Due Date: Monday, February 27.
Please show all work to receive full credit.

1. Formula 39 from the Table of Integrals (with \( a = 1 \)) gives 
\[
\int \sin^2(x) \, dx = \frac{x}{2} - \frac{1}{4} \sin(2x).
\]

However, 
\[
\frac{d}{dx} \left[ \frac{x}{2} - \frac{1}{4} \sin(2x) \right] = \frac{1}{2} - \frac{1}{2} \cos(2x).
\]

Is Formula 39 incorrect, or is \( \frac{1}{2} - \frac{1}{2} \cos(2x) = \sin^2(x) \)?

2. (p. 363 # 41) Evaluate 
\[
\lim_{n \to \infty} \frac{1}{n} \sum_{j=1}^{n} \left( \frac{j}{n} \right)^{3}
\]
by expressing it as a definite integral and then evaluating this integral using the Fundamental Theorem of Calculus.

3. (From the Kiwi files)
   a. Write an expression for the left Riemann sum \( L_n \) for the integral 
   \[
   \int_{0}^{1} e^x \, dx
   \]
   using \( n \) intervals of equal length.

   b. Find a closed-form expression for \( L_n \). (Hint: Use the formula for geometric sums discussed in class. 
   \( 1 + r + r^2 + r^3 + \ldots + r^{n-1} = \frac{1 - r^n}{1 - r} \).)

   c. Use your answer to part b to find 
   \( \lim_{n \to \infty} L_n \).

   d. Evaluate 
   \[
   \int_{0}^{1} e^x \, dx
   \]
   using the FTC. Does your answer agree with your answer to part c?

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4. In class, Chris pointed out that you need to use more terms in a Riemann sum to approximate the integral of a function with a spike or with rapid oscillations than to approximate the integral of a slowly varying function. The purpose of this problem is to explore that idea.

a. Consider the definite integral $I = \int_{0}^{\pi/2} \cos(x) \, dx$. Use Theorem 3 on p. 387 of the textbook to find a value of $n$ for which $|I - L_n| \leq 10^{-3}$.

b. Consider the definite integral $I = \int_{0}^{\pi/2} 5e^{-x} \cos(20x) \, dx$. Use Theorem 3 on p. 387 of the textbook to find a value of $n$ for which $|I - L_n| \leq 10^{-3}$.