Due Date: Monday, February 6.
Please show all work to receive full credit.

1. a. (p. 288 # 66) Use the MVT to show that \(|\sin(b) - \sin(a)| \leq |b - a|\) for all real numbers \(a\) and \(b\).
   b. Use the result in part a to show that \(|\sin(x)| \leq |x|\) for all real numbers \(x\).

2. (p. 313 # 66) Suppose that \(f\) is a continuous function. If the average value of \(f\) over the interval \([0, 1]\) is 2 and the average value of \(f\) over the interval \([1, 3]\) is 4, what is the average value of \(f\) over the interval \([0, 3]\)? Please explain your answer.

3. Let \(f(x) = \sqrt{x}\) and \(g(x) = x\).
   a. Sketch the region in the \(xy\) plane bounded by the curves \(y = f(x)\) and \(y = g(x)\).
   b. Express the area of the region in part a using definite integrals.

4. (p. 321 # 67a, c)
   a. Use the figure below to show that \(\int_0^x \sqrt{1 - t^2} \, dt = \frac{1}{2} x \sqrt{1 - x^2} + \frac{1}{2} \arcsin(x)\) if \(0 \leq x \leq 1\).
      Hint: The area of a circular sector of radius \(r\) and angle \(\theta\) is \(r^2 \theta/2\).
   b. Find the derivative of the function \(F\) given by the formula \(F(x) = \frac{1}{2} x \sqrt{1 - x^2} + \frac{1}{2} \arcsin(x)\).