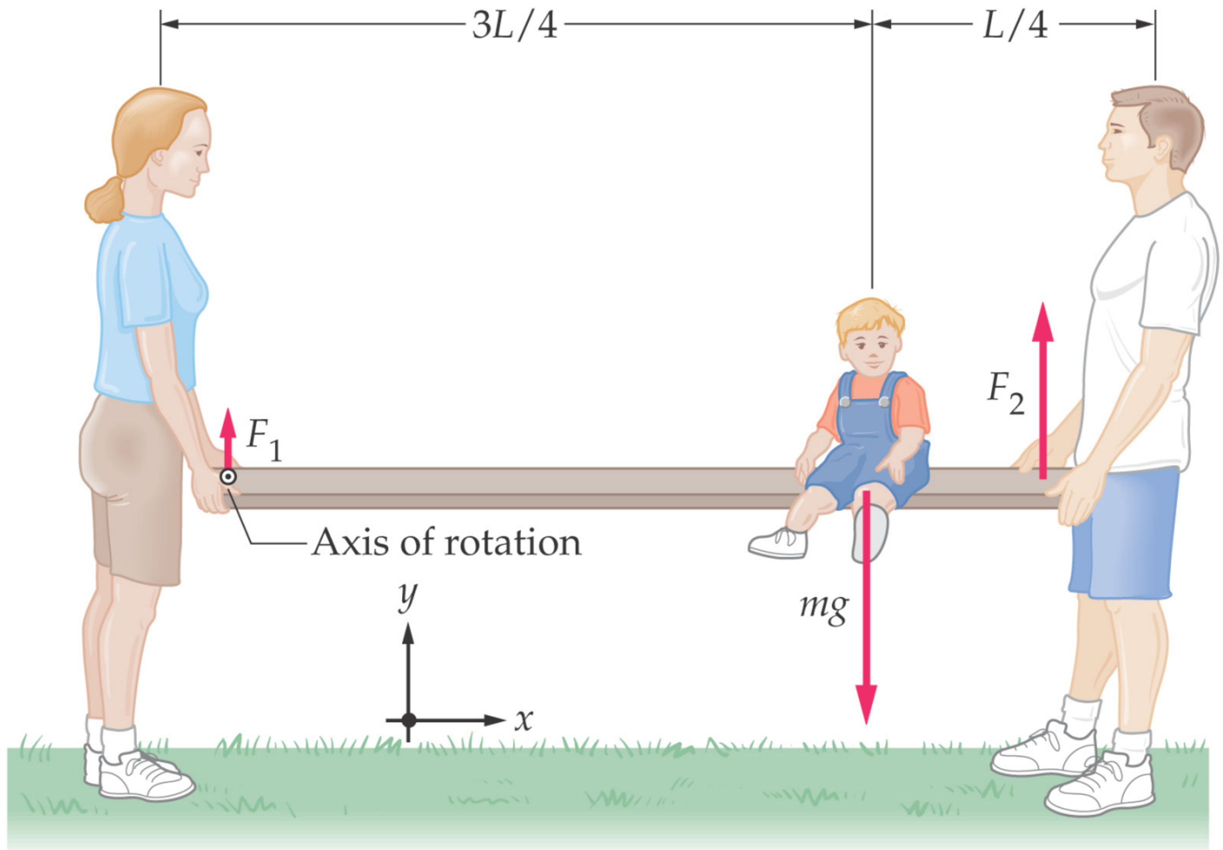
2. Net Torque acting on an object is 0

$$\Sigma \tau = 0$$

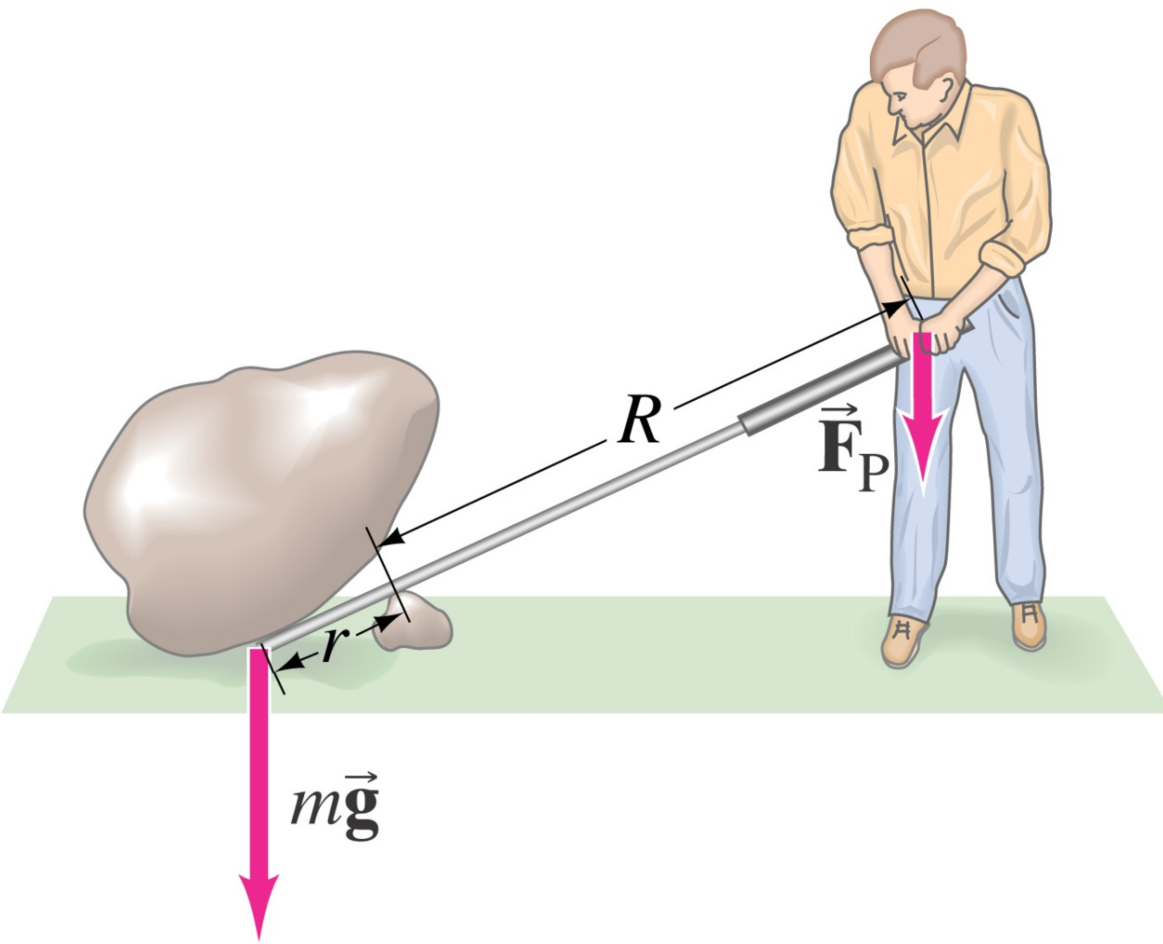
Net Torque is Zero about any Axis!

Otherwise the object would accelerate…

So we get to choose any axis we like. It is often simplest to choose an axis that is coincident with one of the forces, thus removing it from the equation.

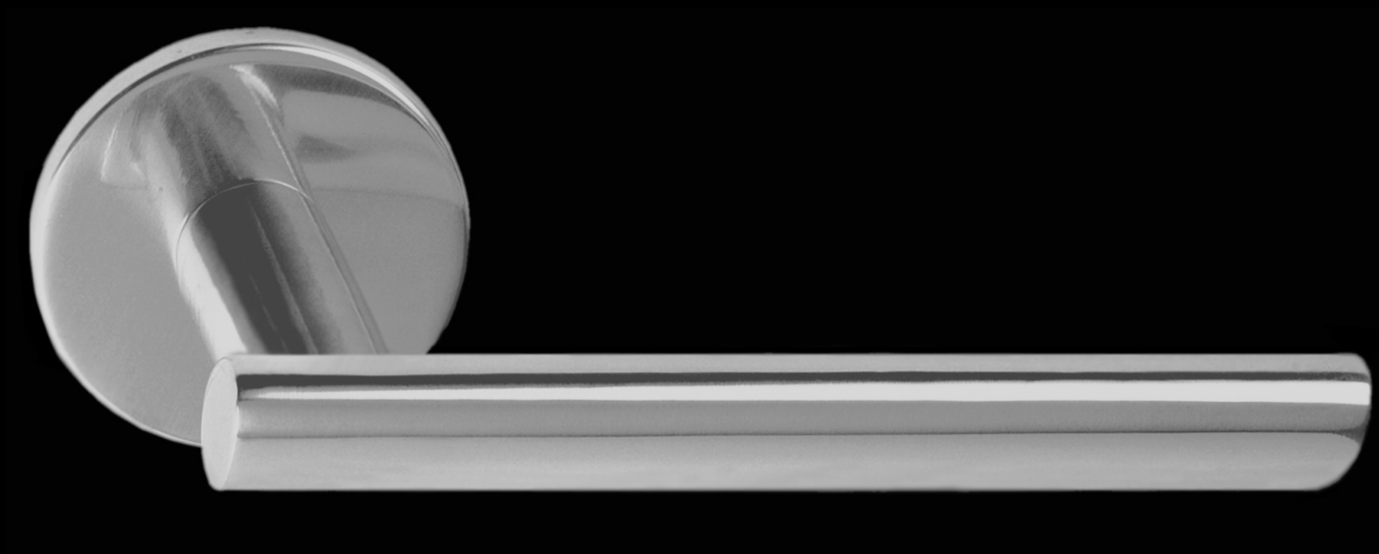


Example: A Simple Lever



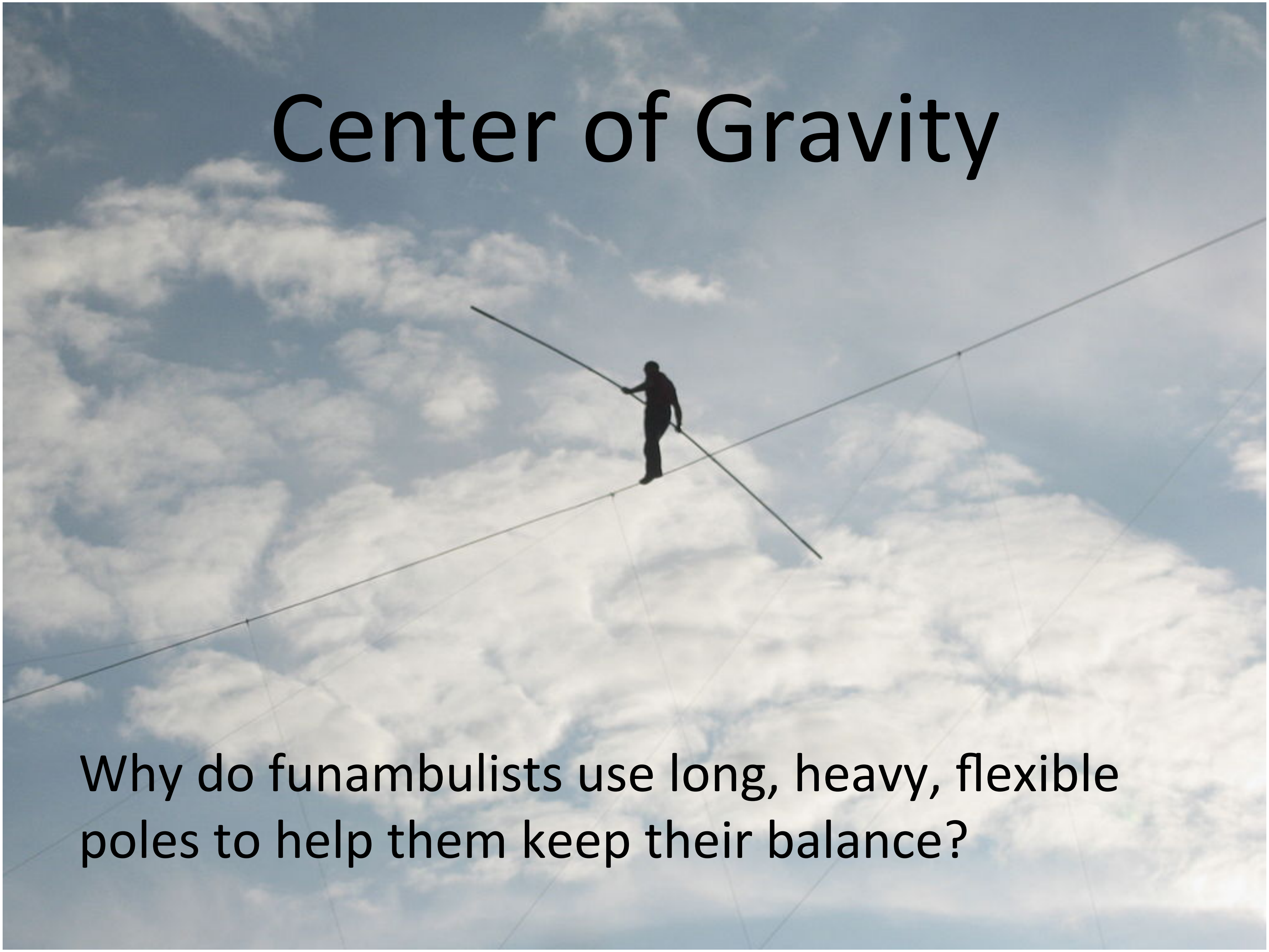
If this poor guy isn't able to lift the rock, what are some ways he could alter his setup to improve his chances?

Levers



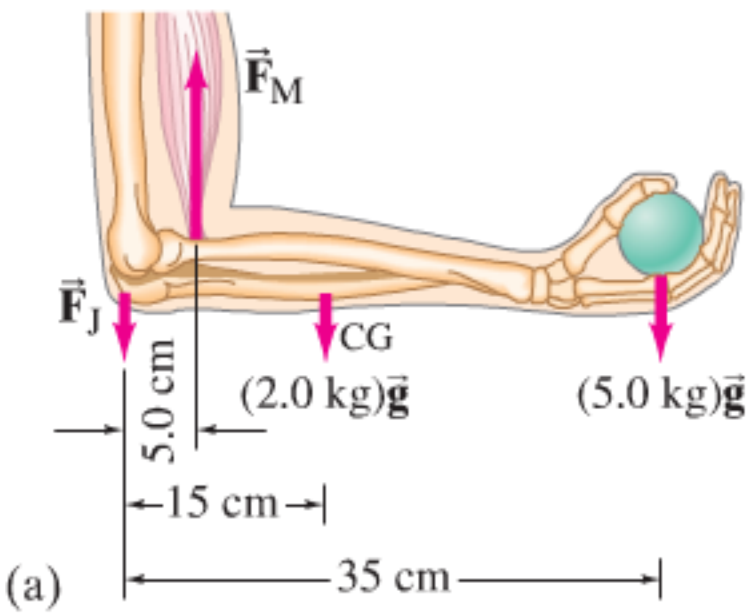
We use levers everyday, without even noticing. A simple door handle is an example of a lever that gives us a mechanical advantage. Throughout your day today, try to notice each time you are using a lever… you may be surprised by how abundant they are (all of you commuters used one with your foot just to get here this morning)!

Center of Gravity



Why do funambulists use long, heavy, flexible poles to help them keep their balance?

Lifting a Weight



Resolve forces vertically:

$F_M = F_J + W_{ball} + W_{arm}$

Resolve Torques 'about' the elbow:

$F_M \times 5cm = W_a \times 15cm \times W_{ball} \times 35cm$

$F_M = 400\ N$

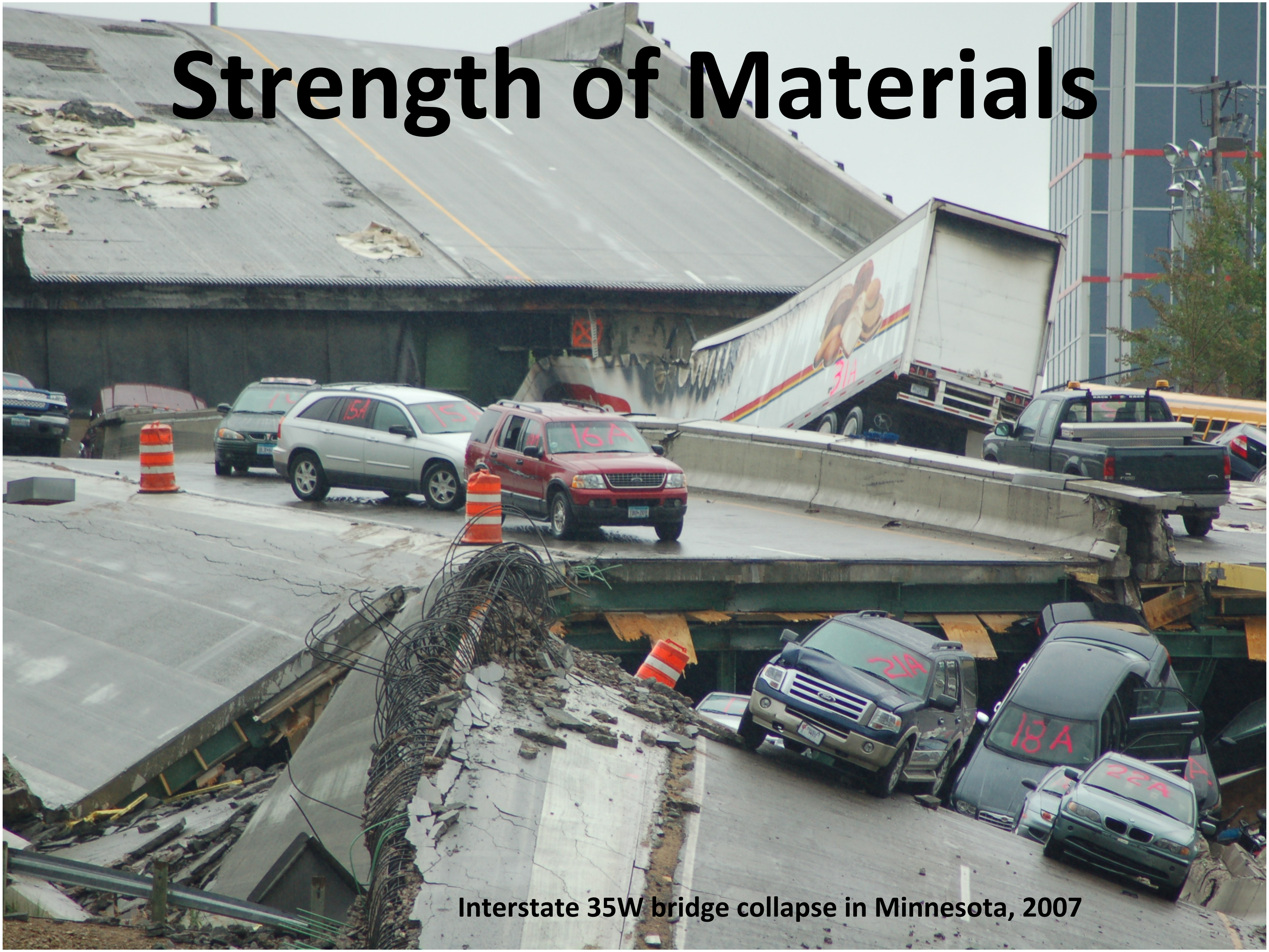
What is F_J ?

Example:
A ladder leaning against a wall

Is it safe for this person to be so close to the top of this ladder?

It depends on what is going on at the bottom…

Strength of Materials

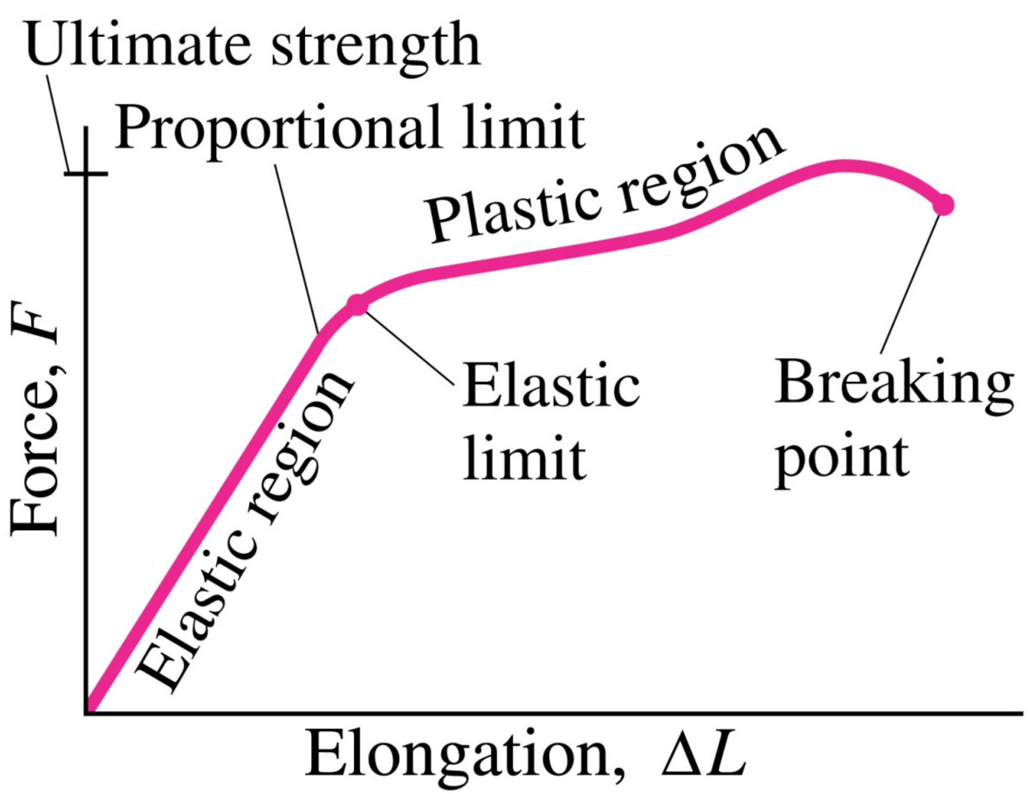


Interstate 35W bridge collapse in Minnesota, 2007

- Define strength!
- Squeeze, Stretch, or Twist?
- Being made of atoms, many materials are spring-like in both compression and tension
- Hooke's Law -> $F = k \Delta L$
  - But what do ΔL and k depend on?
  - Does the law break at some point?

Elasticity and Fracture

This proportionality holds until the force reaches the **proportional limit**. Beyond that, the object will still return to its original shape up to the **elastic limit**. Beyond the elastic limit, the material is permanently **deformed**, and it **breaks** at the breaking point.



Elastic (Young's) Modulus

- $\Delta L \propto F$
- $\Delta L \propto L$
- $\Delta L \propto$ 1/(cross sectional area)

Therefore: $\Delta L \propto FL/A$

Or: $\Delta L = \frac{FL}{EA}$ Where E is Elastic modulus (Young's modulus)

Stress and Strain

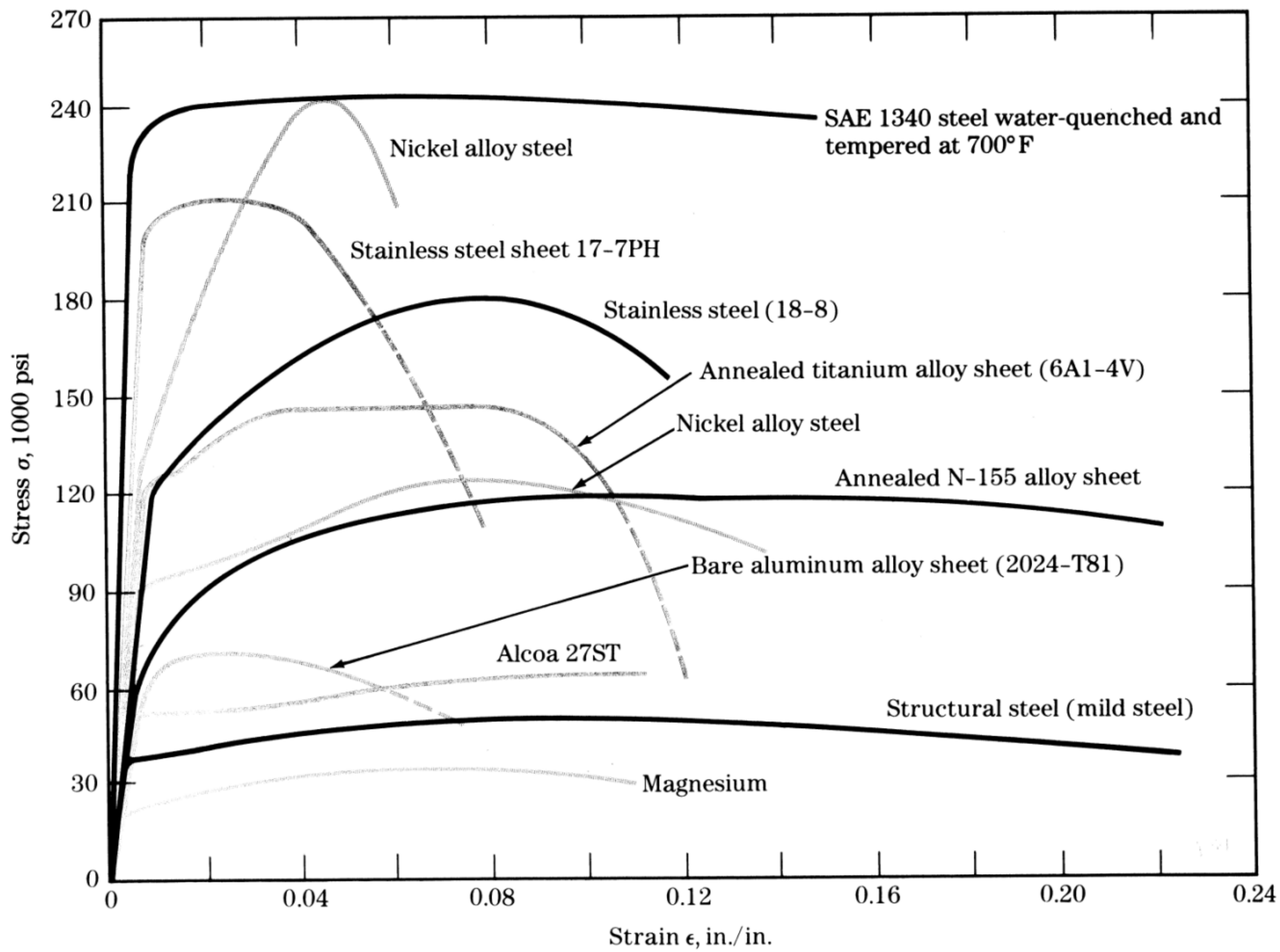
- Force divided by area is Stress
- Change in length divided by original length is Strain
- Strain is the object's response to Stress!

- Note from Definition of Elastic modulus,

$$E = \frac{\text{Stress}}{\text{Strain}}$$

Testing Tensile Strength

Actual Data

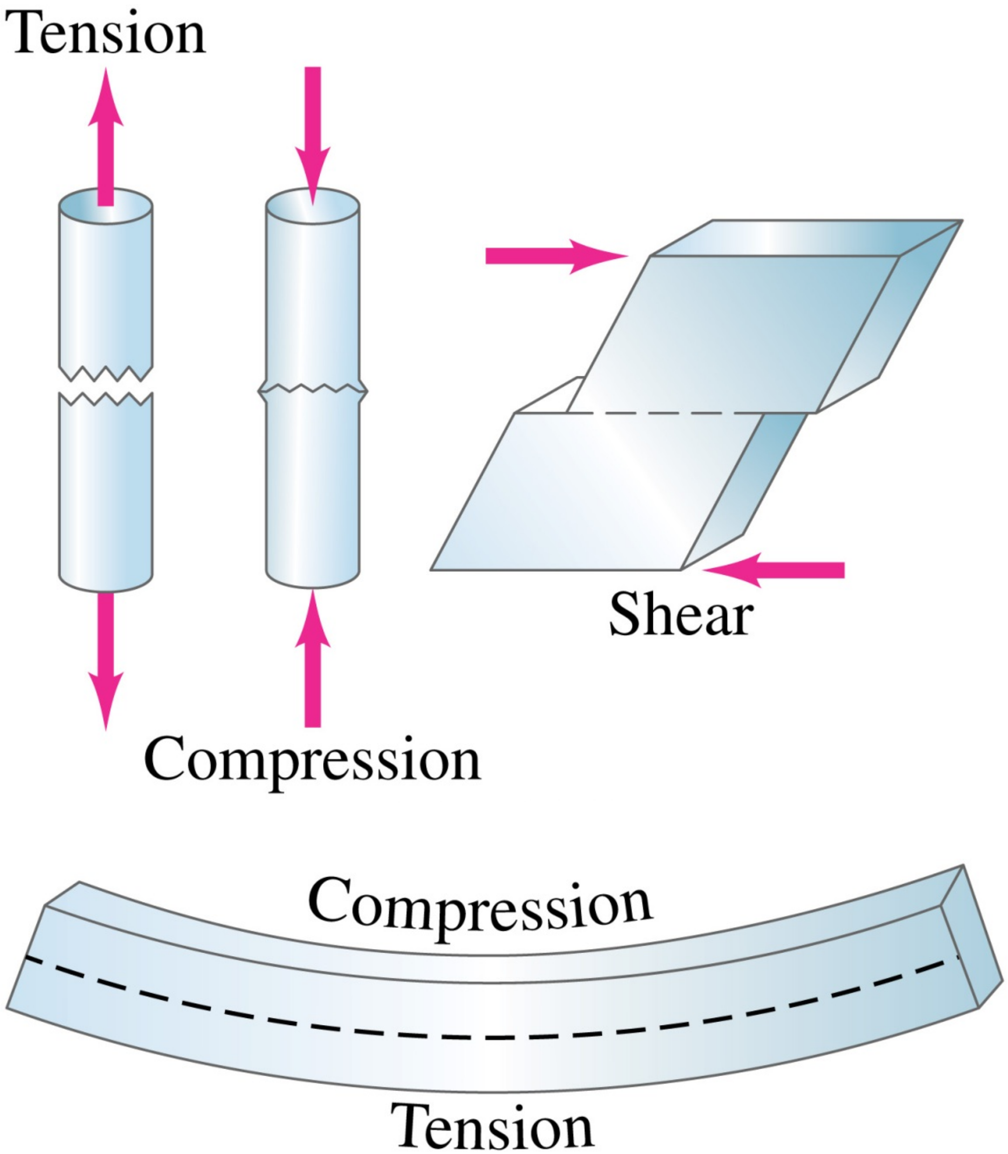


Stress σ, 1000 psi
270
240
210
180
150
120
90
60
30
0
Strain ϵ, in./in.
0
0.04
0.08
0.12
0.16
0.20
0.24
SAE 1340 steel water-quenched and tempered at 700°F
Nickel alloy steel
Stainless steel sheet 17-7PH
Stainless steel (18-8)
Annealed titanium alloy sheet (6A1-4V)
Nickel alloy steel
Annealed N-155 alloy sheet
Bare aluminum alloy sheet (2024-T81)
Alcoa 27ST
Structural steel (mild steel)
Magnesium

TABLE 9–2 Ultimate Strengths of Materials (force/area)

Material		Tensile Strength (N/m^2)	Compressive Strength (N/m^2)	Shear Strength (N/m^2)
Iron, cast		170×10^6	550×10^6	170×10^6
Steel		500×10^6	500×10^6	250×10^6
Brass		250×10^6	250×10^6	200×10^6
Aluminum		200×10^6	200×10^6	200×10^6
Concrete		2×10^6	20×10^6	2×10^6
Brick			35×10^6	
Marble			80×10^6	
Granite			170×10^6	
Wood (pine)	(parallel to grain)	40×10^6	35×10^6	5×10^6
	(perpendicular to grain)		10×10^6	
Nylon		500×10^6		
Bone (limb)		130×10^6	170×10^6	

Summary

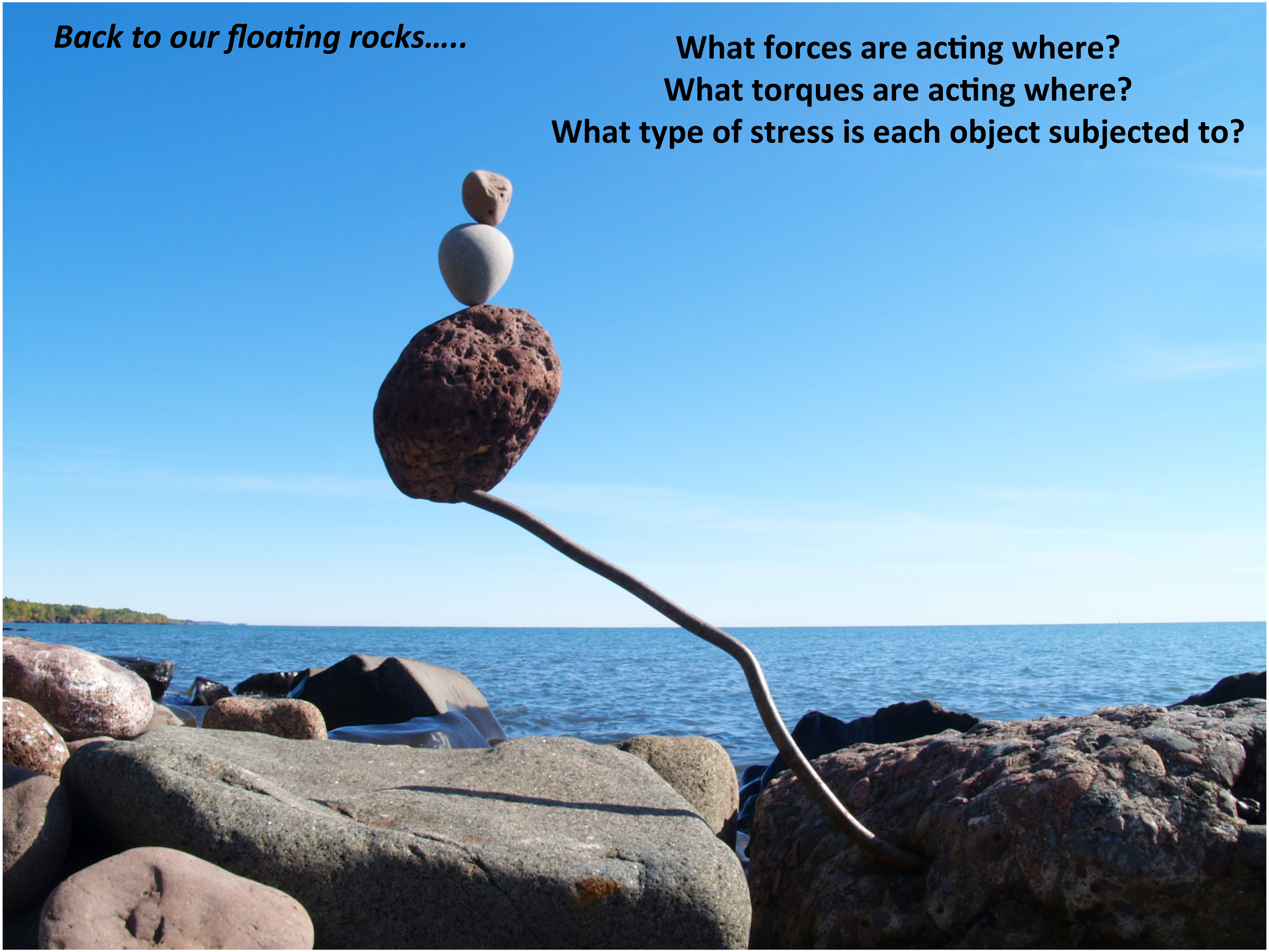


If the **stress** is too great, the object will **fracture**.

When designing a structure, it is a good idea to keep anticipated stresses less than 1/3 to 1/10 of the ultimate strength……

Otherwise…..


Structures will fail…


Back to our floating rocks…..
What forces are acting where?
What torques are acting where?
What type of stress is each object subjected to?