This is an optional assignment for extra homework credit.
Due Date: Tuesday, May 16.
Please show all work to receive full credit.

1. Determine whether each of the following series converges absolutely, converges conditionally, or diverges. Explain your answers.
   a. \[ \sum_{k=2}^{\infty} \frac{1}{k \ln(k)} \]
   b. \[ \sum_{k=0}^{\infty} \frac{(-1)^k}{(k + 1)2^k} \]
   c. \[ \sum_{k=1}^{\infty} \left(1 - e^{-1/k}\right) \]

2. Find the interval of convergence of the power series \[ \sum_{n=1}^{\infty} \frac{(x - 1)^n}{\sqrt{n}} \]

3. Bessel functions are useful in describing physical systems with radial symmetry. These functions are not expressible in terms of elementary functions (algebraic, trigonometric, exponential, and logarithmic functions), but they can be calculated from their series expansions. The series for \( J_0 \), the Bessel function of the first kind of order 0, is
   \[ J_0(x) = 1 - \frac{x^2}{2^2} + \frac{x^4}{2^2 \cdot 4^2} - \frac{x^6}{2^2 \cdot 4^2 \cdot 6^2} + \cdots \]
   a. Find the radius of convergence of the given series.
   b. Use the given series to calculate the value of \( J_0(1) \) with an error less than \( 10^{-5} \). Be sure to explain how you know that your answer has the required accuracy.