Due Date: Monday, April 10.
Please show all work to receive full credit.

1. Let \( f(x) = \int_0^x e^{-t^2} \, dt \)
   a. Find the cubic Taylor polynomial \( P_3(x) \) for the function \( f \) about \( x = 0 \).
   b. Calculate \( P_3(0.1) \).
   c. Use Theorem 2 on page 504 of Ostebee and Zorn to find an upper bound on \( |f(0.1) - P_3(0.1)| \).

2. Let \( f(x) = \sin(x) \).
   a. Find a formula for \( P_n(x) \), the Taylor polynomial for \( f \) about \( x = 0 \).
   b. Use Theorem 2 on page 504 of Ostebee and Zorn to find a value of \( n \) such that \( |f(0.1) - P_n(0.1)| < 10^{-4} \).

3. (Part of problem 16, page 516, in Ostebee and Zorn). Consider the function \( f \) defined by
   \[
   f(x) = \begin{cases} 
   \pi + x & \text{if } -\pi \leq x < 0 \\
   \pi - x & \text{if } 0 \leq x \leq \pi 
   \end{cases}
   \]
   a. Find formulas for the Fourier coefficients \( a_k \) and \( b_k \) of \( f \). You can either do this by hand or with Mathematica.
   b. Extra Credit. (10 pts.) Use Mathematica to plot \( f \), \( q_1 \), and \( q_3 \) on the same set of axes for \(-\pi \leq x \leq \pi \). Here \( q_n \) denotes the Fourier polynomial of degree \( n \):
   \[
   q_n(x) = a_0 + \sum_{k=1}^{n} [a_k \cos(kx) + b_k \sin(kx)].
   \]

4. Extra Credit. (20 pts.) Use Mathematica for this problem.
   a. Find \( S(1.5) \), where \( S \) denotes the natural cubic spline through the points
      \((0,0), (1,2), (2,3), (3,-1), (4,1)\).
   b. Plot \( S \) for \( 0 \leq x \leq 4 \).