Due Date: Monday, March 6.
Please show all work to receive full credit.
Problems 3 – 5 are from the Kiwi files.

1. Consider the definite integral \( I = \int_0^1 \sin(x^2) \, dx \).
   a. Find a value of \( n \) for which \(|I - L_n| < 10^{-4}\).
   b. Calculate the value of \( L_n \) using the value of \( n \) from part a.
   c. Find a value of \( n \) for which \(|I - M_n| < 10^{-4}\).
   d. Calculate the value of \( M_n \) using the value of \( n \) from part c.

2. The purpose of this problem is to derive the formulas for the nodes and weights in the Gaussian quadrature formula \( GQ_2 = w_1 f(x_1) + w_2 f(x_2) \approx \int_{-1}^{1} f(x) \, dx \).
   a. Evaluate the definite integrals \( I_1 = \int_{-1}^{1} 1 \, dx \), \( I_2 = \int_{-1}^{1} x \, dx \), \( I_3 = \int_{-1}^{1} x^2 \, dx \), and \( I_4 = \int_{-1}^{1} x^3 \, dx \).
   b. Use the fact that the formula \( GQ_2 \) gives exact answers for \( I_1, I_2, I_3 \) and \( I_4 \) to derive the equations \( w_1 + w_2 = 2 \), \( w_1 x_1 + w_2 x_2 = 0 \), \( w_1 x_1^2 + w_2 x_2^2 = 2/3 \), \( w_1 x_1^3 + w_2 x_2^3 = 0 \).
   c. Solve the equations from part b. (You can either do this by hand or by using Mathematica.)

3. a. Sketch the graphs of the equations \( y^2 = x + 1 \) and \( y = x - 1 \).
   b. Find the area bounded by these curves by integrating with respect to \( x \).
   c. Find the area bounded by these curves by integrating with respect to \( y \).

4. Find the length of the curve given by \( y = \frac{x^2}{2}, 0 \leq x \leq 1 \). (You might want to use the integral tables for this one.)

5. Use integration to show that the volume of a circular cone of radius \( r \) and height \( h \) is \( \frac{1}{3} \pi r^2 h \). Hint: Draw a picture. Can you represent the cone as a solid of revolution?