1. Prove that for all positive integers $n$, $1^2 + 2^2 + \ldots + n^2 = \frac{n(n + 1)(2n + 1)}{6}$.

2. Define a sequence $a_0, a_1, a_2, \ldots$ by the recursive formula $a_{n+1} = 2a_n - a_n^2$.
   Prove that $a_n = 1 - (1 - a_0)^{2^n}$ for all integers $n \geq 0$.

Reference:

http://zimmer.csufresno.edu/~larryc/proofs/proofs.mathinduction.html